CHAPTER SIX

From Uncertainly Exact to Certainly Vague: Epistemic Uncertainty and Approximation in Science and Engineering Problem Solving

Christian D. Schunn

Contents

Abstract

Epistemic uncertainty is a huge area of scholarship. It has captured the minds of scholars in psychology and many domain-specific studies of reasoning and problem solving. What does it mean to resolve uncertainty? This chapter explores the idea that resolution of uncertainty in complex science and engineering fields frequently ends with approximations rather than precise answers. The chapter begins by examining language to motivate the core

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distinction between uncertainty and approximation. Then, the chapter explores whether the distinction can be defended empirically in reliable and valid coding of speech and gesture data in multiple science and engineering domains. Novice/Expert changes in uncertainty and approximation levels are also explored. Finally, three examinations of temporal patterns of co-occurrence with uncertainty and approximation are presented in multiple problem-solving domains to provide an overall model of uncertainty being transformed to approximation through spatial reasoning and mental simulations.

1. Introduction

Studies of behavior in the real world have consistently found that uncertainty has a large influence on behavior. For example, there is a whole subdiscipline of naturalistic decision making focused on judgment under uncertainty (e.g., Klein, 1989). Indeed, there are many pragmatic implications for better understanding uncertainty. For example, the ways in which experts reason about uncertainty in future forecasts under different actions, the ways in which experts choose to communicate this uncertainty to the voting public or the future voting public (in schools), and the ways in which the public understand the uncertainty will also influence critical decisions being made by politicians today (Friday, 2003). While much progress has been made, there is still much to be learned about how uncertainty influences behavior.

There are several taxonomies of uncertainty types in existence. Some come from psychology judgment and decision-making research (Berkeley & Humphreys, 1982; Howell & Burnett, 1978; Kahneman & Tversky, 1982; Krivohlavy, 1970; Lipshitz & Strauss, 1997; Musgrave & Gerritz, 1968; Trope, 1978). Others come from a broad array of particular disciplines, such as geography (Abbaspour, Delavar, & Batouli, 2003), ecology (Regan, Colyvan, & Burgman, 2002; Regan, Hope, & Ferson, 2002), finance (Rowe, 1994), management (Priem, Love, & Shaffer, 2002), geospatial information systems (Plewe, 2002), law (Walker, 1991, 1998), acoustics (Egan, Schulman, & Greenberg, 1961), medicine (Brashers et al., 2003; Hall, 2002), consumer choice (Sheer & Cline, 1995; Urbany, Dickson, & Wilkie, 1989), driving behavior (Vlek & Hendrickx, 1988), educational research (Webster & Bond, 2002), negotiation (Bottom, 1998), military tactics (Cohen, Freeman, & Thompson, 1998), and statistics. The sheer number of such domain-specific accounts makes clear how complex and central uncertainty resolution is to problem solving. These taxonomies typically emphasize the different sources of uncertainty—reasons why a problem solver might be uncertain.

A different issue from the sources of informational uncertainty (objective ambiguity in the existing information) is psychological uncertainty

(Jousselme, Maupin, & Bossé, 2003), the internal feeling of being uncertain about information which may or may not be objectively uncertain. Presumably, it is this internal state that directly influences behavior: making choices (Kahneman & Tversky, 1982), avoiding situations, or driving new problem solving aimed at reducing the uncertainty levels (Trickett, Trafton, & Schunn, 2009).

Of course the underlying source of informational uncertainty may also influence behaviors aimed at reducing the psychological uncertainty. For example, Lipshitz and Strauss (1997) found that decision makers react differently to three different types of uncertainty: inadequate understanding, incomplete information, and undifferentiated alternatives. Inadequate understanding is addressed by collecting more information; incomplete information is typically addressed through assumption-based reasoning; and undifferentiated alternatives are resolved by weighing pros and cons in more depth. But there still remains the question, what is the psychological nature of the uncertainty itself.

In this chapter, I would like to argue for a distinction not previously emphasized in discussions of uncertainty: the difference between psychological uncertainty and psychological approximation, referred to as uncertainty and approximation for the rest of the chapter. Uncertainty is the lack of knowledge about possible states (e.g., is the temperature 18 °C or 19 °C?). Approximation declares a state as falling with a range (e.g., the temperature is between 18 °C and 19 °C). At first blush, this distinction appears bizarre and without conceptual merit. From an information theoretic or logical perspective, there is no difference between the two. However, I will argue that this distinction is a critical psychological distinction in science and engineering problem solving. I will show that uncertainty and approximation are discriminable constructs in behavior, that they systematically occur in different places, and that common problem-solving strategies in science engineering serve primarily to convert from uncertainty into approximation. Thus, to ignore this seemingly nondistinction is to ignore a core feature of very important types of problem solving. Further, psychological research coding uncertainty from speech or gestures will likely falsely include approximation behaviors with uncertainty behaviors unless the distinction between uncertainty and approximation is salient.

2. Linguistic Pragmatics of Uncertainty and Approximation

To first provide some intuitions regarding the difference between uncertainty and approximation, consider the following everyday conversational examples, focusing on the responses in italics.

(1) Speaker 1: How old is she?
Speaker 2: *40?* She was born in January of 1969.
(2) Speaker 1: How old is she?
Speaker 2: *Early forties.*
(3) Speaker 1: How old is she?
Speaker 2: *Forty plus or minus 2.*
(4) Speaker 1: How old is she?
Speaker 2: *Early forties?*

In (1), speaker 2 has all the information required to provide a precise answer to the question, actually provides a precise answer (40) that is accurate (in 2009), and yet is psychologically uncertain, as noted in providing an answer in a question format. By contrast, in (2), speaker 2 provides an approximate answer (early forties), but with no indicated psychological uncertainty. Example (3) is a more academic-speak response with the same key characteristics as (2): approximation but no indicated uncertainty. Example (4) shows that one can have approximation and uncertainty.

From a pragmatics perspective, speaker 2's responses in (2) and (3) are quite reasonable in that they answer the question with precision that is likely sufficient for speaker 1's needs and they set clear bounds on the possible actual values. By contrast, speaker 2's response in (1) of "40?" does not set bounds on the possible actual values, leaving open the possibility of a much wider range of actual age.

Human languages contain many categorical terms that represent approximations on quantitative entities. For example, 50s, 19th century, teenage, early childhood, average height, room temperature, steep, and next door represent approximate quantities of age, time, height, temperature, slope, and location. Moreover, each of those terms represents approximations that are much more approximate than humans can perceive psychologically. That is, we could think and express ourselves more precisely than with those terms, but we on occasion choose not to.

Interestingly, both uncertainty and approximation can be indicated through the use of hedge words added to more precise terms, although the two use different hedge words. Consider the following two examples.

(5) Speaker 1: How old is she?
Speaker 2: *Maybe 40.*
(6) Speaker 1: How old is she?
Speaker 2: *Almost 40.*

In (5), speaker 2 uses the hedge "maybe" to indicate uncertainty in the precise response with no provided bounds on how far the answer could be off, whereas in (6), speaker 2 uses the hedge "almost" to indicate approximation in the precise response and pragmatic conventions suggest the age is less than 40 and unlikely to be more than 1 or 2 years below 40 (i.e., it might be 38 or 39). Overall there appear to be many more ways of expressing uncertainty through hedge words than through direct terms indicating approximate or uncertainty quantities, perhaps reflecting subdimensions of uncertainty (e.g., probability distributions or average versus peak intensity) or approximations that do not have convenient linguistic labels (e.g., temperatures between 14 and 16 °C, or ages between 43 and 45). As a result, our coding from speech tends to focus on hedge words.

The examples above have generally focused on uncertainty and approximation cases that are not informationally equivalent in that the possible range for the uncertainty cases was larger than the possible range for the approximation cases. There are two important points to note about this observation. First, the definitional difference is NOT about relative ambiguity in quantity. Reverse cases are possible: one could be uncertain whether the temperature is 14 or 15 °C and one could assert an approximation of 13–18 °C. Uncertainty is about psychologically not knowing something, whereas approximation is about asserting a range.

Second, it happens to be the case that problem solving tends to reduce the possible range for which one is uncertain to a smaller range that is the approximation. For example, a problem solver might begin with an uncertainty of a very general form (what is the temperature?) or of a wide range (what is the temperature, but knowing that it is a Fall afternoon in New York) and then through some data collection from various sources and reasoning finish with a smaller possible range of 14–16 °C. In other words, problem solving (especially in engineering and science for which some level of precision is required) serves to move information ambiguity from unacceptable levels to acceptable levels for the task at hand. This point will be further examined in Section 6.

3. Coding Approximation and Uncertainty from Speech

In a different sense of pragmatics, the distinction of approximation versus uncertainty is useful to psychologists (or various other scientists of behavior) only if the distinction can be made reliably from observed behavior and is associated with interesting patterns of behavior. Focusing on the first issue, in a number of projects we have found that uncertainty and approximation can be reliably coded from free speech, either in the form of think-alouds during problem solving or in the form of natural conversations.

3.1. Conversation Coding in Engineering Design Team Meetings

In Christensen and Schunn (2009), we coded for uncertainty and approximation from the many hours of conversation transcripts of an innovative engineering design team during their weekly design team meetings. Our approach to coding uncertainty and approximation was syntactical with verification, building on a hedge-word uncertainty coding approach developed with Trickett, Trafton, Saner, & Schunn (2007). Examples of uncertainty hedge words are "probably," "sort of," "guess," "maybe," "possibly," "don't know," "[don't] think," "[not] certain," and "believe," Examples of approximation hedge words are "pretty much," "virtually," "generally," "frequently," "usually," "normally," "basically," and "'almost." (Actually, we searched for the Danish equivalents of these terms, as the team being studied was Danish.) In either the uncertainty or approximation cases, each instance of the hedge words was examined to make sure it was being used in an uncertainty or approximation sense; if so, the segment containing these hedge words were coded as "uncertainty present" or "approximation present."

Interrater reliability for this approach was extremely high, with kappas of 0.95 for uncertainty coding and 0.96 for approximation coding. As a simple validation of each construct and the distinction between the two, we also looked at the adjacency relationships between codes from one transcript segment to the next. The assumption is that mental states of uncertainty or approximation are "sticky" in that they will tend to continue longer in time than just one segment. Uncertainty and approximation are conceptualized as being about particular quantities and thus co-occurrence will not be perfect, but conversations will tend to continue regarding a given quantity, so there should be some continuity. As can be seen in Table 1, this continuity was clearly shown for both approximation and uncertainty (both trends are statistically significant). Further, taking into account the base rates of uncertainty and approximation, there was no tendency for approximation to immediately follow uncertainty or *vice versa*.

3.2. Conversation and Interview Coding in Science and Applied Science Data Analysis

Another project involved a similar coding procedure applied to two different domains of science and two different domains of applied science (Schunn, Saner, Kirschenbaum, Trafton, & Littleton, 2007; Trickett et al., 2009).

Table 1 Rates of Uncertainty and Approximation in the Next Transcript Segment as a Function of their Presence in a Given Segment.

Current segment	Uncertainty in next segment	Approximation in next segment
Uncertainty (n = 247)	16%	4%
Approximation (n = 308)	3%	8%
Neither (n = 5616)	5%	4%

The first domain involved conversations of earth scientists working at the Jet Propulsion Lab analyzing data as it came down from Mars from two robotic rovers—the Mars Exploration Rovers. The coded conversations were of impromptu meetings held throughout the day between groups of 2–10 scientists from several different disciplines (soil and rock scientists, geochemists, geologists, and atmosphere scientists). There were a number of video cameras off to the sides of the large data analysis rooms. The scientists had given informed consent for this video collection, but the cameras were relatively small, discretely located, and constantly present. Thus, the scientists generally forgot about the existence of the cameras and the transcripts likely capture very typical problem-solving behaviors in this context.

The remaining three domains were 13 cognitive neuroscientists analyzing fMRI data (fMRI), 18 meteorologists making weather predictions (Weather), and 22 navy officers localizing an enemy submarine using only passive sonar (Submarine). These datasets involved cued think-alouds of novices (apprentices in the domain, not random undergraduates), intermediates, and experts. Participants were videotaped as they analyzed their data on computers (their own data in the case of fMRI, canned data in the case of Weather and Submarine). After 30–45 min of data analysis, they were then shown three or four different minute-long snippets of the videotape that corresponded to critical decision-making moments during data analysis. The scientists were asked to explain what they knew and did not know at that moment in time. Sometimes problem solvers given think-aloud instructions fall silent exactly at the interesting moments in time, especially when the task is long and complex. This cued-recall method was designed to capture additional information about these more interesting moments.

Across these four domains, we used the same hedge-word technique for coding uncertainty and approximation from the transcribed speech. In all cases, we obtained interrater reliability kappas of greater than 0.8 for both uncertainty and approximation.

The know/do not know probes in the fMRI, Weather, and Submarine domain studies provide another validation of the distinction between uncertainty and approximation (and coding was done blind to question context). One would expect that there would be more uncertainty speech cues in response to the "what did you not know?" question than in response to the "what did you know?" question. An opposite pattern is expected for approximation. Figure 1 presents the results. In all three domains, the predicted pattern was obtained and statistically significant for both uncertainty and approximation codes.

Thus, uncertainty versus approximation is a distinction that can be made reliably in various science and engineering settings from verbal data in the form of think-alouds or natural conversations. Simple patterns in the data clearly suggest that uncertainty and approximation are temporally coherent

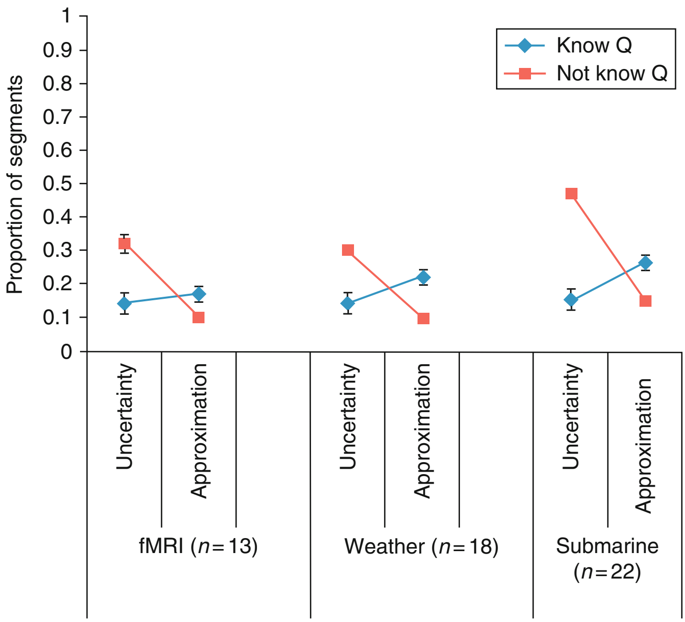

Figure 1 The proportion of speech containing uncertainty or approximation (with SE bars).

within categories and temporally dissociable across categories. Finally, uncertainty and approximation speech appears under expected conditions.

4. Coding Uncertainty from Gestures

In science and engineering, much of the data is inherently visual–spatial or is displayed in spatial format (e.g., graphs of temperature varying with time). Thus, much of the uncertainty and approximation are expressed about visual–spatial quantities. Because science and engineering have formalized much if not all of the quantities and relationships in symbolic formats (e.g., terms for particular quantitative data patterns, equations to represent quantitative data patterns), much can be studied from coding speech from conversations and think-alouds. However, it is likely that considerable representing, reasoning, and problem solving in science and engineering is also happening in a visual–spatial, nonverbal format.

How does one measure internal problem solving on visual–spatial content? All measures of mental representations and problem solving are necessarily indirect. Verbal report is one general source of data regarding mental representation and problem solving. However, for visual–spatial content, it is a suspect source, as verbal data are generally thought to capture the contents of verbal working memory, not spatial working memory (Ericsson & Simon, 1993). Retrospective or intermittent drawings can be another source of data. However many people are not very skilled in drawing, and it is likely that such drawings would influence reasoning more than verbal protocols would because the drawing process is much less automatic and the results of the process are more permanent (i.e., is an object that can be used itself in problem solving). Scientists and engineers do draw (by hand or via a computer) regularly, but not densely enough in time to constitute a good online measure of thinking. A third approach is to use spontaneous gestures. In addition to serving as a communicative act between speaker and listener, spontaneous gestures are thought to be an online measure of mental representations much like verbal protocols (Alibali, Bassok, Solomon, Syc, & Goldin-Meadow, 1999; Alibali & Goldin-Meadow, 1993; McNeill, 1992). In spatial tasks, in fact it is disruptive to the problem solver to prevent gesturing from occurring.

In a later section, I will consider more complex representational content of gestures. But first, I want to focus on gestures as a direct measure of uncertainty or approximation. There are a number of taxonomies of gesture. One common distinction (McNeill, 1992) is between beat gestures (rhythmic, repetitive gestures often co-timed with speech), deictic gestures (pointing to things in the world around the speaker such as the clock on the wall over *there*), iconic gestures (gestures that are literal physical presentations of things absent, such as hand-shape holding an implied glass), and metaphoric gestures (a spatial representation of a nonspatial object, such as pointing behind oneself to represent back in time). All of these gestures can have many phases (McNeill, 2005): preparation (optional), prestroke hold (optional), stroke (obligatory), stroke hold (obligatory if the stroke is static), poststroke hold (optional), and a retraction (optional). Uncertainty gestures are typically wiggling movement in the stroke of an iconic or metaphoric gesture that represents some quantity (i.e., normally would be static). For example, a pinch indicating a size together with wavering the size or wiggling the hand. In this way, the uncertainty gesture is discriminable from a beat gesture in that there is content to the gesture beyond the movement in an uncertainty gesture of this type but the beat gesture does not have content beyond the movement (i.e., the hand does not indicate a size or distance or volume). However, another common form of an uncertainty gesture involves a shoulder shrug. In this case, one must rely on speech or perhaps another gesture to determine which quantity is producing uncertainty.

We have not yet coded approximation gestures, but I could easily imagine width of gestures indicating the approximations on quantities (e.g., between fingers of one hand or between hands). Further, I could easily imagine that some of the wiggling gestures that we previously coded as uncertainty gesture might actually be approximation gestures (e.g., specific movement between particular points).

In this section, the uncertainty gesture data are used as a cross-validation: do uncertainty gestures co-occur with uncertainty speech (and less so with approximation speech)? It is important to note, however, that speech and gesture need not always line up perfectly. Speech–gesture mismatches do happen and are not thought to be simply noise in interpretation; rather they are thought to signal coactivation of competing ideas/strategies (Alibali & Goldin-Meadow, 1993; Alibali et al., 1999).

We examined the overlap between uncertainty gesture and speech in the four science/applied science domains. Figure 2 presents the percentage of segments with uncertainty gestures when the segment has speech uncertainty or speech approximation present/absent in the three domains with cued-recall think-alouds. The first thing to note is that uncertainty gestures are relatively less common that uncertainty or approximation speech codes. The second thing to note is the strong cross-validation across all three domains: uncertainty gestures occurred much more often when uncertainty

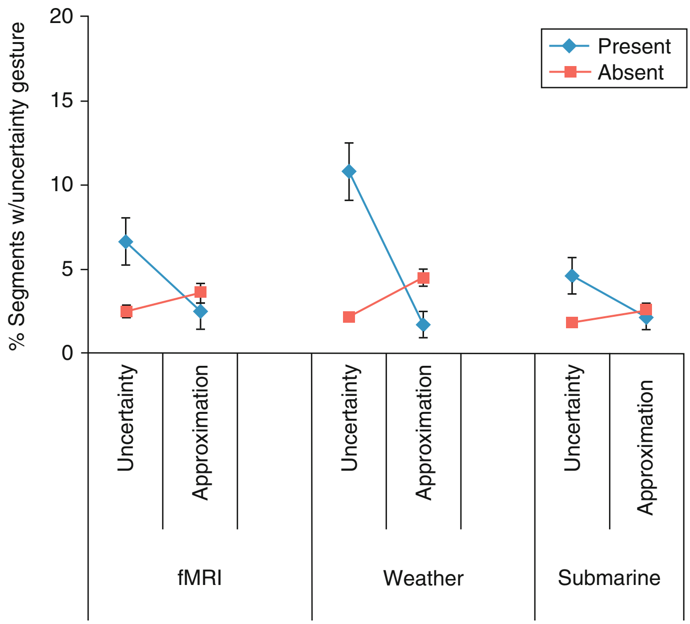

Figure 2 Percentage of segments (with SE bars) with uncertainty gestures as a function of uncertainty and approximation appearing the speech segment for the domains of fMRI, Weather, and Submarine. The "Present" cases each involve approximately 300 segments and the "Absent" cases each involve approximately 1000 segments.

speech occurred ($ps < 0.01$ in all three), whereas uncertainty gestures had no consistent relationship to whether approximation speech occurred (only the Weather difference is statistically significant, $p < 0.05$, and in the reverse direction from the uncertainty speech pattern).

In the naturalistic science conversation Mars data, 5.3% of segments with an uncertainty code had an uncertainty gesture, in comparison to 2.7% of segments without an uncertainty speech code ($X^2(1) = 16.0$, $p < 0.001$)—in other words, uncertainty gestures occur twice as often in the context of uncertainty speech. There is an association between approximation statements and uncertainty gestures ($X^2(1) = 6$, $p < 0.02$), but the association is weaker; uncertainty gestures are only 50% more likely to appear in the context of approximation speech than without approximation speech. Overall, then, uncertainty speech and uncertainty gesture are clearly related, whereas uncertainty gesture and approximation speech have a smaller ambiguous relationship, perhaps reflecting some miscoding of approximation gestures as uncertainty gestures.

To further validate that there is indeed something called an uncertainty gesture that signals an internal state of uncertainty, we can examine gesture data from the fMRI, Weather, and Submarine domains, focusing on the relative frequency of uncertainty gestures in response to the Know and Not know questions. In all three domains, 2% of segments co-occurred with an uncertainty gesture during the response to the Know question. In response to the Not know question, rate of uncertainty gestures increased significantly ($ps < 0.05$) and generally more than doubled (5% fMRI, 8% Weather, and 4% Submarine).

5. Uncertainty, Approximation, and Expertise

With multimodal affirmation of the somewhat surprising distinction between uncertainty and approximation in hand, we can now explore a third pragmatic question: whether the distinction plays a useful role in explaining behavior, in this case behavior of scientists and engineers. One intuition might be that uncertainty and approximation should differ by expertise levels, with experts showing more approximation and less uncertainty. Indeed, some expertise literature focuses on the amazing swiftness with which experts can see problems in terms of solutions features and solve problems (Chase & Simon, 1973; Chi, Feltovich, & Glaser, 1981; Gobet & Simon, 1996; Larkin, McDermott, Simon, & Simon, 1980). However, much of the expertise literature making those claims focuses on well-defined problems such as simple physics problems that are purely education tasks rather than problems an expert would actually encounter. The actual life of an engineer and scientist is much less clear-cut. Indeed, experts in most domains deal with a very uncertain world, hence the large focus on decision making under uncertainty within naturalistic decision-making research. While an expert certainly can produce better solutions and in less time than novices in the much more ill-defined contexts of real science and engineering problem solving (Moss, Kotovsky, & Cagan, 2006; Schunn & Anderson, 1999; Voss, Tyler, & Yengo, 1983), it is not a matter of recognition of simple solutions for the expert. Issues involving uncertainty must be recognized and then resolved through complex processes, like mental simulation. It may be that novices do not even recognize what is uncertain about the current situation, treating initial point estimates as fact rather than estimates.

The fMRI, Weather, and Submarine cued-recall dataset provides an opportunity to look at expertise effects on rates of uncertainty and approximation across domains to look for consistent patterns. We defined novices as those individuals having already learned enough of the task basics to be able to complete the analysis tasks on their own (e.g., analyze an fMRI dataset, make a weather prediction). Experts were those at the top performance levels. Intermediates were those with considerable experience beyond novice levels, but far from expert levels in that domain. In our participant pool for that study, only fMRI involved all three performance levels. The Weather data included novices (juniors and seniors in weather forecasting school) and experts, and the submarine data had only intermediates and experts (both were submarine officers with field experience, but to varying degrees).

Figure 3A presents the levels of uncertainty speech across the expertise levels in each domain, and Figure 3B presents the levels of approximation speech across the expertise levels in each domain. There are a few statistically significant differences, but no consistent differences across the three domains. For example, in the submarine domain, the experts have the highest levels of uncertainty, whereas in the Weather domain they have the lowest. In all three domains, the differences by expertise level are small. The best overall conclusion to draw is that recognizing uncertainty may itself be a kind of expertise and the frequency of uncertainty comments will involve two opposing trends as a function of expertise: (1) experts likely recognize more facets of uncertainty and (2) experts are better able to resolve the uncertainty. How those opposing trends balance in aggregate will depend on the complexities of the task at hand. That is, I doubt that even a whole domain will have general patterns by expertise level on amount of uncertainty as some tasks within the domain will involve more detection challenges and others will involve more resolution challenges.

In support of this idea that there are recognition and resolution elements to uncertainty, one can divide a problem-solving session into two halves (early and late). If experts recognize uncertainty more readily and then are able to resolve it, we would expect their uncertainty levels to go down over

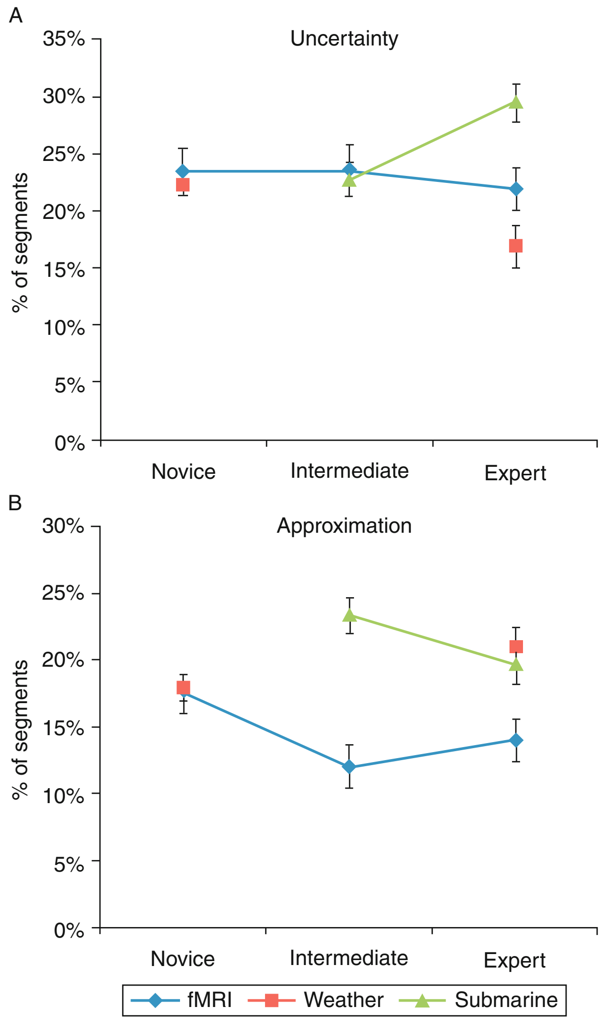

Figure 3 The percentage of segments (with SE bars) showing (A) uncertainty speech and (B) approximation speech as a function of domain and expertise levels.

time. By contrast, if novices are struggling to even see the issues of uncertainty and are less able to resolve these uncertainties, then we would expect novices' uncertainty levels to go up over time. Figure 4 presents relevant uncertainty speech data from the fMRI domain. We see that uncertainty levels do go up for novices and intermediates whereas they go down (directionally but not statistically significant) for experts. Similar (small) interactions of early/late by expertise levels on uncertainty levels could also be seen in the other domains.

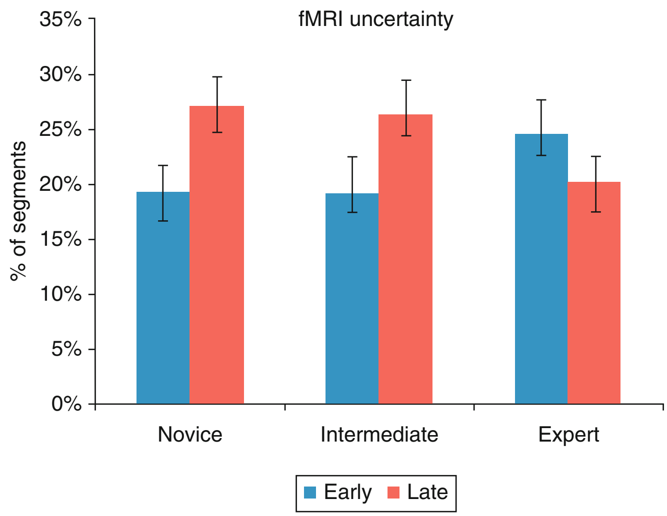

Figure 4 The percentage of segments (with SE bars) in the fMRI domain showing uncertainty speech as a function of early and late minutes of problem solving and expertise levels. *N*s for each percentage vary between 130 and 300 segments of speech.

Of course, more fine-grained coding of uncertainty detection and resolutions' strategies included in this analysis of expertise effects on uncertainty and approximation would provide a more conclusive perspective on why uncertainty is not clearly associated with expertise and appears to be changing in different ways over time. We have done this coding in all four science/applied science domains. We specifically looked at what indicators were used to identify sources of uncertainty. For example, uncertainty becomes apparent when different data sources (as in two weather models) produce conflicting results, or when one data source produces seemingly impossible results (as in brain activation outside the skull). A number of such general indicators could be found. We also looked at the strategies used to resolve the uncertainty. It turns out that there are a very large number of such general strategies that can be observed, some more spatial in form, others less spatial. There are some expertise differences by strategy within each of the domains, but the differences are not consistent across the domains, probably because different strategies are differentially effective within each domain. In sum, uncertainty and approximation have a complex relationship to expertise levels rather than a simple-linear trend relationship, and the relationship likely depends upon the ease in which uncertainty is detectable and resolvable in a given setting given available tools and strategies.

6. From Uncertainty to Approximation via Spatial Reasoning

Thus far, I have focused on the differences between uncertainty and approximations—how they are not the same. Now I would like to focus on the positive relationship that they have to one another. In particular, the theoretical assertion that I would like to make is that uncertainty and approximation have an input/output relationship to one another with spatial reasoning lying in between, at least in science and engineering problem solving. The next three sections build up the evidence for this theoretical assertion. Section 6.1 examines verbal protocol evidence that uncertainty leads to mental spatial transformations. Section 6.2 examines gesture data to examine the relative temporal relationship of uncertainty, approximation, and spatial mental representations. Section 6.3 focuses on a particular kinds of spatial problem solving that appears to be used to move from uncertainty to approximation in problem solving.

6.1. Uncertainty and Verbally Coded Spatial Transformations in Basic and Applied Science

In Trickett et al. (2007), we used the syntactic approach to coding uncertainty in speech and then also coded the speech for the presence of spatial transformations. Spatial transformations are mental operations a person mentally performs on an internal representation or an external visualization (on paper or computer screen). Typical spatial transformations are creating a mental image, adding or deleting features to an image, rotating or moving an object, or making comparisons between different views. Table 2 provides examples of uncertainty codes and spatial transformations from utterances.

In one study, we examined the relative co-occurrence of spatial transformations with uncertainty in speech for an expert (over 16 years of experience) making a weather forecast while giving a think-aloud (approximately 50 min of speech to analyze). We found that the rate of spatial transformations was almost twice as high during speech with uncertainty markers than in speech without uncertainty markers. Follow-up work with more experts and novices (although still trained in weather forecasting) found that both experts and novices showed this pattern but the effect is much larger in experts than in novices.

In the second study of Trickett et al. (2007), a more rigorous test was conducted using the fMRI and Weather cued-recall dataset described earlier, but in a slightly different way. Here, spatial transformations were coded from the think-aloud speech of the problem solver doing the initial fMRI data analysis or weather forecast. Then relative levels of uncertainty were coded

Table 2 Examples of Spatial Transformations and Certain and Uncertain Utterances with Indications of Uncertainty in Bold and Spatial Transformation in Italics (adapted from Trickett et al., 2007).

Utterance	Code	Spatial transformations (ST)
Nogaps [a mathematical model] has some precipitation over the Vancouver/Canada border (while viewing a visualization)	Certain	No ST
This is valid today	Certain	No ST
Possibly some rain over Port Angeles	Uncertain	No ST
And then uh, at Port Angeles, there's gonna be some rain up at the north, and if that **sort of** *sneaks down*, we **could see** a little bit of restriction of visibility, but only down to 5 miles at the worst	Uncertain	ST: mentally moving rain [sneaks down]
I **don't think** the uh front's gonna get to Whidbey Island, but *it should be sitting right about over Port Angeles* **right around** 0Z this evening	Uncertain	ST: mentally moving front/animation

from the responses to the cued-recall questions. A minute of problem solving was determined to be a high-uncertainty minute if the cued-recall phase for that minute generated a high percentage of uncertainty speech codes whereas the minute of problem solving was determined to be a low-uncertainty minute if the cued-recall phase generated a low percentage of uncertainty speech codes. Thus, spatial transformations and relative uncertainty levels were coded from different datasets (and also by coders from different labs). Further, in our prior analyses, uncertainty speech was coded at the utterance-by-utterance level, whereas the underlying uncertainty is likely more pervasive (i.e., the speech codes may be considered as the tip of the uncertainty iceberg). This designation of entire minutes as being high or low uncertainty addresses this issue. Indeed, using this approach to examining uncertainty against spatial transformations, we found that spatial transformations were over four times greater during high-uncertainty minutes than during low-uncertainty minutes.

6.2. Association of Uncertainty and Approximation with Spatial Gestures in Basic Science

In addition to potentially capturing uncertainty or approximation in thinking, gestures can also capture spatial problem solving. If spatial problem solving takes place between uncertainty and approximation, then we should see more spatial gestures between uncertainty and approximation. But what kind of gestures should we expect to see?

There are many different kinds of spatial representations. The spatial reasoning literatures (in cognitive psychology, developmental psychology, and cognitive neuroscience) frequently make distinctions between large scale and small scale, egocentric and exocentric (or allocentric), and between two-dimensional and three-dimensional visual–spatial representations. The work described in the prior section suggests that spatial transformations are frequently used by problem solvers to resolve uncertainty. The cognitive neuroscience literature has suggested for multiple decades that a ventral (‘‘what’’ or object type information) and dorsal (‘‘where’’ or object location information) pathway is a critical distinction in thinking about visual–spatial processing (Ungerleider & Mishkin, 1982). Later work (for a review, see Kosslyn, Ganis, & Thompson, 2001) has suggested that the parietal lobe (part of the where pathway) is heavily involved during spatial transformations (e.g., during mental rotation). Other neuroscience work has suggested that the parietal lobes are specifically involved in small 3D representations of space (Previc, 1998). By inference, one would expect to see high numbers of small 3D manipulation gestures following uncertainty speech and preceding approximation speech if mental transformations are doing the work of going from uncertainty to approximation and these gestures map onto mental transformations of this type.

We have tested exactly this prediction in the Mars data described earlier. In addition to coding uncertainty gestures, we also coded for several other kinds of spatial and nonspatial gestures. The most common spatial gesture was small-scale 3D gestures. Based on a theoretical framework I have developed elsewhere (Harrison & Schunn, 2002), these are called manipulative gestures. Specifically, manipulative gestures are gestures that place objects and activity in a nearby space, such that the problem solver can actually manipulate or place the imaginary objects. Examples of manipulative gestures included one-handed gestures of a brain region (a cupped hand facing up) and two-handed gestures showing dusting billowing over a small crater lip (the left hand flat and held still at an angle to represent the crater lip and the right hand swooping over the left with fingers wiggling to show the billowing dust). Gestures in which the hand shape suggests placing or holding as opposed to strictly pointing were also coded as manipulative.

To examine the relative temporal arrangement of uncertainty speech and manipulative gestures, we divided speech segments into several different types: segments with uncertainty speech (exact), segments that have uncertainty 1–5 segments before the current one (before), segments that have uncertainty 1–5 segments after the current one (after), segments with both before and after relationships, and then segments not near uncertainty speech (distant), which can be thought to establish a base rate of spatial gestures. We then examined the rate of manipulative gestures during each of these segment types. The same analysis was also done for gestures' temporal relationship to approximation speech codes.

Figure 5 presents the results of this analysis. Focusing on manipulative gestures relative to uncertainty speech, the highest rates of manipulative gestures occur when the uncertainty speech occurs before the current segment. The "during" cases (both and exact) have lower rates of manipulative gestures, and the after case has a manipulative gesture rate similar to segments distant from any uncertainty speech. Thus, it appears that uncertainty speech occurs primarily before manipulative gestures and not after. For approximation speech and manipulative gestures, a different pattern appears. Here manipulative gestures are elevated anywhere near approximation speech, but particularly right during it. Thus, the approximation representations appear to occur simultaneously with the spatial transformation work. Overall, these data are consistent with the view that uncertainty leads to spatial transformations that produce approximation results.

6.3. From Approximation to Uncertainty via Mental Simulations in Engineering Design

The Christensen and Schunn (2009) examination of uncertainty and approximation in engineering design also examined the temporal relationships of uncertainty and approximation relative to mental problem solving.

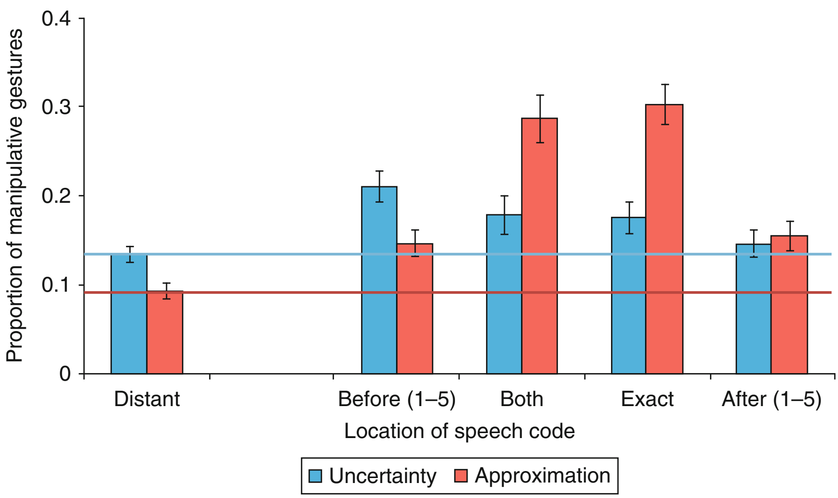

Figure 5 The proportion of speech segments (with SE bars) with manipulative gestures as a function of whether uncertainty speech occurred before (within five segments), after (within five segments), both before and after, or exactly in the segment. Each proportion is based on between 300 and 600 segments of data, except the "distant" proportions, which are based on 1300 segments.

In particular, we focused on a kind of problem solving that was quite frequent in engineering design team meetings: mental simulations. These mental simulations happened approximately once every two minutes on average during the meetings. In the part of the meetings in which the conversation was focused on active design of the product (vs. future planning), the rate went up to one per minute.

The coding scheme for mental simulations was adapted from the coding scheme developed by Trickett and Trafton (2007) for coding scientist mental simulations. A mental simulation is a mentally constructed model of a situation, building upon objects in memory of mental modifications of objects currently present. A defining feature of a mental simulation is that something is "running," that is, that the process alters the representation. The simulation is not just asking a "what if" question. It also provides an answer about whether something will work, what a resulting feature will be, etc. Mental simulations involve a sequence of three critical elements: creating an initial representation, running the representation (elements or functions are changed, added, or deleted), and a final changed representation. Each segment was coded as "mental simulation" or "no mental simulation," along with the separate steps. Table 3 presents an example mental simulation from the transcripts coded into three components. The interrater reliability for coding mental simulations was quite high, kappa = 0.9.

Figure 6 presents the rate of uncertainty and approximation speech as a function of step during a mental simulation. The base rate of (speech coded) uncertainty is 8% in this dataset. The rate of uncertainty speech was statistically significantly higher than the base rate at the initial representation and

Table 3 An Example Mental Simulation from the Engineering Design Domain (from Christensen & Schunn, 2009).

Step	Utterance
Initial representation	Could you add something so that you couldn't close this thing because there would be something in the way when you try to fold this way...
Run	But if this thing goes this way, then it is in a position to allow the ear to enter... But then I just don't know how it should be folded... 'cause if it is folded this way then it will come out here...then it should be folded unevenly somehow...You should fold it oblique.
Changed representation	It wouldn't make any difference one way or the other. It would fold the same way, and come out on this side the same way.

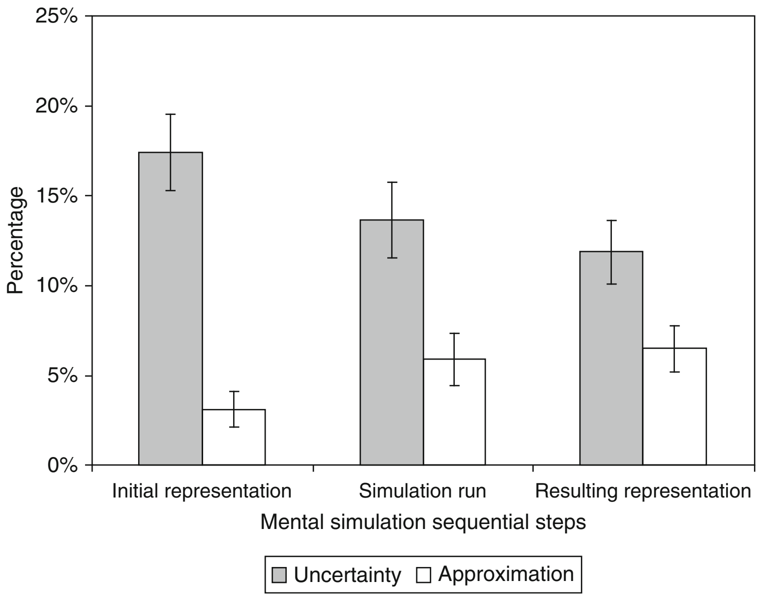

Figure 6 Percentage of segments with uncertainty and approximation by mental simulation sequential step, with SE bars (from Christensen & Schunn, 2009).

during the simulation run, but not during the resulting representation. By contrast, approximation speech was at baseline levels (3%) at the initial representation step, and rose to significantly higher levels by the resulting representation. Thus, the temporal patterns are perfectly consistent with the hypothesis that mental simulations have the effect of turning uncertainty into approximations.

More recently, Linden and Christensen (2009) coded for uncertainty and mental simulations in a different engineering design dataset and found exactly the same results—a reduction of uncertainty in the initial representation down to base levels of uncertainty by the resulting representation state of the mental simulation.

7. Summary and Discussion

Epistemic uncertainty is a huge area of scholarship. It has captured the minds of scholars in psychology and many domain-specific studies of reasoning and problem solving, presumably because uncertainty is ubiquitous or nearly so in real-world problem solving. With all the rich distinctions that could be made about uncertainty, I began this chapter with a different psychological distinction that seemed on first inspection a non-distinction: the distinction between vaguely uncertain and certainly vague.

Indeed, when I began empirical investigations into uncertainty problem solving, I assumed that uncertainty was the start state and precise certainty was the end state. That is, I assume a problem solver moved along a continuum of precision, with initial states involving little precision and final states involving high precision. Yet, examination of problem-solving transcripts hinted at a different transformation: from uncertainty to imprecision, or, as I call it now, approximation.

In the early coding work on the uncertainty/approximation distinction, we had many arguments within the lab about what the distinction even was and how it could be coded with any conceptual integrity. Yet, the initial intuition about the need for such a distinction appeared to have merit. The distinction can be defined psychologically, even though the logical or information theoretic definitions are lacking. More importantly perhaps to researchers who are empirically rather than philosophically oriented, the distinction could be coded reliably in real problem-solving transcripts, and cross-validation investigations were also very successful.

Of course, subtle demand characteristics of context might have created these distinctions in the minds of the coders. For example, it is hard to hide from the coders the context of participants being asked what did they know versus what did they not know. Even with the questions themselves being hidden, the participants often repeat the question verbatim or with minor rephrasings. However, the same pattern was observed in many different datasets, which involved many different coders (spread across labs in different cities), and cross-validations of different forms. Furthermore, we focused on more syntactic approaches to coding uncertainty and approximation to reduce the possible influence of situational demand characteristics significantly determining our results. Finally, we did not find that expertise levels had clear associations with uncertainty levels, even though some of the coders had strong expectations that there would be such patterns. Thus, effects of coder expectations on coding behavior were not strong enough to create results through expectations alone.

Perhaps, most persuasive and interesting are the patterns of uncertainty and approximation against reasoning indicators. We found clear temporal patterns: (1) uncertainty invokes mental spatial transformations; (2) spatial gestures seem to reside between verbal uncertainty and verbal approximation; and (3) mental simulations seem to reside between verbal uncertainty and verbal approximation.

Before declaring victory in this appeal for a new general distinction, I want to return to the information theoretic/logical basis or nonbasis for the distinction. In cognitive science, there is a general view that cognition is but computation. Further, considerable recent theorizing has focused on the optimality or rationality of human cognition (Anderson, 1990; Gigerenzer, 2000; Griffiths & Tenenbaum, 2006). It should make the reader nervous to accept a distinction as the basis of rational, expert problem solving when the computational/logical basis of the distinction is fundamentally flawed.

As I noted before, the definition for uncertainty and approximation could not be made simply on the basis of logical informational ambiguity. That is, one could imagine uncertain cases that had less ambiguity than other approximation cases. However, I also noted that, empirically, problem-solving processes would generally reduce the underlying ambiguity as the problem solver moved from uncertainty to approximation. Therein lies the true rational basis of this mode of processing.

The computational work of Forbus (1997) building running conceptual simulations (called qualitative reasoning) shows how approximate quantitative answers can be derived from incomplete information. To extend a computational framework to the current proposal, the idea is as follows. A problem solver is working on a task and discovers that the informational ambiguity is above some threshold such that a critical decision/inference cannot be made (e.g., will a design choice produce a satisfactory outcome?). A state of uncertainty is thus taken on, which motivates problem-solving processes (such as spatial transformations or mental simulations) to reduce the underlying ambiguity. When the ambiguity is sufficiently reduced to enable decision making, then the resulting ambiguity is declared an approximation.

I should also add an important caveat. While uncertainty frequently resolves in approximation in science and engineering, I am not claiming that it always results in approximation; sometimes it just ends in more uncertainty and the problem solvers move on, and sometimes it even ends in precise certainty. Although the world of scientists and engineers is complex enough that exact, certain values are not the norm, it does happen.

A final caveat involves my focus on science and engineering. Many psychologists avoid rich real domains because of the difficulties in obtaining access to participants and the complexities of studying real tasks. Those psychologists who do study real domains tend to pick a particular domain to study. I have presented data from many different domains, including several basic sciences, several applied sciences, and engineering design. Hopefully, the case is now persuasively made for science and engineering. But certainly the space of domains involving informational uncertainty is much broader still. I suspect that similar distinctions will be relevant in these other domains, but that remains an empirical question for others to examine.

8. Future Directions

I have attempted to provide a simple and rational account for what problem solvers do with uncertain information, but many questions remain. For example, we have at best a very incomplete understanding of how information uncertainty is detected by the problem solver. In science and engineering, the problem solver might encounter hundreds to thousands of quantities, all of which may involve some uncertainty and yet psychological uncertainty is not triggered for all of those values. Most values are simply accepted. What raises the uncertainty hairs of the problem solver in these complex settings?

A related question involves what it means, exactly, to have the uncertainty hairs raised. We know that information ambiguity is troubling to problem solvers. It motivates them to reduce the ambiguity and the ambiguity reduce procedures appear to be useful for problem-solving success. But this behavioral description does not precisely unpack the mental state of uncertainty. Is it purely cognitive or does it have a core emotional component? Does it have underlying phenomenological primitives or is psychological uncertainty a foundational concept? As mentioned in Section 1, we know that uncertainty derived from different factors produces different behaviors, but that, by itself, does not answer the phenomenological question. Cognitive neuroscience may provide some interesting data on this front. We know that relative predictability of outcomes is a key variable in predicting the reactions of certain brain areas (e.g., the anterior cingulate cortex or the basal ganglia), and this relative predictability is heavily implicated in learning.

Another further direction involves the qualitative. Thus far I have emphasized psychological uncertainty about quantitative dimensions. What about qualitative dimensions? Perhaps, the enemy will come by plane or by train. Perhaps, it will snow or it will rain. Psychological uncertainty is clearly relevant to these qualitative ambiguities. What about approximation? Let us briefly consider some of the hedge words that we used for coding approximation in speech: "pretty much," "virtually," "generally," "frequently," "usually," "normally," "basically," and "almost." All of these hedges could be applied to the qualitative ambiguities in enemy transportation method or precipitation type. Semantically, those qualifiers would be ones of probability, which is a quantitative dimension attached to discrete qualitative states. Indeed, many of the things that were coded in our datasets as approximations involved these sorts of probabilistic hedges to qualitative issues. The task for future research is to fathom whether approximation on quantities and approximate probabilities on qualities is actually the same basic thing.

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