Examining the Role of Manipulatives and Metacognition on Engagement, Learning, and Transfer

Daniel M. Belenky[1] and Timothy J. Nokes[1]

Abstract

How does the type of learning material impact what is learned? The current research investigates the nature of students' learning of math concepts when using manipulatives (Uttal, Scudder, & DeLoache, 1997). We examined how the type of manipulative (concrete, abstract, none) and problem-solving prompt (metacognitive or problem-focused) affect student learning, engagement, and knowledge transfer. Students who were given concrete manipulatives with metacognitive prompts showed better transfer of a procedural skill than students given abstract manipulatives or those given concrete manipulatives with problem-focused prompts. Overall, students who reported low levels of engagement showed better learning and transfer when getting metacognitive prompts, whereas students who reported high levels of engagement showed better learning and transfer when getting the problem-focused prompts. The results are discussed in regards to their implications for education and instruction.

Keywords

learning, engagement, transfer, problem solving, manipulatives, metacognition

[1]University of Pittsburgh

Learning math can be hard. One way educators try to aid in math learning is by teaching new concepts using concrete examples. This has been hypothesized to be an effective instructional tactic because it reduces memory load (Sweller, 2006; Sweller, Merrienboer, & Paas, 1998), facilitates understanding by grounding new information in meaningful prior knowledge (Brown, Collins, & Duguid, 1989), and may increase students' motivation to learn and understand the instruction, task, or problem (Cordova & Lepper, 1996; Schraw, Flowerday, & Lehman, 2001). However, there are also potential downsides to this pedagogical strategy. Using highly realistic situations and materials may cause the knowledge to be tied to the particulars of that scenario, making transfer to other scenarios or into abstract terms more difficult (Goldstone & Sakamoto, 2003; Son & Goldstone, 2009a). The relevant features that are key to deep understanding may be less salient. Moreover, the concrete details may distract students from these features (Harp & Mayer, 1998; Son & Goldstone, 2009b). It appears that there are open questions as to the best way to use concrete materials in learning and whether learning with them is different than learning with more abstract materials.

A second way in which teachers try to aid in math learning is by having students engage in valuable and productive activities with the learning materials. This sometimes manifests itself as students being "active" with the materials (Brown, Collins, & Duguid, 1989), such as giving students manipulatives so that they can get "hands-on" experience (Fuson & Briars, 1990). Manipulatives are physical objects that are supposed to help the student concretize his or her knowledge by expressing concepts and performing problem-solving steps with them. It has been hypothesized that being "active" facilitates learning by doing (Anzai & Simon, 1979) and increases attention and engagement (Chi, 2009). However, being active does not necessarily mean that students are engaging in the kinds of cognitive processes that are associated with deep learning (Chi, 2009). Another way teachers may try to engage students is by asking them questions that focus on important aspects of the learning materials (Graesser & Black, 1985). Metacognitive prompts are questions that ask students to reflect on various aspects of the learning materials and problem-solving process and have been hypothesized to facilitate abstraction and learning (Schoenfeld, 1987). We hypothesize that it is not only the content of the learning materials (concrete versus abstract) but also how those materials are used that is critical to learning complex cognitive skills such as those taught in mathematics and science.

The development of a complex cognitive skill is more than simply learning a list of declarative concepts or a set of rote procedures. Ideally, such learning would result in the development of *adaptive expertise* (Hatano & Inagaki, 1986; Schwartz, Bransford, & Sears, 2005), which allows for the flexible application of the knowledge to novel problems. Adaptive expertise is hypothesized to be comprised of two sets of skills: one that deals with procedural knowledge, and the other with conceptual knowledge. Critically, these skills must be integrated and coordinated to facilitate transfer to novel situations. If we

take the goal of instruction as the development of adaptive expertise, it makes sense to consider how different types of learning environments can influence the acquisition and coordination of these skills.

In the current work we examine how different pairings of learning materials (concrete versus abstract) and prompt-based activities (metacognitive versus problem-focused) impact the learning and coordinating of conceptual and procedural skills. We hypothesize that concrete materials are most effective when paired with metacognitive prompts because they ground new information in prior knowledge but also enable students to abstract the critical features through reflection. We are also interested in how student engagement with the learning materials interacts with the kinds of prompts they are given. If students are highly engaged, are some types of prompts more effective than others? If students are already using deep processing strategies, what types of prompts are most effective?

In the next section we briefly review the relevant literature on the effect of different learning materials on the development of procedural and conceptual skill. We focus this review with the hypothesis that it is not only the type of materials but also how those materials are used that determines learning, engagement, and transfer. We then present our study, which manipulated whether students learned probability concepts with either abstract or concrete materials and with either metacognitive or problem-focused prompts.

Prior Work on Abstract and Concrete Learning Materials

Materials can be thought to be abstract[1] if they strip away extraneous details and present information in a decontextualized way. Frequently, this is accomplished by reducing the complexity of the visual stimuli. For example, Goldstone & Sakamoto (2003) used a dynamic computer simulation in the context of ants foraging for food to teach the complex systems principle of "competitive specialization." This is the idea that the parts or agents of a system can start out undifferentiated and then become specialized through simple interactions between those parts. Students could manipulate certain parameters of the simulation using sliders (e.g., the number of ants, the walking rate of the closest ant, the walking rate of the ants which are not closest) and observe the results. All of the participants received instructions that the simulation dealt with ants looking for food and their goal was to maximize the coverage of resources (food). Half of the participants were given depictions of ants and apples, while the other half were given symbols (i.e., dots for the ants, blobs for the food). This learning phase was followed by a transfer task, which was based on the same principle but in a very different surface domain (e.g., machine learning of letter identification). Those participants who had used idealized, symbolic stand-ins for the ants and food did better in this new domain than those who had used small images of ants and apples (see Figure 1a). The concreteness of seeing ants and apples seemed

to have tied any knowledge discovered about the system more tightly to that context, while seeing simple blobs and calling them ants made the knowledge more symbolic and flexible.

Figure 1. Two examples of concrete and idealized (abstract) learning materials. Part A reprinted from "The Transfer of Abstract Principles Governing Complex Adaptive Systems," by R.L. Goldstone and Y. Sakamoto, 2003, *Cognitive Psychology, 46*, p.449, Copyright 2003, with permission from Elsevier. Part B reprinted from "The Advantage of Simple Symbols for Learning and Transfer" by V.M. Sloutsky, J.A. Kaminski, and A.F. Heckler, 2005, *Psychonomic Bulletin & Review, 12*, p. 510, Copyright 2005, with permission from Psychonomic Society Publications.
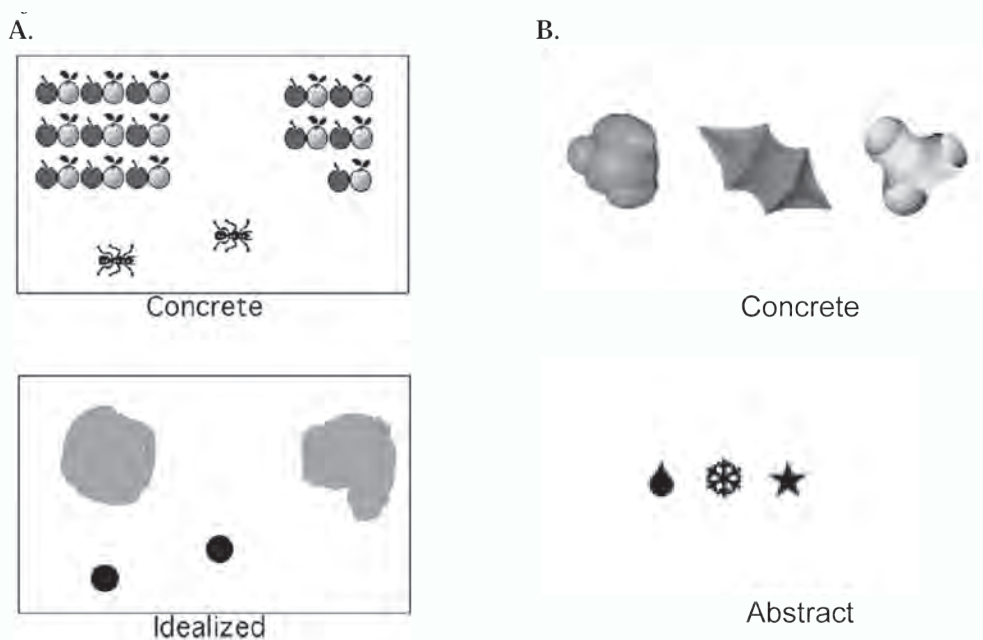
Similarly, a set of basic, mathematical relations (based on a "modular arithmetic" system) was taught to two groups of students (Sloutsky, Kaminski, & Heckler, 2005). One group learned these relations for visually salient stimuli, such as polygons with 3-dimensional depth, which moved and interacted in movie clips, while the other group used visually sparse, static symbols (see Figure 1b). Those who had learned using the more abstract materials transferred those rules to a new system of concrete relations and were better able to use those rules to solve new problems. In sum, these studies show a benefit for learning and transfer when using more abstract materials.

Abstract objects may help guide attention to focus on important relations between objects, rather than the objects themselves (Sloutsky et al., 2005). The result of using more abstracted materials seems to be a more flexible understanding of the underlying conceptual relations. However, this flexibility may have the downside of requiring a longer period of initial learning and poorer ability to apply those concepts in a given scenario. That is,

abstract materials may be helpful in acquiring the conceptual side of adaptive expertise, but adaptive expertise also requires some measure of procedural skill as well. Prior work has shown that the application of abstract declarative knowledge requires more time and cognitive processing than applying procedures acquired by practice on concrete problems (Nokes & Ohlsson, 2005). Abstract materials may require so much processing of conceptual information that procedural skills are left underdeveloped in the target domain. This sort of procedural skill may benefit more from practice with concrete materials.

Materials can be considered concrete when they include details or use vivid stimuli. An example of concrete materials in educational settings is the use of story problems in algebra. Algebra is fundamentally a symbolic, abstract system. However, it is frequently taught and assessed using concrete word problems. For those just learning algebra, these problems are usually easier to solve than symbolic representations of the same problems (Koedinger, Alibali, & Nathan, 2008). This improvement stems from students using more everyday knowledge and more informal problem-solving strategies when given word problems. However, when the problems become more complicated, a more abstract notation is easier, as it may make problem-steps clearer, or may induce a smaller cognitive load.

Concrete materials could also be useful if they highlight salient, important features and relations. Children learning fraction concepts show quicker learning when using pie pieces rather than tiles, because the pie pieces embody the fraction concept and better highlight the nature of the part-whole relationship (Martin & Schwartz, 2005; but see Ma, 1999, for a discussion of the limitations of this technique). Finally, concrete materials can help develop procedural fluency by increasing the interest in and engagement with learning materials (Durik & Haraciewicz, 2007), or by minimizing the amount of cognitive load in the problem solving procedure (e.g., Sweller, 2006).

One movement in schools that is focused on using concrete materials is the push to include manipulatives in pedagogy. Manipulatives are considered helpful in math (Ball, 1992), and reading (e.g., Glenberg, Gutierrez, Levin, Japuntich, & Kaschak, 2004). However, there are some important caveats. Developmental research has shown that young children have trouble when an object is supposed to both represent itself and stand as a symbol for something else (Uttal, Scudder, & DeLoache, 1997). For example, young children have trouble locating an object in a full-sized room when given the corresponding scale model (DeLoache, 1989). This seems to be due to trouble dissociating properties of the scale model, which is a physical entity in its own right, from abstracted information about the other room. Manipulations which make the children believe that, instead of being a scale model, the small room has now been blown up to be the full scale room remove the difficulty (DeLoache, 1995). The lesson for educators from this is that understanding an object to be both itself and a symbol for a more abstract piece of information is not simple for children. This difficulty could have implications for adults as well, who clearly

have no trouble understanding scale models, but who may primarily focus on concrete aspects of an object, or automatically be drawn to perceptual features, drawing processing resources away from the more conceptual level (i.e., Sloutsky et al., 2005).

One potential reason for these difficulties may be because different materials afford the development of different representations. The internal representation of the problem is critical, as it determines which strategies a person will use to solve it (Kotovsky & Fallside, 1989; Nokes, 2009; Nokes & Ohlsson, 2005). Kotovsky and Fallside (1989) showed how the exact same stimuli can produce different mental representations, which affect subsequent problem solving. They asked participants to perform certain problem-solving tasks (isomorphs of the classic "Tower of Hanoi" problem) while systematically varying either the perceptual stimuli they received or the instructions. For example, identical stimuli could either be interpreted as representing an object changing two-dimensional shape, or moving through space and changing 3-dimensional depth. They found that using similar mental representations at learning and transfer was the best predictor of performance, and that the critical variable was not really which stimuli were used, but what mental representation was formed.

Concrete materials, then, may be most beneficial when they facilitate intuitive strategies or make certain aspects of the concepts obvious, such as the case of using pie pieces when teaching fractions. Practice with these sorts of materials should lead to increased procedural fluency, as some of the harder abstract reasoning is unnecessary, and more informal strategies can usually be recruited. However, this focus on concrete knowing and procedural fluency may lead to poor development of abstract, conceptual knowledge without further cognitive processing such as engaging in reflection.

If we know that different materials may be creating different representations, an open question remains about how to make instruction more likely to create appropriate representations that highlight the critical features of the problem. We know that it is not just what kinds of learning materials are used (i.e., concrete versus abstract) but how those materials are used that is critical. The types of activities students engage in when problem solving critically influences what is learned and where that knowledge can be transferred. In the current work we focus on how the type of directive question, whether it is metacognitive or problem-focused, impacts learning and problem solving. In the next section we briefly review the prior work on metacognitive prompting and the predictions of the current study.

Prior Work on Metacognition in Problem Solving

Metacognition is the active process of reflecting, explicitly, on one's own cognitive activity (Brown, 1978; Dunlosky & Metcalfe, 2009; Kluwe, 1982; Schoenfeld, 1987). While problem-solving, this is manifested in monitoring one's progress, evaluating that progress, and

regulating future activity (Zimmerman & Campillo, 2003; Winne & Hadwin, 1998). It is also a process which allows for the creation of new knowledge, as monitoring and evaluating can lead one to notice important deficits in knowledge, leading one to address that deficit by taking steps like consulting examples or self-explaining (Chi, 2000).

Research has shown benefits for receiving metacognitive prompts while learning by problem solving (Berardi- Coletta, Buyer, Dominowski, & Rellinger, 1995; Chi, de Leeuw, Chiu, & LaVancher, 1994; Schoenfeld, 1987). Specifically, the act of reflecting on one's problem-solving *process* within the domain (metacognitive prompts) is more important to developing conceptual understanding than focusing on the particular pieces of domain knowledge (problem-focused prompts) (Berardi-Coletta et al., 1995). This research suggests that pairing concrete materials with metacognitive prompts should facilitate procedural fluency as well as conceptual understanding and transfer. The concrete materials should support intuitive strategies, reduce cognitive load, and highlight important features of the problem for learning procedural skills. Prompting for metacognitive reflection of those skills should facilitate sense-making processes for determining how the highlighted features relate to one another, improving conceptual understanding, abstraction, and the development of flexible representations.

Predictions

Given this existing literature on the effects of different learning materials (abstract versus concrete), as well as the separate studies showing a benefit for metacognitive prompts versus problem-focused, we predict the following: 1) Concrete materials paired with metacognitive prompts would result in the acquisition of adaptive expertise (i.e., acquiring both procedural fluency and conceptual understanding); 2) Concrete materials would help participants develop procedural fluency, while abstract materials would help participants develop better understanding of the underlying concepts; 3) Concrete materials would be the most engaging for participants, and that engagement with the learning materials would result in improved learning. This last prediction was based in the view that concrete, visually salient materials would be more motivating to use for participants. More engagement with and better affect toward learning materials would lead to better learning outcomes (Cordova & Lepper, 1996).

Methods

Participants

Ninety University of Pittsburgh students participated (mean age = 19.25, *SD* =2.14) in this experiment and received partial course credit. The majority of the participants (71) reported not having any prior knowledge of the target concepts (permutations and com-

binations) as assessed by a background questionnaire. Nineteen participants reported having some prior experience with the probability concepts being covered. Because these participants were evenly distributed across the conditions, we did not exclude them from the subsequent analyses.

Design and Materials

We used a 3 (manipulatives: concrete, abstract, none) X 2 (prompt-type: metacognitive, problem-focused) between-subjects design with participants randomly assigned to one of six learning conditions. Materials were presented in separate learning and test packets. The materials consisted of a demographic sheet, a questionnaire, a talk out-loud practice sheet, the learning materials, a second questionnaire, test problems, and a final questionnaire.

Questionnaires

The demographic questionnaire consisted of three open-ended questions that assessed prior mathematics experience (e.g., "Please list all of your high school mathematics courses."). There were also three questionnaires administered during the study including: a pre-learning questionnaire, an after-learning questionnaire, and an after-test questionnaire. The pre-learning questionnaire assessed participants' attitudes toward math in general and general learning strategies (see Table 1, Part A for example questions). The post-learning questionnaire assessed participants' subjective experiences while going through the learning packet, as well as their perceptions of the materials (see Table 1, part B). The post-test questionnaire also assessed participants' subjective experiences, as well as attitudes about the experimental materials and their mathematical abilities (see Table 1, part C). These were all assessed on a 5-point Likert scale, ranging from "Strongly Disagree" (1) to "Strongly Agree" (5). All questionnaire items are listed in the Appendix.

Table 1. Sample items from pre-learning, post-learning, and post-test questionnaires.

A. Initial Questionnaire	B. Post-Learning Questionnaire	C. Post-Test Questionnaire
I am interested in learning about probability.	*The problem scenarios were easy to imagine.*	*I would like to learn more about probability in the future.*
When I learn something new, I try to relate the material to what I already know.	*I was engrossed in the materials while going through the packet.*	*During the experiment, I thought I was improving, even if I was making mistakes.*

Learning materials

Problems. The target learning concepts were permutations and combinations with the order of presentation counterbalanced across conditions. The first page of the learning packet was a brief instruction on probability notation (i.e., "The probability of any given

number coming up on a six-sided die is 1/6") and on factorial notation (i.e., "4! = 4 X 3 X 2 X 1"). There were also simple, comprehension-check questions such as "What does 5! equal?" If participants displayed any difficulties with these, the experimenter would clarify the concepts.

The next page presented a simple probability problem (either combinations or permutations) without instructions about how to solve it (i.e., the "discovery problem," see Table 2). Participants could solve the problem with a counting method (e.g., for permutations make one valid order, record it, make another valid order and record that, until all possibilities have been exhausted). The purpose of the problem was to see if participants knew the formula to calculate the solution, and if not, whether they could invent a way to solve it given the materials they had for that condition.

The next problem students completed was the modeled problem, which also used small, computationally tractable values and could be completed by a counting method. The first few steps of a counting method were demonstrated for the participant (a more detailed description of this demonstration is provided in the procedure), and the participant then solved the problem by herself.

The next page presented a worked example that walked participants through solving a problem vis-à-vis the formula (see Figure 2). Participants had to fill in blanks as they proceeded, which were designed in such a way as to focus attention on important conceptual features of the formula. The purpose of the worked example was to bridge the discovery and modeled problem learning experiences and intuitive strategies to the formal computational method for calculating the solution. The worked example was then followed by two practice problems that also provided the formula. These problems included higher values that were not easily calculated via the counting methods that were demonstrated earlier.

The next page presented the second probability concept (permutations or combinations depending on the condition), and replicated the order and types of materials for that concept. All problems in each section used the same general cover story (i.e., permutations problems dealt with puppies, combinations with CDs, see Table 2). Several problems were adapted from Ross & Kilbane (1997), and all problems instructed participants to find the probability of one scenario out of all the possibilities.

Prompts. Two types of prompts were used during the experiment (see Table 3 for example prompts used in each condition). Metacognitive prompts were designed to focus attention on the conceptual relations between the objects and activities in the story problems and the variables and values in the formulas. They were designed to have participants reflect on the problem-solving processes involved in calculating permutations and combinations. In contrast, the problem-focused prompts were designed to focus attention on participants' current goals and tasks.

These prompts were based on the prompts used in Berardi-Coletta et al. (1995). The

Table 2. Example discovery, modeled, and practice problems for each concept, along with task demands for the participant (some problems adapted from Ross & Kilbane, 1997).

Problem Type	Concept		Task Demands
	Permutations	*Combinations*	
Discovery	You have three puppies. You are training them to get ready to go for a walk. When you say "walk" the puppies scramble for the door and get in a single file line. In how many ways could the puppies line up in a single file at the door?	You have 5 CDs in jewel cases. How many different ways can you make groups of 3 CDs from the 5 you have?	If prior knowledge exists, recall the correct formula to solve the problem. If no prior knowledge, invent a correct method to count all the possibilities.
Modeled	The Puppy Pound Palace has 4 puppies. Bobby and Timmy each come to get a friendly puppy to take home. They each have a hard time deciding on a puppy, so they decide to choose randomly. They decide to let Bobby choose first, and then Timmy will choose. In how many different ways could Bobby and Timmy pick their puppies?	You have 6 CDs. Your friend asks to borrow 2 CDs for a party he is throwing. He does not have any preferences as to which CDs he gets. From 6 CDs, how many different combinations of 2 CDs can you make?	Use a simple counting method modeled for the participant to count all possibilities.
Worked Example	The Puppy Pound Palace was having an open house to help find homes for their 5 new puppies. The 4 children who came were very excited about getting friendly puppies. To be fair, the children each randomly chose a puppy. The choosing went by size with the smallest child choosing first. What is the probability that the smallest child got the smallest puppy, the second smallest child got the second smallest puppy, the third child got the third smallest puppy, and the fourth child got the fourth smallest puppy?	You have 7 CDs. However, you have not kept them all in the right cases, and now the artist on the outside of the case may not match the CD inside of it. You open 4 at random. What is the probability that the 4 you open are the 4 that contain the CDs that match their cases?	Follow instructions to solve the problem using the correct formula (see Figure 2).
Practice	The Puppy Pound Palace was having an open house to help find homes for their 6 new puppies. The 2 children who came were very excited about getting friendly puppies. However, the children couldn't decide which puppy to take, so the manager decided randomly for each of them, choosing for the youngest child first. What is the probability that the youngest child got the youngest puppy, and the second youngest child got the second youngest puppy?	Jeff recently received a 4-disc CD changer. He owns 6 CDs, and wants to choose some to put into the changer. Unfortunately, he has not kept them in the correct cases, so he opens 4 at random. What is the probability that the 4 he chooses are the 4 Rock-and-Roll CDs he owns?	Use the formula provided by the worked example to correctly solve the problem.

Figure 2. The worked example which introduced the combinations formula, adapted from Ross & Kilbane (1997).

Now that you have seen how to calculate the number of possible combinations by hand, this section will introduce a formula that will help you solve problems involving the probability of a certain combination occurring.

COMBINATIONS

You have 7 CDs. However, you have not kept them all in the right cases, and now the artist on the outside of the case may not match the CD inside of it. You open 4 at random. What is the probability that the 4 you open are the 4 that contain the CDs that match their cases?

The set of items being selected from is the set of CDs. How many CDs are available for you to select from? _____. This number is j.

The subset that you must select for is the number of CDs you open. How many CDs do you open? _____. This number is h.

The subset of CDs that is NOT selected by you is the third set. How many available CDs are left after you open the cases? _____. This number is (j-h).

j = _____ (The number you are choosing from)

h = _____ (The number you are choosing)

(j-h) = _____ (The number left after you have chosen)

These numbers can now be used to calculate the number of combinations.

The first step in finding the probability that the 4 correct CDs are selected is to find out how many possible subsets of 4 CDs can be derived from the 7 available CDs.

$$j!/[h! \, (j-h)!] = \underline{\quad\quad} / \underline{\quad\quad\quad}$$

The second step is to find the probability that a specific 4 CDs will be selected. Since you now know the number of different subsets of 4 CDs out of 7 total CDs that are possible, the probability that a specific subset of 4 CDs will be selected is just 1 divided by that number, or

$$\frac{1}{j!/[h! \, (j-h)!]}$$

In the example problem,

$$\frac{1}{j!/[h! \, (j-h)!]} = \underline{\quad\quad\quad\quad} .$$

This is the probability that the 4 CD cases selected contain the correct disc.

Table 3. Example metacognitive and problem-focused prompts (adapted from Berardi-Coletta et al., 1995). Different initial and variable prompts were given every 45 seconds. Variable prompts refer to specific variables in a given formula (i.e., "h" and "r" are variables which represent the number of options available). Reflection prompts were given after participants had looked over the solution for 30 seconds.

Focus	Prompt Type		
	Initial	*Variable*	*Reflection*
Metacognitive	What is your approach to solving this problem?	How did you decide what value "h" is (or "r")?	How does the solution relate to what you did?
Problem-Focused	What is your current goal?	What does "h" (or "r") tell you?	What does the correct solution tell you?

experimenter gave a different prompt every 45 seconds while participants were working on the discovery, modeled, and practice problems. No prompts were given during the worked example, nor during the test phase. The reflection prompt was given after each problem had been solved, or time had run out for that particular problem (see third column of Table 3).

Manipulatives. All problems in the learning packet had cover stories dealing with either puppies or CDs. Participants in the concrete manipulatives condition received manipulatives that matched the contexts of the problem (see Figure 3). That is, problems about puppies were matched with small toy puppies, and problems about CDs were matched with CD cases. Participants in the abstract manipulative condition received the same number of colored blocks. These are considered abstract in that they share no direct relation to the contexts of the problem. Moreover, those in the abstract manipulative condition

Figure 3. Examples of abstract and concrete manipulatives.
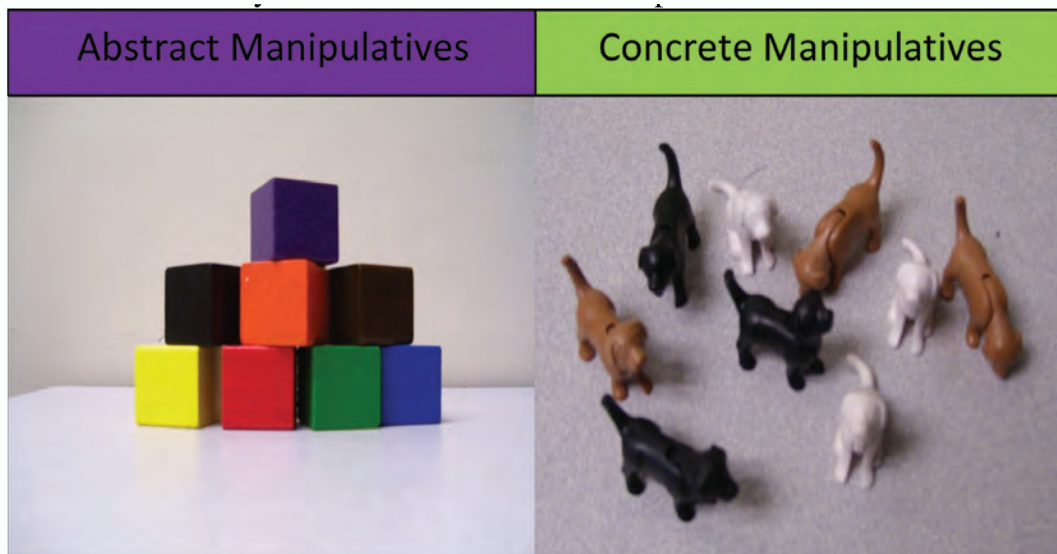
used the same objects in both contexts. However, this manipulation is still more concrete than receiving no manipulatives; this condition requires participants to generate and use some kind of symbolic notation to solve the problem.

Test materials

The test consisted of 12 word problems, two "write-your-own" problems, where participants were presented the formula and asked to write a word problem that could be solved using that formula, and eight conceptual questions, which asked participants about various features of the formula and their underlying conceptual understanding.

Packets were counterbalanced so that half the participants received a combinations problem first and the other half received a permutations problem. This first problem had the same cover story as in the learning phase, though the problem itself was new (i.e., permutations problem dealt with puppies, combinations with CDs, but with new values). The participants were not given the formulas during this problem. This problem assessed how sensitive participants were to the learning context. If what they learned was tied to the particular story context they should be more likely to access that knowledge on a problem using the same story context.

The next three problems were also presented without the formula, but these were presented in new cover story contexts (airplanes in a hangar, knights at a joust, and cars at a factory). These problems always dealt with the concept which did not apply to the first problem (i.e., if the first problem was a permutations problem, the next three were combinations problems). These problems assessed transfer to a new context. In addition, participants were given access to new manipulatives during these problems. Each story problem could be matched with either a concrete manipulative that matched the story context (e.g., airplanes), an abstract manipulative which did not (i.e., colored tiles), or no manipulatives. This was done to see if participants who had used manipulatives had learned particular strategies that were more accessible when they had access to manipulatives.

The next 8 word problems were multiple choice problems (4 permutations and 4 combinations). Each problem presented both formulas, and the participants' task was to first choose the correct formula and then calculate the solution. These problems separated concept access (determining which formula applies) from procedural application (choosing what values to map onto the variables). Even if the participant had chosen the conceptually wrong formula, which would not give them the correct answer for the problem, they could still extract meaningful values from the problem and use it to come to a solution. This will be discussed in detail in the results section.

Procedure

Participants were run individually in 90-minute sessions. The experimenter was present throughout, administering all of the materials and delivering all of the prompts. Participants worked through, in order, a demographics sheet, a questionnaire, talk out-loud practice (to

get them used to talking aloud as they worked), the learning materials, another questionnaire, the test materials, and a final questionnaire.

The three questionnaires took approximately two minutes each to complete. After filling out the demographics sheet and the pre-learning questionnaire, participants were given instructions and practice for talking aloud during the learning. The learning session was audio- and video-recorded. Each of the two concepts in the learning packet took approximately 20 minutes to complete.

Participants were given four and a half minutes to solve the first (discovery) problem. After they generated a solution or time elapsed they were presented the second (modeled) problem. For this problem, the experimenter first had the participant read the problem aloud and then modeled a procedure for calculating the solution (i.e., a simple counting method), using the manipulatives. The experimenter arranged the manipulatives, illustrated two possible arrangements, and then told the participant to continue counting all possibilities in the same way. Participants in the no-manipulatives condition were also shown a counting method, but the method was introduced via a paper-based worked example representing the objects on the page instead of with manipulatives, and with no experimenter input. The instructions on the page mirrored what the experimenter told participants in the manipulative conditions. Participants had four minutes to solve this problem. Participants were then given four and half minutes to read and complete the worked example, after which they were given 30 seconds to look over the solution.

Participants were then given two and half minutes for the first practice problem (with 30 seconds to look over the solution), and another two and half minutes for the second practice problem (with 30 seconds to look over the solution). For all of these problems, the participants would circle the word "Finished" at the bottom of the page when they were done. After participants marked finished, the experimenter waited 30 seconds before allowing them to move on. During this time participants were encouraged to go back, check their work, and change their solution if they wished. Participants were prompted every 45 seconds as they worked on all of these problems, except for the worked example, which they completed on their own. Responses to the prompts, as well as the talk-aloud protocols of their problem solving, were captured by an audio recording.

The test phase took approximately 40 minutes total, with participants having up to two and a half minutes for each of the word problems, three minutes for each of the "write-your-own" problems, and five minutes for the short answer conceptual questions. Participants did not talk aloud during the test phase.

Results

The results are divided into two main sections of test performance and engagement. First, we examine how the learning materials and prompts impact participants' test perfor-

mance by examining conceptual and procedural accuracy. Then, we examine participants' self-reported engagement with the learning materials and its effect on subsequent test performance.

Test Performance

Recall that our predictions were that concrete manipulatives would lead to better procedural skill in choosing variables, which will be referred to in this section as "Procedural Use," while abstract manipulatives would benefit the ability to choose the applicable concept, which will be referred to as "Concept Access." We also predicted an interaction between these factors, such that the best overall learning would result from the combination of concrete manipulatives and metacognitive prompts.

A successful solution to the test problems required participants to choose the appropriate formula (permutations or combinations), extract the relevant numbers for each of the variables (all of the problems had distracter information), and instantiate those variables correctly in the formula. A unique property of these concepts is that one can assign the correct numbers even when dealing with the wrong formula (i.e., knowing that the larger number is the one being chosen or selected from). Participants' extraction of relevant variables from the problem was not contingent upon them choosing the conceptually relevant formula. Because of this dissociation, we could differentiate more conceptual-level knowledge (which concept and corresponding formula applies) from procedural skill (how to assign variables from text to the appropriate variables).

Reported here is the performance on the first 12 items. There were no differences in performance on the first 4 problems, which had participants generate the formula, and the last 8, which asked participants to choose the correct formula, $F(2, 84) < 1$, *ns*, so all 12 items are combined and reported together. All of these problems required participants to determine the applicable formula, find the correct variables from the problem statement, and then compute the solution. First, we describe overall accuracy in solving the problems (correct formula, variable instantiation, and computation). Then, we split our accuracy scores into the sub-skills of "Concept Access," i.e., choosing the correct formula, and "Procedural Use", i.e., instantiating variables. All analyses are reported in terms of proportion correct.

Accuracy. Accuracy assessed participants' ability to solve each problem completely and correctly (i.e., correct formula, variable instantiation, and computation). This was coded as 0 for incorrect answers and 1 for correct answers. Because some participants had trouble with the calculations involved with the formulas, this measure provides different information than simply combining the "Concept Access" and "Procedural Use" measures. Group means were generally low; only about 4.5 problems out 12 were solved correctly, on average (see Figure 4).

We conducted a 3 (manipulatives: abstract, concrete, none) X 2 (prompt type: metacognitive, problem focused) between-subjects ANOVA to investigate the effect of manipu-

Figure 4. Mean proportion (+/- one standard error) of problems correctly solved for each training group, or overall accuracy.
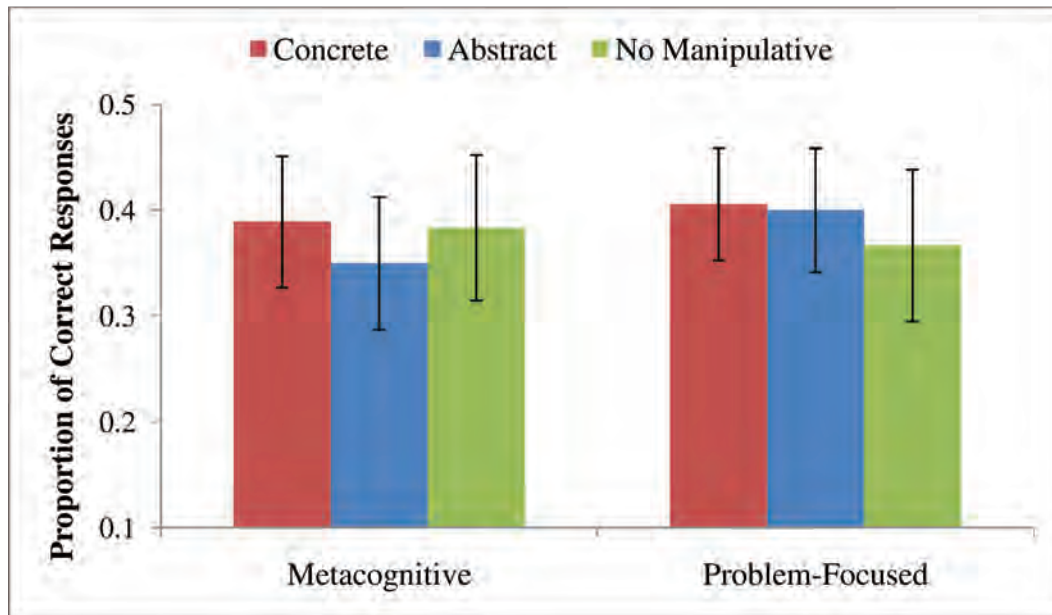
Figure 5. Mean proportion (+/- one standard error) for correctly selecting the correct principle for each training group, or "Concept Access." The dashed line represents performance at chance.
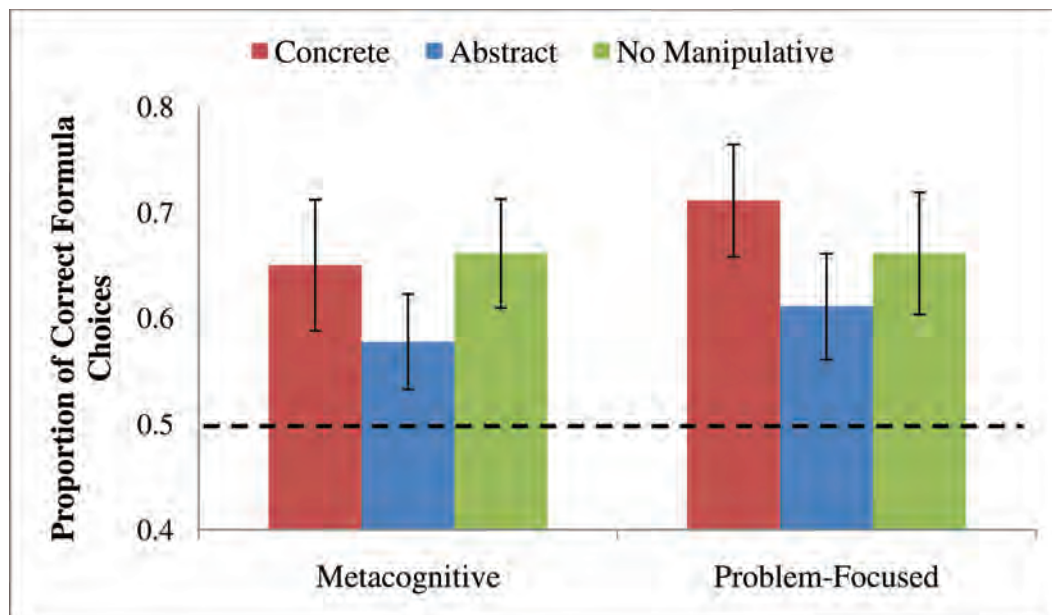
latives and prompt type on overall accuracy. Analysis revealed no effect of manipulative, F (2, 84) = .07, *ns,* prompt type, F (1, 84) = .09, *ns,* or an interaction, F (2, 84) = .12, *ns.* This shows no overall performance differences on problem solving when collapsing across sub-skills. Next we look at whether groups differed on the sub-skill of concept access.

Concept Access. Concept access assessed participants' ability to correctly choose which formula applies on a given problem. Participants were given a score of 0 if they chose the incorrect formula, and given a score of 1 if they chose the correct formula. Figure 5 shows means and standard errors for this measure.

We conducted a 3 (manipulatives: abstract, concrete, none) X 2 (prompt type: meta-cognitive, problem focused) between-subjects ANOVA to investigate the effect of manipulatives and prompt type on concept access. Analysis revealed no effect of manipulative, $F(2, 84) = 1.42$, *n*s, prompt type, $F(1, 84) < 1$, *ns,* or an interaction, $F(2, 84) < 1$, *ns.* It appears that our experimental manipulations did not differentially influence participants' ability to decide which formula applied. However, some learning did occur, as every group performed better than chance, all *t*'s (14) > 1.75, p ≤ .05 (see Figure 5). This result suggests that all conditions were somewhat successful in facilitating learning of the concepts and the ability to transfer and access those concepts in novel scenarios (all test problems but one had new problem scenarios).

Procedural Use. Procedural use measured participants' ability to correctly assign the relevant values from the word problems to the appropriate variables in the equation. Correct instantiation of the variables could be completed regardless of whether the right concept was selected because there were two variables for any given equation—the number of items being chosen from and the number of items being chosen. Each word problem also included distracter information, so this process involved more than simply finding numbers in the problem and plugging them into the formula (randomly plugging numbers from the word problem into each variable would result in only a 7% chance, on average, of getting both variables correct). Participants were given scores of 0, 1, or 2, based on whether they correctly assigned 0, 1 or 2 of the variables. Figure 6 shows means and standard errors of each group on this measure.

A 3 x 2 ANOVA was conducted with procedural use as the dependant variable. Analysis revealed no effect for manipulatives, F (2, 84) <1, *ns,* prompt type, F (1, 84) =1.59, *ns,* or an interaction, F (2, 84) = 1.23, *ns.* However, planned comparisons for the effect of prompts on the students using concrete manipulatives revealed a significant difference between the problem-focused versus metacognitive prompt conditions. That is, among all participants who received concrete materials, those who received metacognitive prompts had a higher percentage of variables correct than those who received problem-focused prompts, *t*(28) = 2.44, *p* < .05, *d* = .93. These results are consistent with our hypothesis that the concrete materials would facilitate procedural fluency and the reflective prompts would facilitate

Figure 6. Mean proportion (+/- one standard error) of variables correctly instantiated for each training group, or "Procedural Use."
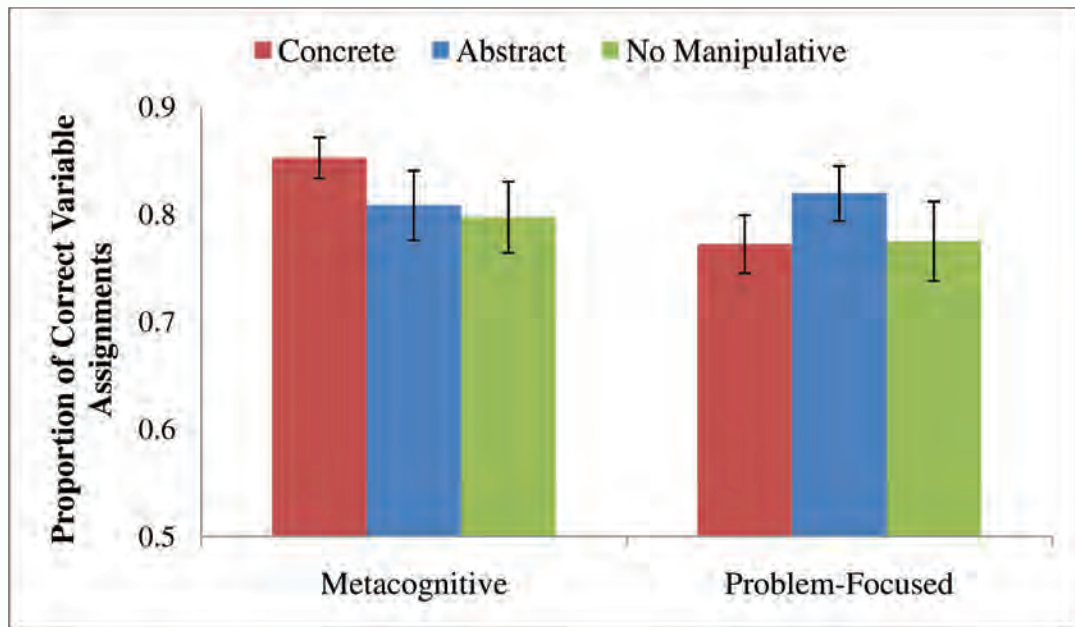
Figure 7. The interaction between prompt type and engrossment level on test performance. Error bars represent +/- one standard error.
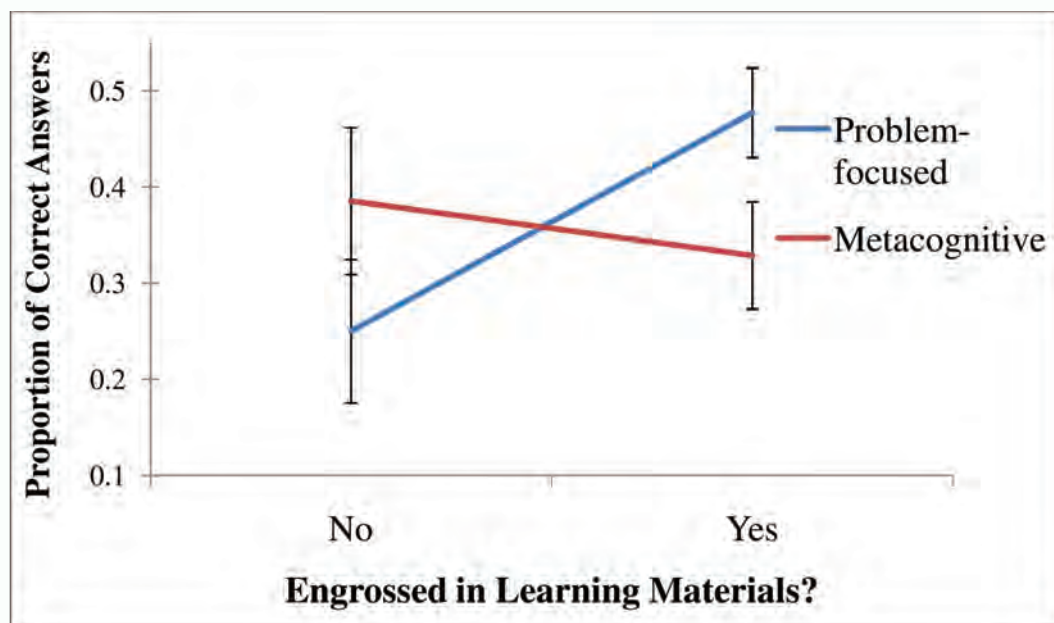
Table 4. Each value reflects % of the group who was coded as having a correct answer, with standard error in parenthesis, for the Concrete Manipulative (CM), Abstract Manipulative (AM) and No Manipulative (NM) groups, within each prompt type.

Conceptual Problem	Metacognitive			Problem-Focused		
	CM	AM	NM	CM	AM	NM
In this formula, what does "... (n – r + 1)" mean? What does it tell you in terms of solving the problem?	.87(.09)	.80(.11)	.73(.12)	.80(.11)	.67(.13)	.67(.13)
Suppose you had a problem in which n = 4 and r = 2, and another problem in which n = 5 and r = 3. What is the relationship between the two problems and their solutions? If you know the answer to the first one, is there a shortcut to find the answer to the second one?	.40(.13)	.33(.13)	.47(.13)	.27(.12)	.33(.13)	.33(.13)
What is 1 (in the given formula)? *	.73(.12)	.60(.13)	.33(.13)	.73(.12)	.60(.13)	.40(.13)
What are the similarities and differences between combinations and permutations? How are these reflected in the formulas?	.53(.13)	.20(.11)	.20(.11)	.47(.13)	.40(.13)	.53(.13)

the abstraction of those skills so they could transfer to new contexts (conceptual learning advantages are discussed in the next section).

Conceptual Problems. For the conceptual, open-response questions, a solution rubric was used to classify the solutions as either correct and incorrect. These problems asked participants to explain features and applications of the formulas they had learned, such as what the variables stand for, and to compare and contrast the permutations and combinations formulas. Results for the conceptual questions reported here are in terms of correct or incorrect on four problems (see Table 4); the other four problems were at ceiling for all conditions. These four ceiling problems asked participants to describe what variables in the formula stood for (i.e., "What is n?" "What is r?" "What is j?" "What does (j-h) mean?"). Most participants could answer these in simple terms, such as "r is the number of items being chosen."

To test the effect of the pairing of concrete manipulatives with metacognitive prompts, we computed a total correct and incorrect sum across the four conceptual problems, and submitted these to Chi-Square analyses. We compared performance of the groups who received concrete manipulatives to the other two manipulative types. Within the meta-cognitive group, the concrete manipulatives group had a higher percentage of correct responses (63.3%) than the abstract and no manipulative groups combined (45.8%), X^2 (1, N = 180) = 4.91, $p < .05$. Within the problem-focused group, there was no difference between concrete (56.6%) and other types of manipulatives (49.2%), X^2 (1, N = 180) = .91, ns. Similar comparisons of abstract and no manipulatives against the performance of the other two groups were not significant, all X^2's (1, N = 180) < 2.50, ns. It appears that the pairing of concrete manipulatives with metacognitive prompts produced better conceptual knowledge than other combinations of materials and prompts.

Engagement

We hypothesized that concrete materials would be the most engaging for participants, and that engagement would lead to deeper processing. To assess engagement, we asked participants to complete a questionnaire immediately after the learning phase. The two most direct measures of engagement and interest asked participants how much they agreed to statements "I was engrossed in the materials as I went through the packet" and "I thought the materials were interesting" on a 5-point Likert scale. Different materials did not produce any differences in response to these questions, as demonstrated in a one-way ANOVA, F's (2, 85) < .90, ns. Engagement in this population was not easily manipulated through the use of different materials. It is possible that these materials were not attractive to the age group in our study, even as useful aids in their learning. Similarly, the type of prompts did not lead to different responses on these items, t's(86) < .78, ns.

Engagement/Prompt Interaction. Further analysis into the effect of engagement on subsequent test performance revealed an interaction within experimental conditions on our measure of accuracy, or the ability to solve each problem correctly. Among those people who reported feeling engrossed in the materials, an interaction emerged with the type of prompt, such that the effect of being engrossed or not was different for each prompt type. Specifically, among those participants who reported feeling engrossed (4 or 5 on the scale), problem-focused prompts resulted in more correct answers. Conversely, when participants reported not being engrossed (1 or 2 on the scale) metacognitive prompts led to better test results. This interaction was significant, F (1, 63) = 5.04, $p < .05$, $d = .57$ (see Figure 7).

No such interaction was observed for different types of manipulatives in a 5 X 3 ANOVA, F(8, 73) < 1, ns, nor for a 3-way interaction between prompts, manipulatives, and engrossment level, F(6, 61) < 1, ns. Among those participants who subjectively reported feeling engrossed in the learning materials, those who were prompted to focus on the

particulars of the problem performed better ($M = .48$, $SD = .22$) on the test than those who received more abstraction-based metacognitive prompts ($M = .33$, $SD = .23$), $t(31) = 2.7$, $p < .05$. Conversely, among those participants who did not have the affective experience of being engrossed in the learning materials, the metacognitive prompts ($M = .38$, $SD = .31$) tended to improve subsequent test performance compared to problem-focused prompts ($M = .25$, $SD = .25$), although this difference was not significant, $t(25) = 1.22$, ns. We will discuss potential reasons for these differences in the following section.

Discussion

Our manipulations did not produce any differences in participants' overall problem accuracy. We also did not see any differences in participants' ability to correctly choose which formula applied in a given problem. However, all groups showed strong evidence of learning, performing better than chance in choosing which formula applies as well as answering about 40 percent of the computationally complex problems correctly. Participants also developed the skill to correctly instantiate variables based on the word problems, even amongst distracter information. All groups demonstrated above chance ability on this measure. Critically, we observed a benefit for concrete materials paired with metacognitive prompts over problem-focused prompts.

We had predicted that concrete materials paired with metacognitive prompts would lead to the most robust learning, that abstract materials would aid in conceptual understanding, and that concrete materials would aid in the development of procedural skill. Our results did not support the last two predictions, but we did see evidence for the benefit of concrete materials paired with metacognitive prompts. In contrast to our predictions, participants who learned with abstract materials trended toward having the lowest levels of concept access on the subsequent test, suggesting that they actually took attention away from developing conceptual knowledge necessary to choose which formula applies in a given situation. It is possible that the materials were too abstract for participants to see their utility in facilitating understanding and problem-solving in our experiment. Similarly, concrete learning materials on their own did not facilitate procedural use, or correct variable instantiation, on the test. It is possible that these materials were not seen as a useful model of the real-world applications of the concepts, and so the variable assignment aspects of the formula were insufficiently grounded in their everyday reasoning.

However, we did see that concrete materials paired with metacognitive prompts during learning led to better procedural use, the subskill of finding the correct variables in a given problem. This may be because grounding new learning in concrete examples results in a reduced cognitive load (Sweller, 2006), relative to instruction without such resources. This reduction in cognitive load may allow deeper, more effortful reflection to occur in response to metacognitive prompts, leading to improved ability to reason about

the problem (Sweller, 1988). Additional evidence for this possibility comes from the higher scores on the conceptual test items for those students who received concrete manipulatives and metacognitive prompts. Another possible account for the benefit of concrete learning materials with metacognitive prompts has to do with the representations formed by this pairing. As noted earlier, the cognitive representation acquired will have a critical role in how knowledge is used in problem solving (Kotovsky & Fallside, 1989). This pairing may be particularly well-suited to creating a flexible representation, which is based in concrete, real-world knowledge but, after reflection, abstract enough to apply in many situations and to give insight into the concepts underlying the formulas.

These results provide evidence that materials by themselves do not necessarily improve the ability of students to reason deeply about the topic. This work adds to a growing body of research that points to weaknesses in the pedagogical assumption that making materials as concrete and real-world as possible will automatically increase student learning. Research on text-based interest (Harp & Mayer, 1998), perceptual richness (Sloutsky, Kaminski, & Heckler, 2005; McNeil, Uttal, Jarvin, & Sternberg, 2009) and narrative voice (Son & Goldstone, 2009b) have all pointed to the limitations of such a view. In many of these cases, manipulations are made to instructional materials and the effect on student learning is observed. The current study has illustrated the benefit of examining manipulations to both instructional materials and student processing, such as responding to prompted questions. However, work clearly remains to be done to map out all of the ways in which materials and student cognition interact, and how this varies across populations. Some of this research is done with young children (i.e., DeLoache, 1989) while some is done with adults (i.e., Goldstone & Sakamoto, 2003), so results need to be considered in a developmental framework.

Perhaps our most surprising findings dealt with the role of engagement with learning materials on subsequent test performance. While we had predicted that concrete materials would be the most engaging for participants, we instead found no relationship between type of materials used and how engrossed participants reported feeling. This was most likely due to the population used, college students, who did not seem to consider the manipulatives useful tools for their understanding, even though relying solely on their internal representations did not lead to optimal performance. The types of prompts students received did not lead to differences on the questionnaire items dealing with engagement. We did find, however, an interaction between level of engagement and type of prompt, such that those who felt engrossed benefited from receiving simpler, more problem-focused prompts. Those who did not feel engrossed showed no difference across prompts, but a trend of a benefit from receiving more reflective, difficult metacognitive prompts.

This suggests the possibility that the subjective feeling of engrossment occurs when students are deeply processing the material, such that further attempts to have them reflect are distracting and harmful for learning. In such a case, simple prompts that keep

them focused on the particulars of the problem may be the best learning tool. However, if a student is not feeling engaged in the materials, asking harder, more reflective questions may facilitate deeper processing of a problem which they may otherwise have not engaged. This view is consistent with research on text processing, which has found that more engaging materials direct attention to more abstract, comprehension-related cognitive processes, while less engaging materials keep attention focused on more perceptual-level details, such as features of the font and spelling (McDaniel, Waddill, Finstad, & Bourg, 2000). There is also evidence that students interested in developing a deep understanding of a topic or who already have some domain knowledge are harmed by interventions aimed to make the materials more attractive (Durik & Harackiewicz, 2007), while those who are not interested do benefit from such interventions (Durik & Harackiewicz, 2007; Son & Goldstone, 2009b) and from more exploratory, open-ended and self-guided instructional methods (Belenky & Nokes, 2009).

The educational implications of these findings are for instructors to focus on monitoring how absorbed students are in the materials and then tailoring instruction to match the motivational / affective state. Based on these and related findings, it would seem that if a student is already engaging, it is best to stay out of her way, and not interrupt her self-guided activity to focus on a conceptual-level question. With the development of intelligent tutoring systems with built-in detectors of affect based on student behaviors (i.e., Baker, Rodrigo, & Xolocotzin, 2008), this possibility will be open to empirical investigation. This work also demonstrates a need for educators to consider not only the ways in which materials are presented, but also the myriad ways in which students can engage with them. An overreliance on monitoring behavioral activity (such as using physical materials) could lead educators to pay less attention to the cognitive activity students are undertaking. Such a view could lead to educational practices that seem successful at first glance, but ultimately result in poorer learning (Mayer, 2004).

Future work in dealing with the effect of affect and subjective experience on learning is needed. Assumptions such as "if a person is motivated, they will learn better" are insufficient, at best, and incorrect, at worst. Research examining different types of processing triggered by various levels of engagement and affective response will provide a crucial next step in better understanding the role of subjective experience on learning.

Appendix

Initial Questionnaire
When I am learning about something new, I try to make sure I understand it.
When I learn something new, I try to relate the material to what I already know.
I am interested in learning about probability.
I am interested in learning about math.

I think I would do better in math classes if they did not have as many calculations.

In math classes, I understand the concepts well but have problems with their problem solving.

When I have a chance to learn something new, I usually take it.

When I am studying, I ask myself questions to make sure I understand the material.

Activity Questionnaire

The explanations in the examples were easy to follow.

The information in the **worked** examples was well organized.

The information in the **practice** problems was well organized.

The problem scenarios were easy to imagine.

The materials helped me to organize the information.

I was engrossed in the materials while going through the packet.

How do you feel about your ability to solve the problems that were presented? (1-3 scale)

Post-Test Questionnaire

I thought the materials were interesting.

I got caught up in the materials without trying to.

I think others would find these materials interesting.

I would like to learn more about probability in the future.

I found it easy to pay attention to the materials.

I found it easy to understand the materials.

I have used materials like these before.

During the experiment, I thought I was improving, even if I was making mistakes.

How do you feel about your ability to solve the problems that were presented? (1-3 scale)

*Unless noted, all questions were assessed on a 5-point Likert scale ranging from "Strongly Disagree" (1) to "Strongly Agree" (5).

Endnote

1. It is important to note that the terms abstract and concrete always describe a *relation* to a "norm" or another "situation / object / feature" in the world. When we use the terms "abstract" or "concrete" we are saying more "abstractly related" or more "concretely related" to the particular learning scenario. We view this as a continuous dimension that is relative to some comparison scenario. Abstraction also has another meaning that we do not address in the current work that refers to the class of objects the concept points to, e.g., the concept of mammal is more abstract than the concept of dog.

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