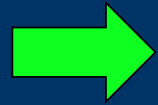


Course Business

- Midterm assignment due on Canvas next Monday at 1:30 PM
 - Unsure if an article is suitable? Can run it by me
- Add-on packages to install for today:
 - **performance** (may have gotten this last week)
 - **emmeans**
- Lab materials on Canvas

Week 8.1: Post-Hoc Comparison



- Unbalanced Factors
 - Weighted Coding
 - Unweighted Coding
- Post-Hoc Comparisons
 - Tukey Test
 - Estimated Marginal Means
 - Comparing Marginal Means
- Lab



Unbalanced Factors

- Sometimes, we may have **differing numbers of observations** per level
- Possible reasons:
 - Some categories naturally more common
 - e.g., college majors
 - Categories may be equally common in the population, but we have sampling error
 - e.g., ended up 60% female participants, 40% male
 - Study was designed so that some conditions are more common
 - e.g., more “control” subjects than “intervention” subjects
 - We wanted equal numbers of observations, but lost some because of errors or exclusion criteria
 - e.g., data loss due to computer problems
 - Dropping subjects below a minimum level of performance



Week 8.1: Post-Hoc Comparison

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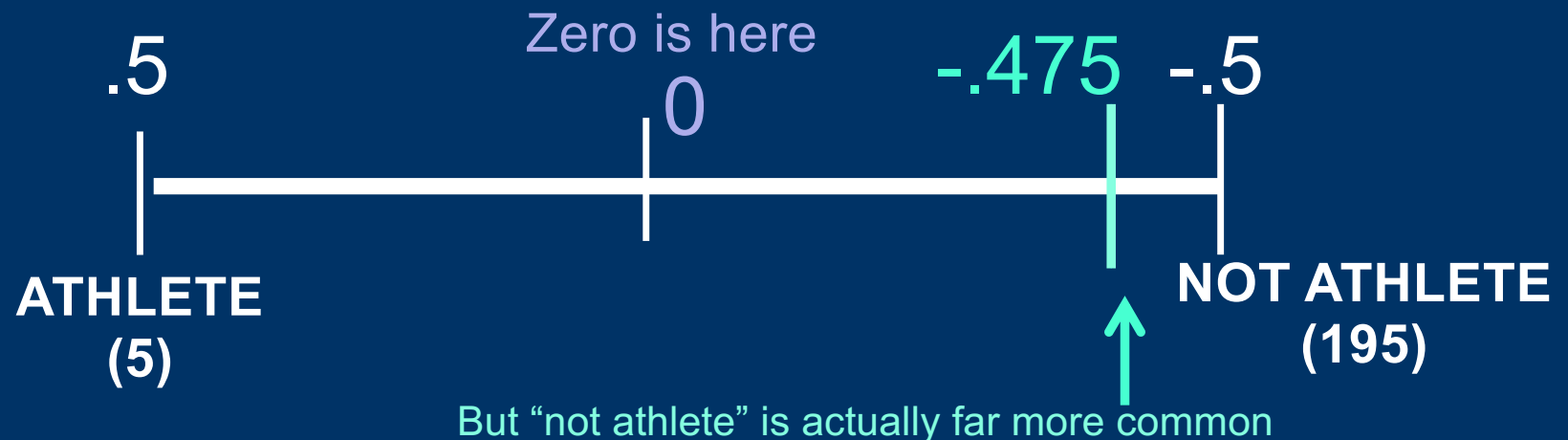


Weighted Coding

- “For the average student, does course size predict probability of graduation?”
 - Random sample of 200 Pitt undergrads
 - 5 are student athletes and 195 are not
- How can we make the intercept reflect the “average student”?
 - We could try to apply effects coding to the **StudentAthlete** variable by centering around the mean and getting $(0.5, -0.5)$, but...

Weighted Coding

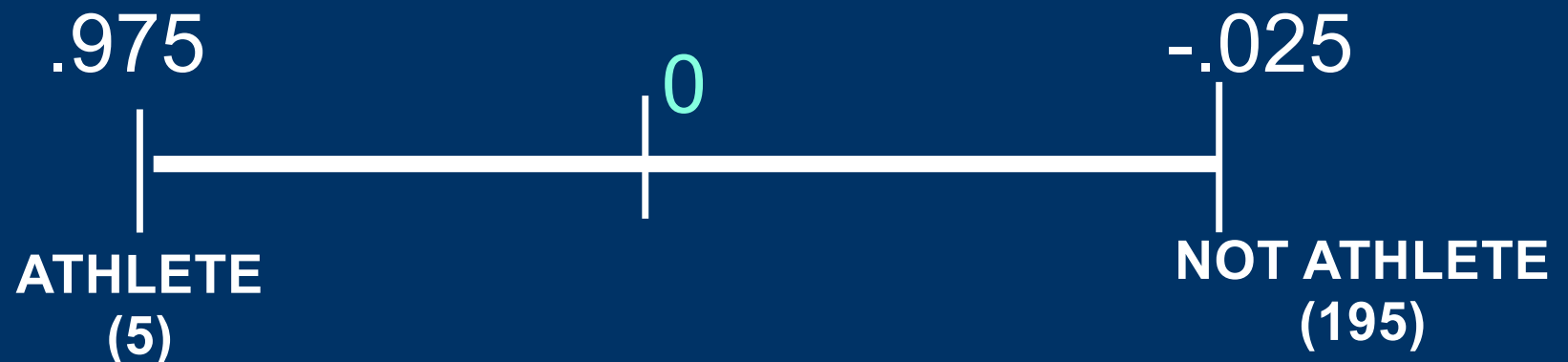
- An intercept at 0 would no longer correspond to the overall mean



- As a scale, this would be totally unbalanced
- To fix balance, we need to assign a heavier weight to Athlete



Weighted Coding



- Change codes so the mean *is* 0
 - `c(.975, -.025)`
- `contr.helmert.weighted()` function in my `psycholing` package will calculate this

Weighted Coding

- **Weighted coding:** Change the codes so that the mean is 0 again
 - Used when the imbalance reflects something *real*
 - Like Type II sums of squares
- “For the average student, does course size predict graduation rates?”
 - Average student is *not* a student athlete, and our answer to the question about an “average student” should reflect this!


Week 8.1: Post-Hoc Comparison

- Unbalanced Factors
 - ✗ Weighted Coding
 - ➔ Unweighted Coding
- Post-Hoc Comparisons
 - Tukey Test
 - Estimated Marginal Means
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- Lab

Unweighted Coding

- Last week we looked at `aphasia.csv`:
 - Response times (**RT**) in a sentence verification task
 - Effect of **SubjectType** (aphasia vs. control)
 - Effect of **SentenceType** (active vs. passive)
 - And their interaction

Unweighted Coding

- Oops! Our experiment loaded up the wrong image for one of our Passive sentences (“Groceries”)
 - It may have been sabotaged 
 - `UsableItem` column is `No` for this item
- First, can we remove this from our data?
- Some possibilities:
 - `aphasia %>% filter(UsableItem == 'Yes') -> aphasisa`
 - `aphasia %>% filter(UsableItem != 'No') -> aphasisa2`
 - etc.

Unweighted Coding

- Oops! Our experiment loaded up the wrong image for one of our Passive sentences (“Groceries”)
- Now, there’s an imbalance, but it’s an accident and not meaningful
 - In fact, we’d like to get rid of it!

```
> summary(aphasia2)
  Subject      Item      SubjectType  SentenceType
S1       : 31  Astronaut: 30  Aphasia:465  Active :480
S10      : 31  Bear      : 30  Control:465  Passive:450
S11      : 31  Boy       : 30
S12      : 31  Breakfast: 30
S13      : 31  Burglar  : 30
S14      : 31  Cheese   : 30
(Other):744  (Other)  :750
```

Unweighted Coding

- Oops! Our experiment loaded up the wrong image for one of our Passive sentences (“Groceries”)
 - Now, there’s an imbalance, but it’s an accident and not meaningful
 - In fact, we’d like to get rid of it!
 - Retain the **(-0.5, 0.5)** codes
 - Weights the two conditions equally—because the imbalance isn’t meaningful
 - Like Type III sums of squares
 - Probably what you want for factorial experiments
-
-

Unbalanced Factors: Summary

- Weighted coding: Change the codes so that the mean is 0
 - Use when the imbalance reflects something *real*
 - Can be done with `contr.helmert.weighted()`

Mean across each individual:



- Unweighted coding: Keep the codes as -0.5 and 0.5
 - Use when the imbalance is an *accident* that we want to eliminate

Mean of the two levels:

Mean of the
active sentences

Mean of the
passive sentences

- With balanced factors, these are identical
-
-

Week 8.1: Post-Hoc Comparison

✘ Unbalanced Factors

✘ Weighted Coding

✘ Unweighted Coding

➔ Post-Hoc Comparisons

- Tukey Test
- Estimated Marginal Means
- Comparing Marginal Means

- Lab

Post-hoc Comparisons

- Maximal model for the aphasia data was:
 - `model.Maximal <- lmer(RT ~ 1 + SentenceType * SubjectType + (1 + SentenceType|Subject) + (1 + SubjectType|Item), data = aphasia)`
- This didn't converge:
 - `boundary (singular) fit: see ?isSingular`

```
Random effects:
```

Groups	Name	Variance	Std.Dev.	Corr
Item	(Intercept)	44860.91	211.804	
	SubjectTypeAphasia	71.18	8.437	-1.00
Subject	(Intercept)	43286.19	208.053	
	SentenceTypePassive	1350.57	36.750	0.18
Residual		6861.01	82.831	

Number of obs: 930, groups: Item, 31; Subject, 30

Probably overparameterized

Post-hoc Comparisons

- Maximal model for the aphasia data was:

- ```
model.Maximal <- lmer(RT ~
 1 + SentenceType * SubjectType +
 (1 + SentenceType|Subject) +
 (1 + SubjectType|Item),
 data = aphasia)
```

- So, let's simplify:

- ```
model2 <- lmer(RT ~  
  1 + SentenceType * SubjectType +  
  (1 + SentenceType|Subject) +  
  (1|Item),  
  data = aphasia)
```

- Doesn't seem to harm model fit— $p > .20$

- ```
anova(model.Maximal, model2)
```

|               | npar | AIC   | BIC   | logLik  | deviance | Chisq  | Df | Pr(>Chisq) |
|---------------|------|-------|-------|---------|----------|--------|----|------------|
| model2        | 9    | 11214 | 11257 | -5597.9 | 11196    |        |    |            |
| model.Maximal | 11   | 11216 | 11269 | -5596.8 | 11194    | 2.2473 | 2  | 0.3251     |

# Post-hoc Comparisons

- With treatment coding, we get estimates of simple-effects:

Intercept: RT for healthy controls, active sentences

```
Random effects:
Groups Name Variance Std.Dev. Corr
Item (Intercept) 43090 207.58
Subject (Intercept) 43286 208.05
 SentenceTypePassive 1348 36.72 0.18
Residual 6879 82.94
Number of obs: 930, groups: Item, 31; Subject, 30

Fixed effects:
 Estimate Std. Error df t value Pr(>|t|)
(Intercept) 1716.01 74.88 56.54 22.916 < 2e-16 ***
SentenceTypePassive 553.44 75.60 30.23 7.321 3.57e-08 ***
SubjectTypeAphasia 84.52 76.35 28.00 1.107 0.278
SentenceTypePassive:SubjectTypeAphasia 188.30 17.27 28.00 10.904 1.38e-11 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Correlation of Fixed Effects:
 (Intr) SntnTP SbjcTA
SntncTypPss -0.465
SbjctTypAph -0.510 -0.011
SntncTP:STA -0.049 -0.114 0.095
```

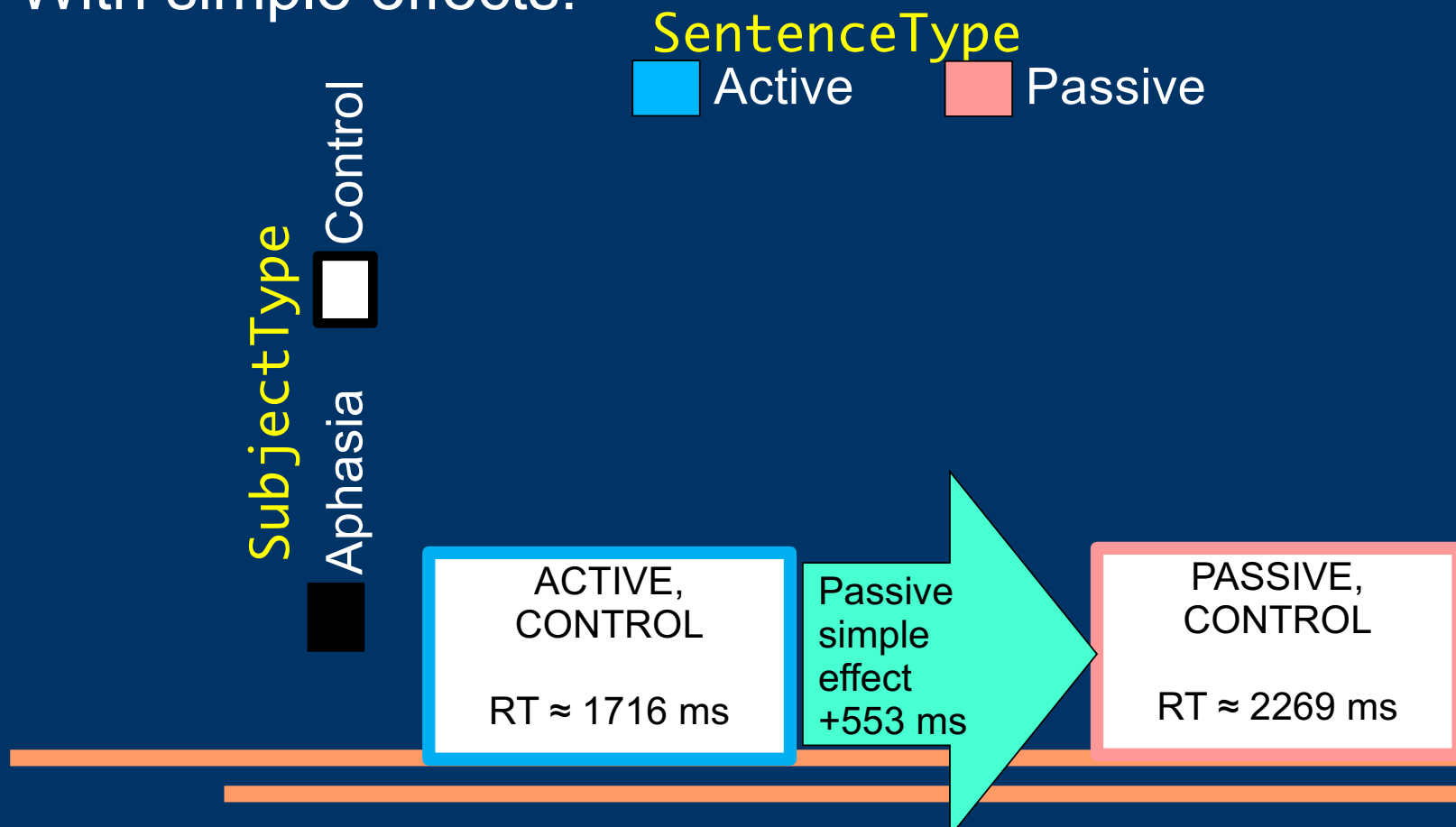
Significant RT difference for passive sentences (among healthy controls)

Not a significant RT difference for aphasics (among active sentences)

Significant special effect of aphasia + passive sentence

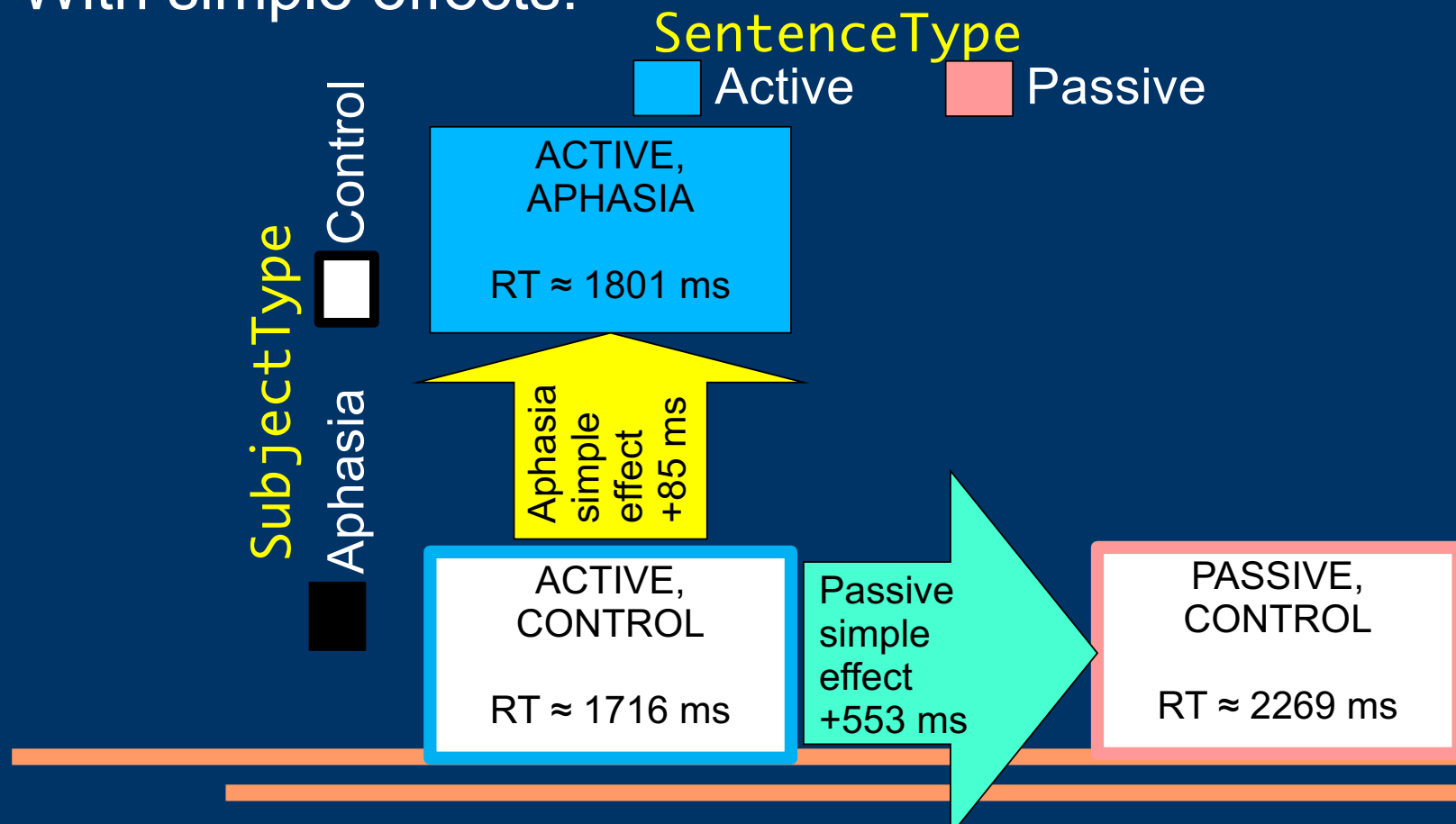
# Post-hoc Comparisons

- The estimates from a model are enough to *fully describe* differences among conditions
- With simple effects:



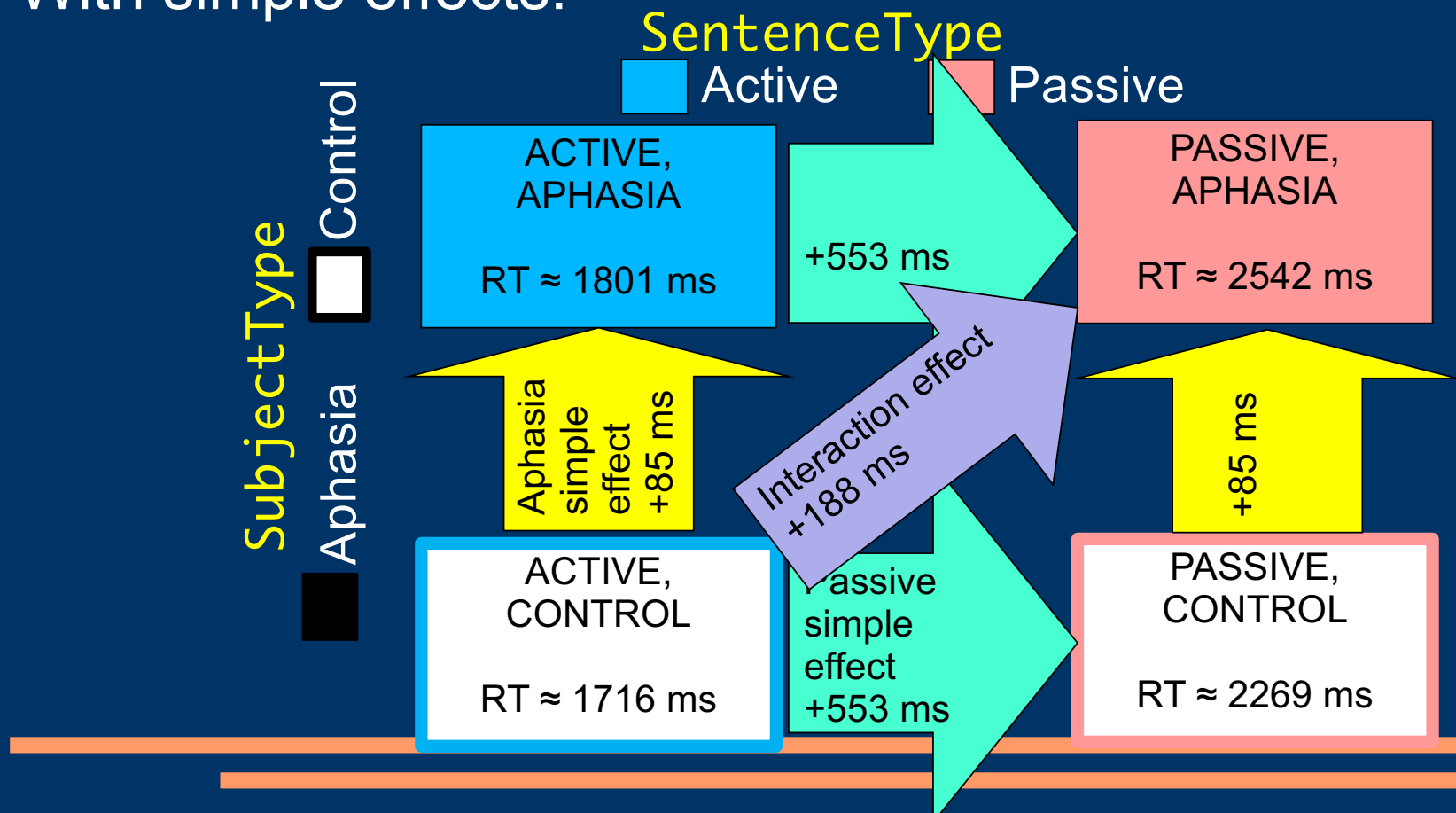
# Post-hoc Comparisons

- The estimates from a model are enough to *fully describe* differences among conditions
- With simple effects:



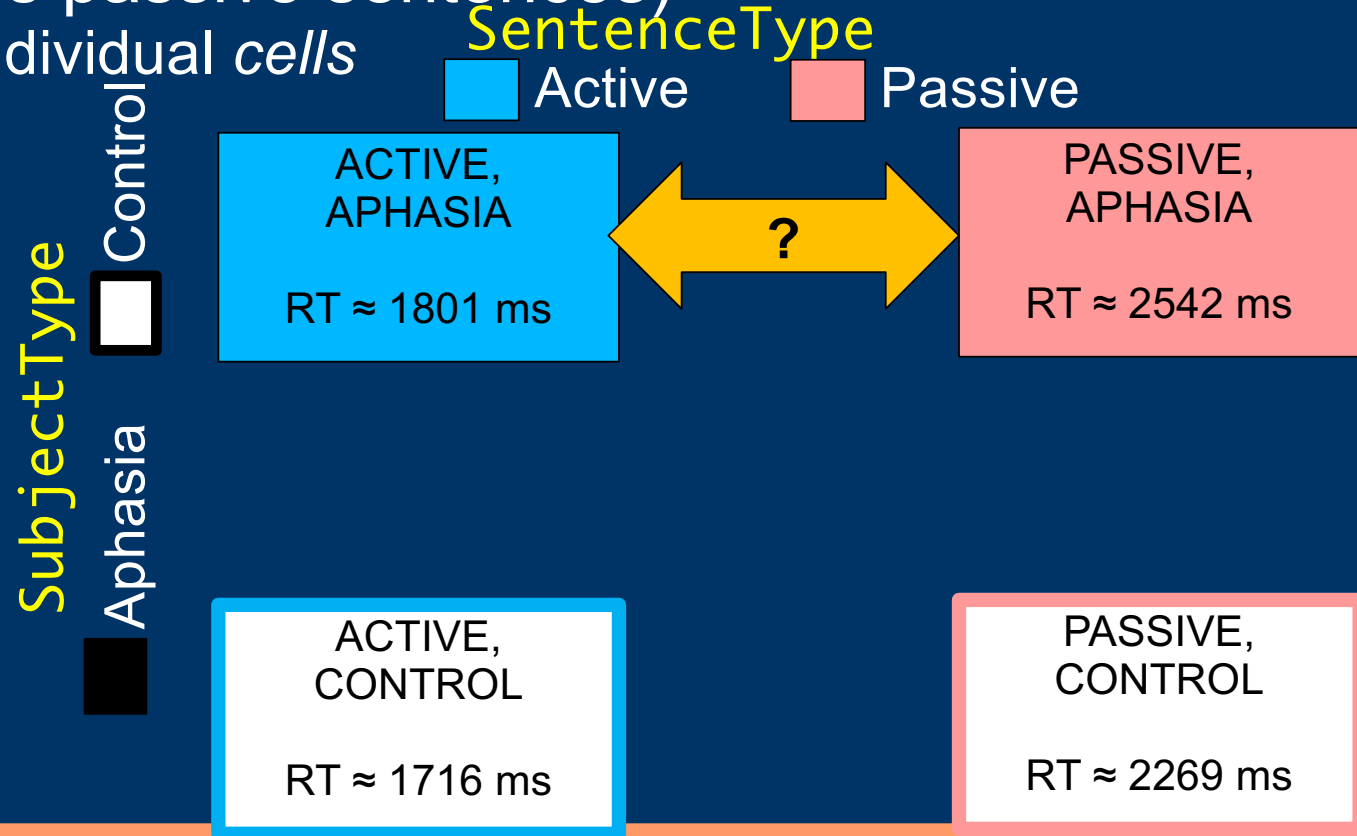
# Post-hoc Comparisons

- The estimates from a model are enough to *fully describe* differences among conditions
- With simple effects:



# Post-hoc Comparisons

- But, sometimes we want to compare individual combinations (e.g., people w/ aphasia seeing active vs passive sentences)
  - i.e., individual cells



# *Week 8.1: Post-Hoc Comparison*

- ✘ Unbalanced Factors
  - ✘ Weighted Coding
  - ✘ Unweighted Coding
  - Post-Hoc Comparisons
    - ➔ Tukey Test
      - Estimated Marginal Means
      - Comparing Marginal Means
  - Lab



# Post-hoc Comparisons: Tukey Test

- But, sometimes we want to compare individual combinations (e.g., people w/ aphasia seeing active vs passive sentences)

- i.e., individual *cells*

Name of the *model* not the original dataframe

- `emmeans(Model.Maximal, pairwise~SentenceType*SubjectType)`

- Requires `emmeans` package to be loaded

The independent variables (for now, all of them)

- `library(emmeans)`

- Uses Tukey test to *correct* for multiple comparisons so overall  $\alpha$  still = .05

```
$contrasts
contrast estimate SE df t.ratio p.value
Active Control - Passive Control -553.4 77.1 29.9 -7.179 <.0001
Active Control - Active Aphasia -84.5 76.4 28.0 -1.107 0.6885
Active Control - Passive Aphasia -826.3 108.0 57.2 -7.650 <.0001
Passive Control - Active Aphasia 468.9 108.1 57.2 4.339 0.0003
Passive Control - Passive Aphasia -272.8 79.9 28.0 -3.415 0.0100
Active Aphasia - Passive Aphasia -741.7 74.1 29.9 -10.010 <.0001
```

Comparisons of each pair of cells

- Which two cells *don't* significantly differ?



# *Week 8.1: Post-Hoc Comparison*

- ✘ Unbalanced Factors
  - ✘ Weighted Coding
  - ✘ Unweighted Coding
  - Post-Hoc Comparisons
    - ✘ Tukey Test
    - ➡ Estimated Marginal Means
      - Comparing Marginal Means
- Lab



# Estimated Marginal Means

- `emmeans` also returns estimated means and std. errors for each cell of the design
- EMMs represent what the means of the different groups would look like *if* they didn't differ in other (fixed or random) variables

```
emmeans
SentenceType SubjectType emmean SE df lower.CL upper.CL
Active Control 1716 75.6 56.4 1565 1867
Passive Control 2269 78.6 56.3 2112 2427
Active Aphasia 1801 74.2 56.1 1652 1949
Passive Aphasia 2542 77.1 56.0 2388 2697
```

## *Estimated Marginal Means*

- As an example, let's add **SentenceLength** as a covariate
  - Task was to read the sentence & judge whether it matches a picture, so length of sentence would plausibly affect time needed to do this
- *Not* perfectly balanced across conditions

|   | SentenceType | SubjectType | SentenceLength_M | RT_M         |
|---|--------------|-------------|------------------|--------------|
|   | <fct>        | <fct>       | <dbl>            | <dbl>        |
| 1 | Active       | Control     | 10.2             | <u>1716.</u> |
| 2 | Active       | Aphasia     | 10.2             | <u>1801.</u> |
| 3 | Passive      | Control     | 9.53             | <u>2269.</u> |
| 4 | Passive      | Aphasia     | 9.53             | <u>2542.</u> |

# Estimated Marginal Means

- New model & estimated marginal means:
  - `model.Length <- lmer(RT ~ 1 + SentenceType * SubjectType + SentenceLength + (SentenceType|Subject) + (1|Item), data=aphasia)`
  - `emmeans(model.Length, pairwise~SentenceType*SubjectType + SentenceLength)`

| \$emmeans    |             |                |        |      |      |          |          |
|--------------|-------------|----------------|--------|------|------|----------|----------|
| SentenceType | SubjectType | SentenceLength | emmean | SE   | df   | lower.CL | upper.CL |
| Active       | Control     | 9.9            | 1676   | 54.3 | 28.7 | 1564     | 1787     |
| Passive      | Control     | 9.9            | 2313   | 56.8 | 28.7 | 2196     | 2429     |
| Active       | Aphasia     | 9.9            | 1760   | 54.3 | 28.7 | 1649     | 1871     |
| Passive      | Aphasia     | 9.9            | 2585   | 56.8 | 28.7 | 2469     | 2702     |

What we'd expect if the sentences were all of equal length

| SentenceType | SubjectType | SentenceLength_M | RT_M  |
|--------------|-------------|------------------|-------|
| <fct>        | <fct>       | <dbl>            | <dbl> |
| 1 Active     | Control     | 10.2             | 1716. |
| 2 Active     | Aphasia     | 10.2             | 1801. |
| 3 Passive    | Control     | 9.53             | 2269. |
| 4 Passive    | Aphasia     | 9.53             | 2542. |

Our raw data (where the average sentence length differs slightly across conditions)

# Estimated Marginal Means

- EMMs are a hypothetical
  - What the means of the different groups would look like *if* they didn't differ in this covariate
  - Like unweighted coding in the case of missingness
- Based on our statistical model, so if our model is wrong (e.g., we picked the wrong covariate), the adjusted means will be too
  - Unlike the raw sample means
  - Need to use some caution in interpreting them
  - Be clear what you are reporting




# Estimated Marginal Means

- Also possible to *test* whether each of these estimated cell means significantly differs from 0
  - `ls_means(model.Length)`
  - Silly in case of RTs, but could be relevant for some other DVs (e.g., preference)

Least Squares Means table:

|                                        | Estimate | Std. Error | df   | t value | lower    | upper    | Pr(> t )  |     |
|----------------------------------------|----------|------------|------|---------|----------|----------|-----------|-----|
| SentenceTypeActive                     | 1717.838 | 38.670     | 29.4 | 44.423  | 1638.801 | 1796.876 | < 2.2e-16 | *** |
| SentenceTypePassive                    | 2448.983 | 40.440     | 29.4 | 60.559  | 2366.324 | 2531.641 | < 2.2e-16 | *** |
| SubjectTypeControl                     | 1994.074 | 55.080     | 28.4 | 36.204  | 1881.312 | 2106.837 | < 2.2e-16 | *** |
| SubjectTypeAphasia                     | 2172.747 | 55.080     | 28.4 | 39.447  | 2059.985 | 2285.509 | < 2.2e-16 | *** |
| SentenceTypeActive:SubjectTypeControl  | 1675.578 | 54.338     | 28.7 | 30.836  | 1564.398 | 1786.757 | < 2.2e-16 | *** |
| SentenceTypePassive:SubjectTypeControl | 2312.571 | 56.833     | 28.7 | 40.690  | 2196.283 | 2428.860 | < 2.2e-16 | *** |
| SentenceTypeActive:SubjectTypeAphasia  | 1760.099 | 54.338     | 28.7 | 32.392  | 1648.919 | 1871.279 | < 2.2e-16 | *** |
| SentenceTypePassive:SubjectTypeAphasia | 2585.394 | 56.833     | 28.7 | 45.491  | 2469.106 | 2701.683 | < 2.2e-16 | *** |


# *Week 8.1: Post-Hoc Comparison*

- ✘ Unbalanced Factors
  - ✘ Weighted Coding
  - ✘ Unweighted Coding
  - Post-Hoc Comparisons
    - ✘ Tukey Test
    - ✘ Estimated Marginal Means
    -  Comparing Marginal Means
  - Lab

# Comparing Marginal Means

- `emmeans` can also test marginal means:
  - `emmeans(model2, pairwise~SubjectType)`

Now, include just *one* variable (for which we want marginal means)



- Effect of one variable *averaging over* the other
  - e.g., aphasic participants (averaging over all sentence types) vs. controls (averaging over all sentence types)
  - These are what *main effects* are testing

```
Semmeans
SubjectType emmean SE df lower.CL upper.CL
Control 1993 66.4 49.5 1859 2126
Aphasia 2171 66.4 49.5 2038 2305

Results are averaged over the levels of: SentenceType
Degrees-of-freedom method: kenward-roger
Confidence level used: 0.95
```



# *Week 8.1: Post-Hoc Comparison*

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