Science
Education

LEARNING

Modeling Scientific Processes With Mathematics Equations Enhances Student Qualitative Conceptual Understanding and Quantitative Problem Solving

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ABSTRACT: Amid calls for integrating science, technology, engineering, and mathematics (iSTEM) in K–12 education, there is a pressing need to uncover productive methods of integration. Prior research has shown that increasing contextual linkages between science and mathematics is associated with student problem solving and conceptual understanding. However, few studies explicitly test the benefits of specific instructional mechanisms for fostering such linkages. We test the effect of students developing a modeled process mathematical equation of a scientific phenomenon. Links between mathematical variables and processes within the equation and fundamental entities and processes of the scientific phenomenon are embedded within the equation. These connections are made explicit as students participate in model development. Pre–post gains are tested in students from diverse high school classrooms studying inheritance. Students taught using this instructional approach are contrasted against students in matched classrooms implementing more traditional instruction (Study 1) or prior traditional instruction from the same teachers (Study 2). Students given modeled process instruction improved more in their ability to solve complex mathematical problems compared to traditionally instructed students. These modeled process students also show increased conceptual understanding of mathematically

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modeled processes. The observed effects are not due to differences in instructional time or teacher effects. *Sci Ed* 1–31, 2015

INTRODUCTION

There have been many calls for integrating science, technology, engineering, and mathematics (STEM) instruction in K–12 schools to enhance student learning (Honey, Pearson, & Schweingruber, 2014). Cited reasons include (1) make mathematics and basic science appear more relevant to students to improve motivation during learning and thereby broaden participation in STEM (NGSS, 2013; PCAST, 2010); (2) produce STEM undergraduates who are better able to apply what they learn in mathematics to science, and in mathematics and science to engineering (Apedoe, Reynolds, Ellefson, & Schunn, 2008; Fortus, Dershimer, Krajcik, Marx, & Mamlok-Naaman, 2004; Litzinger, Lattuca, Hadgraft, & Newstetter, 2011); and (3) produce a general citizenry and workforce who are more technologically fluent through improved understanding of the scientific and engineering basis of modern technologies (PCAST, 2010; Peters-Burton, 2014). Unfortunately, given the generally siloed nature of instruction, particularly in high school, there are still questions about what constitutes productive models of STEM integration (Morrison, 2006; Peters-Burton, 2014). In this paper, we will present one instance of an integrated STEM (iSTEM) unit taught within a high school science class and examine its effect on quantitative problem solving and qualitative conceptual understanding.

The iSTEM unit integrates all four areas of STEM. An engineering design task motivates and deepens the learning, while technological advances in molecular biology allow students to visualize the normally invisible and indirectly measured objects of inheritance. The primary focus of the unit and our analysis in this paper, however, is the integration of mathematics with science. We assess whether a particular form of integration of mathematics with science, done via modeling of a process, enhances students' ability to solve problems in and improve their understanding of inheritance.

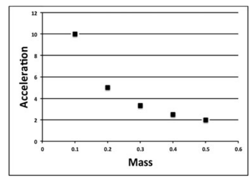
Forms of Embodiment of Mathematics in Science Education

Mathematics has long been a part of science education, particularly in chemistry and physics. There are different ways in which mathematics can be integrated into science education (Table 1). We review forms that are more typically present and then turn to alternative approaches that may be more productive for learning.

Mathematics as Data Presentation and Calculated Procedures. Two of the most common embodiments of mathematics in science education are as a summary of data and as a calculated procedure. As an example of data presentations, students might plot data on a graph from an experiment on mass and volume. As a common example of calculated procedures, students in physics are asked to memorize the equation for calculating the change in position of an accelerating object ($\Delta x = 1/2at^2 + v_0 t$), and taught to plug in the values for acceleration (a), time (t), and initial velocity (v_0) to get the answer. In biology, calculated procedures are less common, but still exist. For example, students are taught how to use a Punnett square to calculate the probability of a set of parents generating an offspring with a specified gene combination (Appendix, Table A1).

Both of these forms of mathematics are experienced by most students as relatively meaningless symbol manipulation (J. Stewart, 1983; Walsh, Howard, & Bowe, 2007). They are missing either data (calculated procedure) or operation (data representation)

TABLE 1
Embodiment of Mathematics in Science Education

	Calculated Procedure	Display of Data	Modeled Process
Example representation	$\Delta x = 1/2at^2 + v_0t$ $P_1 * P_2$ where P_n is the probability of getting genotype of gene *n*,		$2H_2 + O_2 \rightarrow 2H_2O$ $\frac{W1 * W2}{(\#\text{ egg types})(\#\text{ sperm types})}$ where W_n is the number of ways of getting a genotype combination of gene *n*,
Example operation	To produce a numerical answer	To display data and the relationships between variables	To express and test ideas about scientific processes
Connections to science	Variables and mathematical processes do not have to be connected to entities and processes within phenomenon	Variables are linked to entities within phenomenon	Variables and mathematical processes correspond to entities and processes within phenomenon

(Larkin & Simon, 1987). There has been increasing awareness of the shortcomings of the embodiment of mathematics as symbol manipulation. For example, when student problem-solving strategies in chemistry were examined, more than half of the students failed to use reasoning about content together with their equation-based approach and none of these students could successfully solve a conceptual transfer problem (Gabel, Sherwood, & Enochs, 1984). The authors argued that student "reliance on algorithms is a substitute for understanding the concepts" (Gabel et al., 1984, p. 232). Other researchers have replicated this finding in chemistry and physics (Chi, Feltovich, & Glaser, 1981; Mason, Shell, & Crawley, 1997; Nakhleh & Mitchell, 1993; Salta & Tzougraki, 2011; Tuminaro & Redish, 2007; Walsh et al., 2007). Students tend to ignore connections to underlying concepts that could allow them to transfer their understanding to superficially different, but structurally similar problems (Chi et al., 1981). Simply exposing students to more problems does not increase conceptual understanding (Byun & Lee, 2014; Kim & Pak, 2002). It is becoming increasingly apparent that a more productive method for incorporating mathematics in science is needed—one that allows students to learn science concepts and transfer problem-solving strategies to novel problems.

Mathematics as a Modeled Process. Rooted in the theory of scientific models and modeling (Buckley et al., 2004; Giere, 2004; Hestenes, 2010; Svoboda & Passmore, 2013), a rarely used third embodiment of mathematics in science education holds promise for improving student understanding: treating mathematics as a model of a scientific process. Specifically, including mathematics as a modeled process of a scientific phenomenon involves linking both the variables in the mathematical representation and the mathematical operations to entities and processes in the modeled phenomenon. Such a representation includes both data (mathematical variables connected to scientific entities) and operations

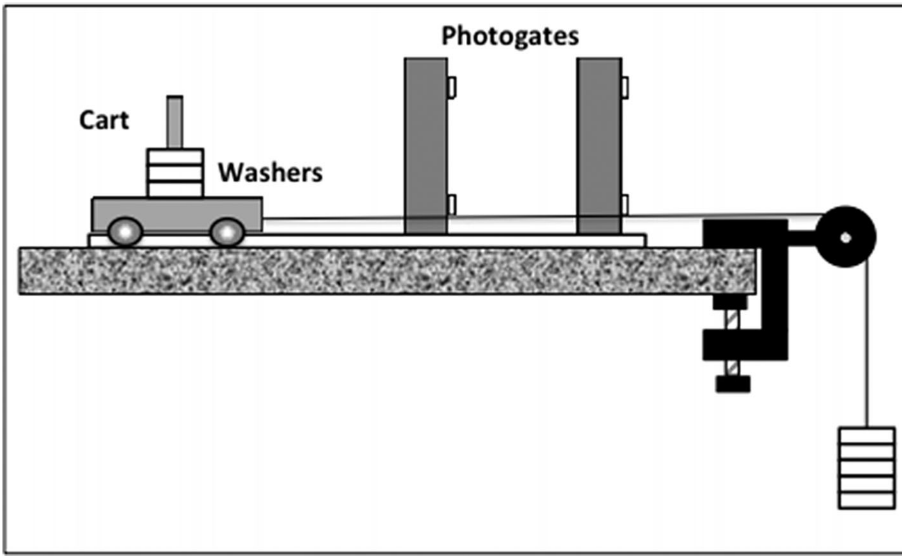


Figure 1. Measuring the effect of mass (determined by the number of washers) on acceleration of the cart.

on that data (mathematical operations paralleling scientific processes) (Larkin & Simon, 1987). In chemistry and biology, the ubiquitous chemical equation, although more often presented as a calculated procedure, can be an example of this modeled use. Consider the balanced equation for producing water (H_2O) from hydrogen (H_2) and oxygen (O_2), $2H_2 + O_2 \rightarrow 2H_2O$. From this equation, a student can calculate how much water will be produced if given a certain amount of oxygen or hydrogen (data). More interestingly, however, the equation describes an operation: Separate hydrogen and oxygen molecules are combined (the combination process is indicated by the plus sign) to produce (as indicated by the arrow sign) a new molecule containing both oxygen and hydrogen. Student engagement with this aspect of the equation could increase conceptual understanding.

The critical distinction between the use of mathematics in science as a modeled process versus either a summary of data or a calculated procedure is that the modeled process contains links to scientific entities (variables) and processes (operations), encouraging students to engage in meaning making (Hestenes, 2010; Sherin, 2001). Meanwhile, mathematics as summary of data and calculated procedure too often devolves into manipulation of symbols with little link to the underlying science. Therefore, even though the latter two embodiments of mathematics in science education are more common and still have a purpose in science education, it seems likely that when the goal is learning about the phenomenon, converting mathematics use in science education to modeled processes might help students learn scientific concepts as well as improve their problem-solving abilities.

Exemplifying the Embodiment of Mathematics in Science Education. We argue that it is possible to transform the use of mathematics for a particular topic from data presentation or calculated procedure to modeled process, rather than simply assuming that specific science topics require calculation or data summary approaches. To illustrate, consider Newton's second law (conceptually: more effort is required to get a heavier object into motion than a lighter one). Students can investigate this phenomenon by using a string with weights to exert a constant force on a cart on a frictionless track (Figure 1).

TABLE 2
Example Data Table Used in Science Instruction

Force (N)	Mass (kg)	Acceleration (m/s^2)
1	0.1	10
1	0.2	5
1	0.3	3.3
1	0.4	2.5
1	0.5	2

Different amounts of mass can be added to the cart, and sensors are placed on the track so that acceleration of the cart can be determined from the time required to travel the distance between the two sensors, using the calculated procedure equation: $a = 2*\Delta x/t^2$. This is a calculated procedure for two reasons: (1) the equation is only being used to derive a quantity, rather than being used as part of some sense-making process and (2) the constant multiplier "2" and the operation time squared have no clear process meaning. For example, there is no entity that is time squared; instead the representation is shorthand for the more meaningful equation, $\Delta x/t/t$, which represents the change in velocity per unit time.

By changing the mass on the cart and calculating the resulting acceleration, students can produce a data table, as shown in Table 2.

The table reveals that acceleration decreases as the mass increases for a fixed force. This statement captures the core phenomenon, but provides no hints about the underlying causal mechanism. By contrast, a student could present their understanding of this phenomenon with the following statement: As mass increases, the force is distributed over more mass, diluting the resulting acceleration. This idea could be represented mathematically by acceleration = Force/mass (i.e., $a = F/m$). This kind of equation is a modeled process. The symbolic form of the equation (Sherin, 2001) matches a conceptual understanding of the physical phenomenon. First, each variable represented in the equation has meaning in the phenomenon. Acceleration is the amount of time it takes for an object to go from the velocity at sensor 1 to the velocity at sensor 2. Mass is the amount of stuff on the cart. Force is the pull exerted by the string. Second, the mathematical operation (division) has meaning as well: The pull of the string is getting distributed over the amount of stuff of the cart. The equals sign describes the result of a physical process applied to inputs (the effects of a force applied to a mass), rather than simply noting a mathematical equivalency that is convenient for calculation (i.e., the force happens to be equal to the acceleration times the mass). Such connections of variables and operations in the equation to the entities and processes in the scientific phenomenon frame the equation in such a way that students may be more likely to engage in physical mapping between the mathematics and science (Bing & Redish, 2008). Participation in problem solving using this equation may therefore tend to occur more often through the more productive epistemic game of mapping meaning to mathematics as opposed to recursive plug and chug (Tuminaro & Redish, 2007).

Contrast this approach with the way the relationship between force, mass, and acceleration is often presented in a textbook. The equation is rewritten as $F = m*a$, and students are asked to memorize this equation as a way to calculate the force exerted by a given object. It is difficult to reason how or why mass should be multiplied by acceleration to give a greater force. It is possible to see that each particle within the object will contribute its own acceleration—but then why is the function not addition rather than multiplication? It is also difficult to reason how acceleration causes a force. Because the equation is presented to rather than derived by students and the variables and mathematical processes within the equation have little connection to the entities and processes within the physical phenomenon, this use of a mathematical equation has much more of the flavor of a calculated procedure. Continued presentation of physics as mathematical formulas to be memorized is one possible explanation for why students have a hard time transferring ideas in physics (Sherin, 2001; Tuminaro & Redish, 2007) and applying their understanding to engineering problems (Litzinger et al., 2011).

Mathematics Linked to Science Concepts Facilitates Problem Solving

Students who are able to solve more complex problems in physics and chemistry have not only an understanding of how to use the mathematics but also an understanding of how that mathematics is linked to the concepts (Bing & Redish, 2008; Chi et al., 1981; Taasoobshirazi & Glynn, 2009; Walsh et al., 2007). Bing and Redish describe an attempt at problem solving by upper level physics majors where the students, stuck in computing the mathematics and failing to engage in connection of the mathematics to the system of interest, are unable to solve the problem, despite their obvious facility with mathematics. It is not until one student asks for the relationship between the equations and the physics particles that the group is able to progress (Bing & Redish, 2008). In chemistry, a student who successfully uses a conceptually based strategy to solve a thermochemistry problem talks about the problem-solving process in terms of the concept first, "... I need to find the heat gained by the water first" and then applied the mathematics, whereas an unsuccessful student expresses his algorithmic approach in this way, "I just came up with an equation to solve for the problem, but I think I plugged in the wrong values or something..." (Taasoobshirazi & Glynn, 2009, p. 184).

Several approaches elevating the contextual element of mathematics within a K–12 scientific curriculum (e.g., problem-based learning, qualitative explanations of problem solving, analogies, model development) have improved student conceptual understanding and/or problem solving. (Dori & Kaberman, 2012; Lehrer & Schauble, 2004; Litzinger et al., 2011; Novick, 1988; Savery, 2006; Wells, Hestenes, & Swackhamer, 1995). However, these studies did not test the effect of embedding understanding of scientific entities and processes within a modeled process mathematical equation, as opposed to simply embedding the equation in a scientific context). The current study focuses specifically on this equation as a modeled process approach.

Mathematics in Biology Education

Almost all of the research that has been discussed so far has revolved around the use of mathematics in chemistry and physics, likely because mathematical representations of phenomena have been a part of physics and chemistry instruction for a longer time (Steen, 2005). However, over the past two decades, rapid changes in biology understanding combined with advances in research technologies (i.e., new measurement tools and computer simulations) require that biology students, not just physics students, learn how to reason in the language of mathematical symbols (Bialek & Botstein, 2004). Furthermore, the most recent scientific standards (*Next Generation Science Standards*) identify using mathematics as a core practice of science that all students should learn (NGSS, 2013). Since so many students take high school biology (Lyons, 2013), it is particularly important that mathematics becomes a greater part of the high school biology curriculum. Thus, there is a need both for biology curricula that incorporates mathematics as modeled processes into instruction, and research into the effect of this approach on student understanding. We seek to determine whether students become better problem solvers and better understand underlying biological processes.

Inheritance and Mathematics as a Modeled Process. Inheritance presents a good opportunity for researching the effects of introducing mathematics as a modeled process. Inheritance instruction has typically involved predicting the probability of getting a particular type of offspring from a set of parents. That is, mathematics has been at least a small part of high school instruction in heredity for a long time, and therefore we can test the effects of changing the approach to mathematics rather than simply adding (any form of) mathematics. Moreover, teachers report that inheritance is one of the hardest topics for students to understand (J. H. Stewart, 1982), so there is great need and opportunity to improve instruction on this topic.

Previous research suggests that when studying inheritance, students have difficulty understanding the underlying biological processes of inheritance (meiosis and fertilization) and how these processes affect the units of inheritance (alleles) that are counted in the mathematical procedures (Moll & Allen, 1987; J. Stewart, 1983; Tsui & Treagust, 2010). Students can also struggle to connect the appearance of an organism with the underlying combination of alleles, particularly across generations (Tsui & Treagust, 2010). One approach that has been pursued is to explicitly develop and connect the process of meiosis with inheritance either through the use of a computer simulation (Buckley et al., 2004) or through tracing the movement of alleles using drawings (Moll & Allen, 1987). The results of these interventions have been mixed. When college students are instructed in how to trace alleles through drawings of meiosis, over half continue to use an algorithmic method to solve genetics problems (Moll & Allen, 1987). Students who draw out meiosis are more successful at solving problems involving one gene than students who use an algorithmic approach, but not more successful at solving problems involving two genes (Moll & Allen, 1987). As the authors point out, drawing out meiosis is a relatively labored and detailed procedure as compared to the speed of the algorithmic approach (Moll & Allen, 1987). The computer-based intervention resulted in higher posttest scores than traditional instruction (Buckley et al., 2004), but studies on a different population of students using the same computer program suggested that genetic reasoning was only improved for the easier types of problems for most students and that a key variable was the mindfulness of student interaction with the different representations of inheritance (Tsui & Treagust, 2003). The computer-based intervention also requires that students have sustained access to computers during class time, a resource that may not be available to most schools.

All of the approaches to modifying inheritance instruction have focused on enhancing student understanding of the biological processes of inheritance. None have suggested fundamentally altering the embodiment of mathematics within the inheritance curriculum. Currently, similar to the worst examples of math–science integration in physics and chemistry, the textbook mathematical expression for predicting inheritance outcomes embodies mathematics as calculated procedure and is devoid of any meaningful connection to biological entities or processes. Instead, as is exemplified in a commonly used high school biology textbook (*BSCS Biology: A Molecular Approach Blue Version*, 2001), students are exposed to a short didactic introduction to probability, which reminds students, “Probability is usually expressed as a fraction. The chance of the coin landing heads up is one out of two, or ½,” (p. 351). There is no exploration of why probability is expressed as a fraction, or how this fractional representation relates to the entities of inheritance. Students are then shown how to use an algorithmic procedure, the Punnett square, for determining how the parental genes for a single trait will recombine in the offspring

(Appendix, Table A2). When students progress to considering inheritance of two gene combinations, they are told that they multiply the probability for getting a particular genotype from each separate gene because "and" means multiply. At this point, there is no biological correlate to the probability for getting a particular genotype from each separate gene and there is no connection drawn to the biological process by which a combination is formed; that is, both the constituent probabilities and the mathematical operator on them is not biologically motivated. Unsurprisingly, studies on how students solve inheritance problems show that they tend to use an algorithmic (calculated procedure) method whether they use a pictorial representation followed by counting (the Punnett square), or the mathematical probability method outlined above (Moll & Allen, 1987; J. Stewart, 1983). Students struggle to extend what they have learned from simple to more complex genetic probability problems and show little ability to connect the mathematics with the biology (Cavallo, 1996; Moll & Allen, 1987; J. Stewart, 1983).

We have developed a new inheritance unit that changes the way mathematics is embodied from calculated procedure to modeled processes (summarized in Figure 2). Following the modeling cycle (Halloun, 2007; Passmore, Stewart, & Cartier, 2009), students engage in scientific practice by analyzing modern technology-based data (e.g., polymerase chain reaction [PCR] data) to develop a model of inheritance, which includes modeled process mathematical representations (i.e., equations that capture data patterns but also reify the underlying genetic process; Figure 3). Prior research in the physical sciences presented above suggests that some form of embedding mathematics in a scientifically rich context could improve problem solving and understanding of scientific content. We theorize that by specifically changing the use of mathematics from (teacher-presented) calculated procedures to (student-developed) modeled processes that embed biological concepts within the mathematics, students will be better able to solve inheritance problems and will also demonstrate better understanding of the mathematically modeled processes. We assess the benefits of instruction via modeled process equations on student learning. Specifically, we asked two questions:

1. Is conceptual understanding improved when students are taught mathematics in science as a modeled process rather than a calculated procedure?
2. Is quantitative problem solving also improved when students are taught mathematics in science as a modeled process rather than a calculated procedure?

The benefits on both conceptual understanding and quantitative problem solving are examined in terms of breadth and scope of benefits to help frame the extent of benefits and the likely mechanisms of change (e.g., general engagement effects of using an iSTEM unit vs. specific modeling of particular processes; general benefits on quantitative reasoning vs. specific benefits to more difficult transfer problems).

The iSTEM Unit and Mathematical-Modeled Processes of Inheritance

The iSTEM inheritance unit begins and ends with an engineering challenge: design a breeding plan to develop a rare gecko so that a zoo can attract visitors (Figure 2). The initial exposure to the design challenge is designed to help students see genetics knowledge as useful in a real-world context and therefore serve as a motivation to understand the phenomenon of inheritance. The unit is constructed as a modeling cycle (Halloun, 2007; Passmore et al., 2009) to develop increasingly complex conceptual models, interconnected ideas, and representations that describe or explain a simplified version of the phenomenon which can be used to make predictions (Etkina, Warren, & Gentile, 2006).

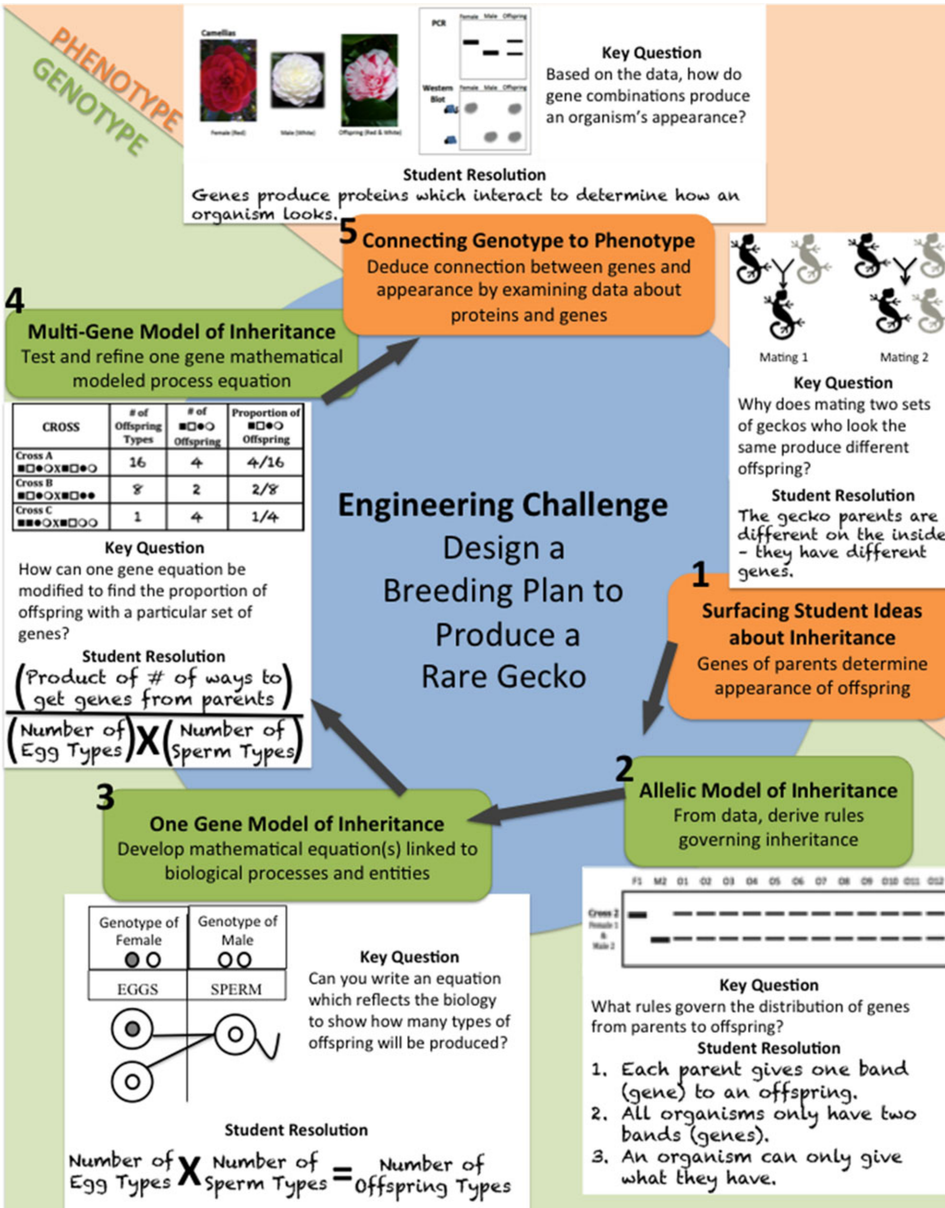


Figure 2. Unit overview of iSTEM inheritance unit. The unit begins and ends with the engineering challenge that is revisited at the end of Tasks 2, 3, and 4. The colored boxes show the product of each task. The numbers indicate their order in the unit. The white boxes show a phenomenological representation provided to students, the key questions students engage with and the target student resolution.

Each nascent model is developed through analysis of data, followed by argumentation with peers to resolve differences in interpretation and representation and reach a consensus (i.e., Task 2 in Figure 2). Revisiting the engineering challenge after the development of each model permits students to test the model's sufficiency. For example, the design challenge specifically asks for a rare gecko to push students beyond a simple one gene breeding design

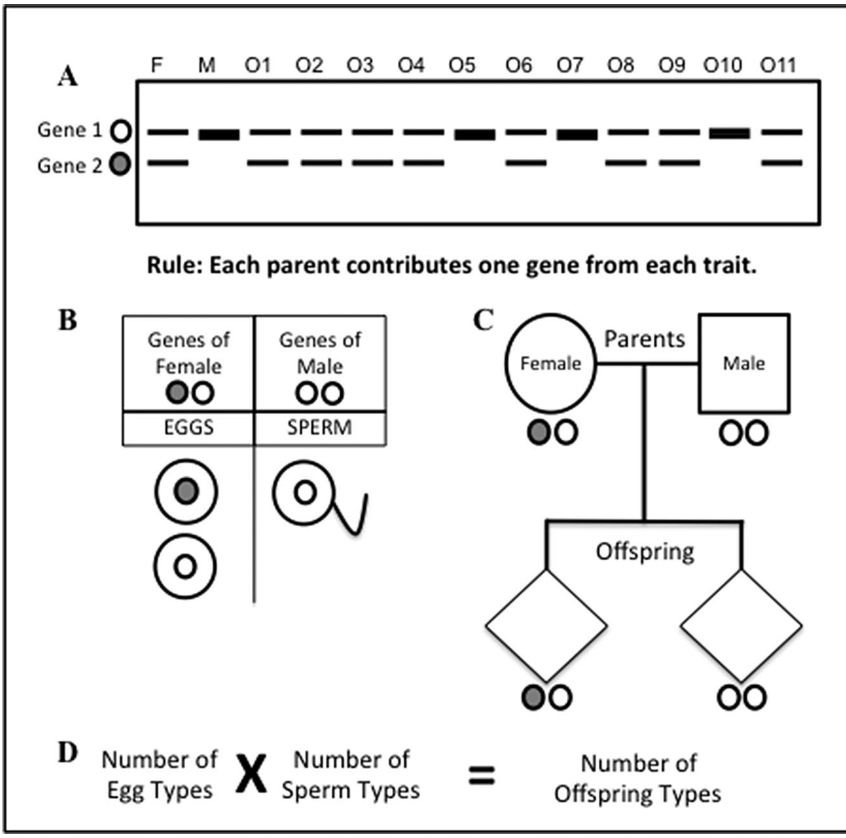


Figure 3. (A) PCR diagram, (B) egg/sperm table, (C) prediction pedigree, and (D) initial mathematical modeled process equation. F, Female; M, male; O, offspring.

to more complex multigene models. This move to multigene modeling of outcomes makes the need to quantitatively predict outcomes more salient and thus serves as a motivation for mathematical representation becoming a key part of the inheritance models (Tasks 3 and 4 in Figure 2).

The development of this mathematical representation occurs in Tasks 3 and 4. However, the groundwork is laid in Task 2, when students are shown the physical entities of inheritance (the genes), which are revealed in parents and their offspring via a technological application (polymerase chain reaction or PCR) (Task 2 in Figure 2). Students are asked to analyze the gene patterns they observe and derive basic qualitative rules that summarize the way in which genes are transferred from one generation to the next. These rules both preview the predictability of inheritance patterns and encapsulate part of the biological processes that will later be represented mathematically (Figure 3A).

Using their newly generated rules, students are directed to work with manipulatives depicting biological entities of inheritance, such as sperm, eggs, and genes, to make predictions about the outcomes of breeding two parents. The manipulatives are designed to enable students to see the relationships between the entities of inheritance (genes, eggs, and sperm), the processes of inheritance (i.e., the packaging of genes and joining of egg and sperm) and the quantitative inputs (number of genes in parents) and outputs (number of offspring types). However, using manipulatives to make predictions is relatively time consuming. The unit asks students to recognize this constraint and introduces the affordances of developing a mathematical model of the process to make predictions. The relationships between entities, processes, inputs, and outcomes are maintained through the inclusion of pictorial representations (Figures 3B egg/sperm table and 3C prediction pedigree).

In their first attempt at modeling the process mathematically, students are directed to examine a data table showing the offspring outcomes for three different combinations of parents. They are then instructed to develop equations that fit the data available to them and map on to the biological processes and entities that they have represented pictorially. Only two possible equations fit these requirements: number of different offspring outcomes = (number of egg types) * (number of sperm types) or number of different offspring outcomes = (number of gene types for trait in female) * (number of gene types for trait in male) (Task 3, Figure 2). The pictorial representations shown in Figure 3 make connections to the symbolic form of the mathematical equation (Sherin, 2001). Specifically, they support connections of mathematical operations (multiplication as combination) to the biological entities and processes (the sperm can join with either egg to produce two new entities).

The engineering design challenge is designed to push students to consider multiple traits, which then encourages refinement of the mathematical model. As part of the application of the single gene model to multiple genes, students are expected to deduce that the equation, number of different offspring outcomes = (number of sperm types)*(number of egg types), is the only one which generalizes, because in the inheritance process, the genes for each trait are packaged independently into sperm and eggs before they are combined in an offspring. This process of testing and subsequent refinement of mathematical representations for inheritance is thus supposed to allow students to gain a deeper understanding of one of the fundamental processes of inheritance. The unit then asks students to recognize that the probability of a desired event is equal to the number of desired outcomes as a proportion of the total number of possible outcomes, to allow the development of the final equation shown in Figure 3, Task 4.

Table A1 (Appendix) compares the modeled process equation to the calculated procedure methods that are used in traditional inheritance instruction, which uses the Punnett square. In this traditional instruction, there is little connection provided to the underlying biology as no biology is needed to teach the approach or solve a given problem. The purpose is not to model an idea about how inheritance of genes occurs, but rather only to calculate the correct answer.

In contrast, the modeled process equation makes explicit connections between the biology and the mathematical process (Figure 3). For example, the variables in the equation are expressed as eggs and sperm, entities in the inheritance process. Egg types are multiplied by sperm types, because each egg could theoretically join with each sperm. Furthermore, the explicit purpose of the equation within the unit is to model ideas about how inheritance occurs and therefore multiple equations are initially developed and tested against additional data, allowing students to refine their ideas about the biological process of inheritance.

It is important to note that the context of the mathematical representation is a big determinant of whether it is a calculated procedure or a modeled process. The modeled process inheritance equation could be a calculated procedure if students were just shown the equation and taught a formulaic approach for plugging in the variables. The embodiment of mathematics in science and education is not simply about the structure and use of the mathematics, but rather about how it is taught to and taken up by students.

We present two studies that examine the effects on student learning (conceptual understanding and problem solving ability) of changing from traditional instruction to using an iSTEM unit. The first study involves comparison between teachers implementing the iSTEM or traditional instruction and the second study focuses on teachers implementing

TABLE 3
Characteristics of Participants Within Each Group

	iSTEM Unit	Traditional Instruction
Participants	12 Teachers, 745 students	6 Teachers, 321 students
Percent of students eligible for free/reduced lunch program	38%	41%
Grade and biology level	Ninth and tenth grade, First-year biology	Ninth and tenth grade, First-year biology
Teacher biology education	80% masters or undergraduate degree in biology	80% masters or undergraduate degree in biology
Years teaching biology	80% more than 6 years	80% more than 6 years
Professional development	Yes (4–25 hours)	None
Instructional hours	820 minutes (4 weeks, planned)	890 minutes (4.5 weeks, average)

both the iSTEM unit and traditional instructional approaches. The iSTEM unit involves several types of instructional changes (e.g., inclusion of engineering challenges and use of technologies like PCR), and thus the intervention is broadly labeled iSTEM. However, in this paper, we focus our analytic lens on changing the treatment of mathematics in inheritance instruction from a calculated procedure to a modeled process. This focus is achieved by examining in detail the nature of changes on student learning (e.g., broadly on all aspects of inheritance or more narrowly on aspects of inheritance most directly connected to the modeled processes).

STUDY 1

Methods

Participants. All teachers were from public school districts in a midwestern state, drawn from urban, suburban, and rural areas. A local educational agency sent out notices inviting teachers to an exposure meeting. Teachers who attended this meeting signed up to participate in professional development. A subset of the teachers who finished professional development volunteered to implement the iSTEM unit in their classrooms and participate in our study. These volunteers recruited additional teachers from their schools as implementers (Table A2, Appendix). The implementing teachers helped to recruit other teachers within their school to serve as controls, using their usual instructional unit for inheritance (described below). Characteristics of the iSTEM and traditional samples are shown in Table 3. Generally, the teachers and students were well matched. Both groups taught honors and nonhonors classes for ninth- and tenth-grade first-year biology students. Additional individual teacher and school characteristics (including standardized test scores) are shown in Table A2 in the appendix. Professional development was conducted by the research team and primarily focused on teachers experiencing the unit as learners, although some pedagogy was covered in the longer professional development sessions.

Teachers who implemented the iSTEM unit received a curricular plan that included daily instructions for lessons, and teachers were observed at least once. Teachers who engaged in traditional instruction kept a daily lesson journal consisting of a two to three sentence summary of the day's events for each class. Five of six teachers submitted a journal. An analysis of these journals revealed that the traditional teachers were indeed engaging in inheritance instruction as usual:

- All five teachers showed or instructed students on how to set up Punnett squares to solve probability problems (e.g., "students were shown how to do single trait crosses using Punnett squares").
- The phrases used by all five teachers suggested that the inheritance laws were learned as a set of dictates handed down by Gregor Mendel (e.g., "We revisited the notes and added to them with Mendel's laws of segregation and independent assortment.").
- Four of five teachers did not mention basic objects and processes of inheritance (including eggs, sperm, fertilization, and gamete formation) in their journals, let alone linking them with mathematical solutions.

Instruments.

Assessments. Pre- and posttests were administered to students to examine the effects on student learning. To allow for a sufficiently broad set of questions for each knowledge subcategory but still use only one class period for the assessment, a matrix sampling protocol was used, drawing from a pool of 42 inheritance questions (genetics terminology, genetic processes, genetic probability) and 11 mathematical probability questions. The question categories were chosen a priori for the reasons outlined below. An exemplar question from each category is shown in Table 4.

Genetics Terminology. Because terminology changes were not part of the intervention, genetics terminology questions provide convergent evidence that teaching ability and student ability were roughly equivalent across conditions.

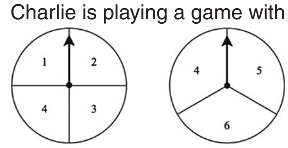
Genetic Processes. Genetics process questions assessed whether students qualitatively understood genetic processes, and were divided into two subtypes: processes that were mathematically modeled (packaging of genes into sperm and eggs and combining eggs and sperm to form offspring, Tasks 2 and 3 in Figure 2) and processes that were not modeled mathematically (how an organism's appearance is determined by its genes, Task 5 in Figure 2). Larger condition effects for the processes that were mathematically modeled would provide evidence in favor of the effects of modeling mathematical processes in particular.

Genetic Probability. Genetic probability questions required students to make probabilistic predictions in the context of inheritance. These questions were also subdivided into two categories: Simple genetic probability questions asked about simple probability in a genetics context; and complex probability questions required students to apply compound probability to a genetics context, which is then necessarily more complex.

Mathematical Probability. Because students' ability to make predictions in a genetics context might be influenced by their understanding of probability in a mathematics context, a category of questions assessing students' understanding of and skill with simple and compound probability in a mathematical context was included. Based on state standards, simple and compound probability had been covered by ninth grade (Comparison of Mathematics Michigan K–8 Grade Level Content Expectations (GLCE) to Common Core Standards, 2010), but that did not mean their performance was universally high.

Because no single previously published assessment contained a sufficient number of questions in all of the categories, the pool was constructed by aggregating questions from previously published assessments (Adamson et al., 2003; Blinn, Rohde, & Templin, 2002; delMas, Garfield, Ooms, & Chance, 2007; Garfield, 2003; Nebraska Department of

TABLE 4
Question Categories on Pre- and Post assessments

Category		Number of Questions[a]	Example Question
Genetic processes	Mathematically modeled	7	The genotypes of the sperm from one male and the genotypes of the eggs from one female are shown below Male Sperm: FQ, fQ Female Eggs: FQ, fQ, Fq, fq Which answer lists all the possible genotypes that could be expected in the offspring: a. FFQQ, ffQQ b. FFQQ, FfQQ, FFQq, FfQq, ffQQ, ffQq c. FFQQ, FfQQ, FfQq, ffQq d. FFQQ, FFQq, FFqq, ffQQ, ffQq, ffqq, FfQQ, FfQq, Ffqq
	Unmodeled mathematically	20	Flies can have red or brown eyes and straight or curly wings. Red eyes (R) is dominant to brown eyes (r) and straight wings (S) is dominant to curly wings (s). If you have a fly which has brown eyes and straight wings, what combination of genes could it have? a. Rrss c. rrss b. RrSs d. rrSs
Genetic probabilities	Simple	3	In dogs, the gene allele (e) for drooping ears is recessive to E for erect ears. A male dog with genotype Ee was mated to a female dog with genotype ee and gave birth to a litter of 10 puppies. What is the expected proportion of drooping-eared puppies (ee) in the litter? a. 1/4 c. 1 b. 1/2 d. Don't know
	Complex	7	If organisms of type BbSs and type bbSs are crossed, what is the probability that the offspring would be BbSs? a. 1/16 d. 1/2 b. 1/8 e. 9/16 c. 1/4
Genetics terminology		5	The appearance resulting from a given gene combination is referred to as the: a. Genotype d. Allelotype b. Prototype e. Stereotype c. Phenotype
Mathematical probabilities		11	Charlie is playing a game with two spinners.  All four sections on spinner A are the same size, and all three sections on spinner B are the same size. Charlie spins Spinner A and Spinner B one time and adds his results. What is the probability of getting a sum of six? a. 1/12 c. 1/4 b. 1/6 d. 1/2

[a]Number of questions refers to the number of questions in the pool.

Education, 2010; "Project 2061: AAAS Science Assessment Beta," 2013; Tobin & Capie, 1984; Tsui, 2002)

Based on two posttests of 26 and 27 questions, average KR-20 is 0.72 (average discrimination = 0.46; average difficulty = 0.50). For a subset of students ($N = 365$), there were no student identifiers on pretests; therefore, it was not possible to match up student posttest score with student pretest score, even though a teacher average could be calculated. The deidentified pretest scores were calculated using multiple imputation. The coefficients for multiple imputation were based on hierarchical linear modeling (HLM; Raudenbusch & Byrk, 2002) results involving the variables that best predicted student posttest scores: (1) membership in an honors biology class and (2) participation in unit implementation as well as (a) student posttest score, (b) the average of student pretest scores for each teacher, and (c) the difference between the teacher's average posttest scores and average pretest scores. When deidentified pretest scores were imputed using these variables, the observed average difference between the observed mean student pretest score for each teacher and the mean calculated from averaging the identified and imputed student pretest scores for each teacher was only 0.0023 (range: –0.03 to 0.05, $SD = 0.02$).

Mathematics in Biology Survey Questions. As part of a larger survey asking students about their attitudes toward the unit, students were asked two questions about the use of mathematics in the unit. Matrix sampling was used with the four survey versions distributed equally across all implementing teachers. Out of the approximately 630 students who took the survey, one quarter of them (157) answered a survey containing these two questions about mathematics use: (1) Did your group find math to be useful in solving the design challenge? YES or NO. (2) If yes, list examples of the types of math you used. The examples the students provided were content coded by two independent raters into biology connected mathematics versus unconnected mathematics with 91% agreement. Table 7 in the results section provides code definitions and example statements.

Results

Overall Effects on Student Problem-Solving Ability and Understanding of Science Content. Generalizability of the effects of an intervention can either be assessed by examining consistency of patterns across teachers and students, as is typically done, or by examining consistency of patterns across test questions. Statistically significant results can derive from effects limited to one strong teacher or subgroups of students (e.g., only the more interested students or only students in honors sections) or to a few particular questions within a conceptual subgroup of questions. More persuasive results are ones that show consistent and significant effects across students and teachers and across questions. We use both analytic approaches, but with statistical methods adapted to each given the constraints of the matrix sampling approach (e.g., individual students can have topic means but not question means) and the nature of the contrast (e.g., students are nested within teachers, but questions are not nested within teachers).

For the analysis of cross-question generalizability, a percent correct score was calculated for each teacher for each question, pre and post. An analysis of covariance (ANCOVA) was conducted examining the effects of instructional condition on posttest scores within each test category, using category pretest score as a covariate. All critical assumptions for ANCOVA were met, including independence of variables, homogeneity, normality, and homoscedasticity.

Both instruction conditions generally showed gains in understanding from pre to post (Figure 4, dark bars compared to light bars). However, students from iSTEM teachers

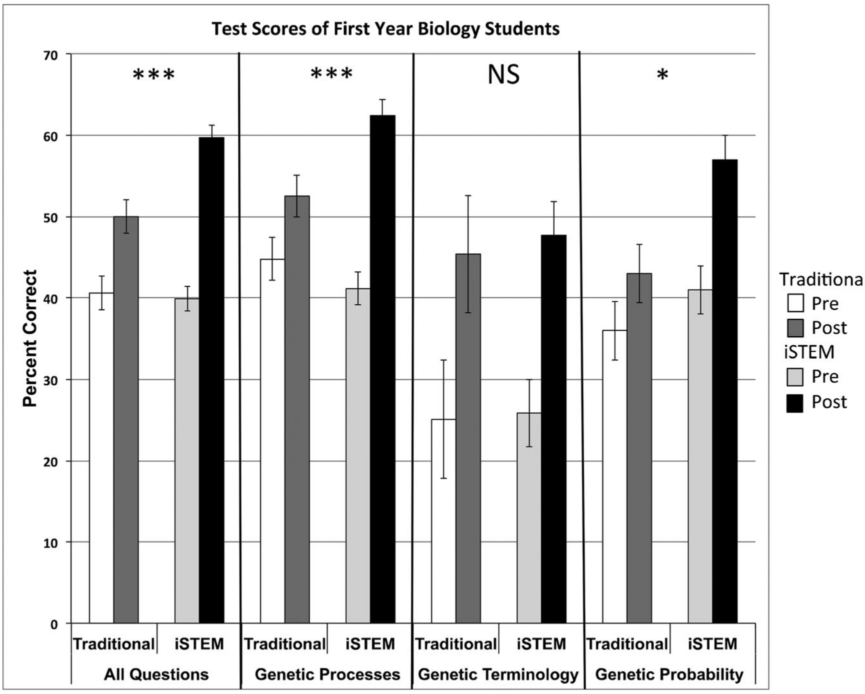


Figure 4. Pre–post teacher-level means (with *SE* bars) within each instructional condition for student problem solving and understanding of different forms of biology content knowledge. NS >.1, **p* <.05, ****p* <.001.

showed significantly greater adjusted posttest scores in their ability to make quantitative predictions about genetics outcomes ($F(1, 146) = 6.4$, $\eta^2 = 0.03$, $p = .015$; Figure 4, genetic probability). Students who received instruction in the iSTEM unit had an average 16 point gain on genetic probability questions, approximately two times the 7 point gain showed by students who were taught using traditional curricula.

Given the iSTEM unit's focus on mathematical modeling, the increased gain in quantitative problem solving is perhaps not surprising. But, we also theorized that mathematically modeling scientific processes by explicitly connecting mathematical symbols and functions with scientific entities and processes would help students understand scientific processes better (i.e., influence nonquantitative questions). When compared to teachers who used traditional curricula to teach genetics, students of iSTEM teachers showed significantly greater adjusted posttest scores for understanding of inheritance processes ($F(1, 419) = 23.1$, $\eta^2 = 0.045$, $p < .001$; Figure 4, genetic processes). The average gain in understanding of inheritance processes for traditional teachers was 8 points, whereas classes taught by iSTEM teachers had an average gain of 21 points, an almost three-fold improvement.

The adjusted posttest scores for understanding of genetics terminology for both traditionally instructed and ISTEM groups of students was approximately equal (corresponding to gains of approximately 20 points; Figure 4, genetic terminology), suggesting that both sets of teachers were similarly effective at helping students learn basic new material.

Specificity of Problem-Solving Benefits. Others have found that students taught using traditional instruction do not struggle with calculating simple genetic probability (Moll & Allen, 1987; J. Stewart, 1983), whereas they often do struggle to transfer this ability to

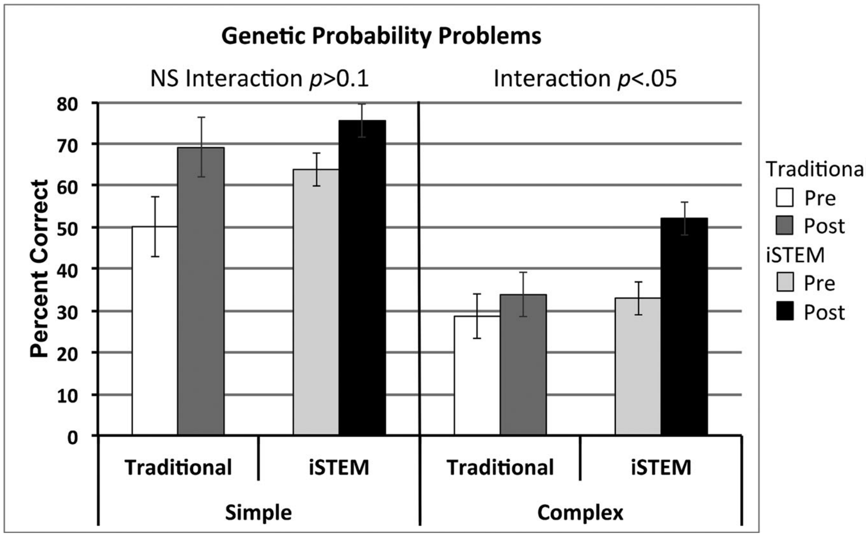


Figure 5. Mean pre- and posttest scores (with *SE* bars) within each instructional condition on simple and complex genetic probability problems.

more complex genetic probability problems (Moll & Allen, 1987; J. Stewart, 1983). Thus, it is likely that the problem-solving benefits of the iSTEM instruction were only found in more complex probability problems. However, there are too few questions within subtypes to use the generalizability across question analytic approach. To approach this more fine-grained analytic question, we (1) switch to a two-level HLM analysis (733 students nested within 12 teachers) examining student means on simple and complex probability problem categories and (2) include as an additional covariate a measure of ability to solve probability problems in general (i.e., with no biology content). Because of the sparse matrix sampling protocol for probability problems, individual students' pretest scores in mathematics with only a few questions each were not meaningful. Therefore, in this HLM analysis, we use a teacher mean score for mathematical probability, obtained from averaging all of the students' scores for the teacher. Five implementing teachers were dropped at this stage of the analysis because the version of the posttest that was administered to these classes had too few questions to generate reliable genetic probability scores for each student. To further reduce noise across the posttest variations used in the matrix sampling protocol, posttest scores were standardized within each test version. The variables included in the analysis were (1) instructional condition, (2) mean pretest mathematical probability score of each teacher's students (Teacher Pretest Probability Score), (3) honors designation, and (4) each student's pretest score (Student Pretest Score). Condition and honors variables were left uncentered; all other variables were grand mean centered. All key statistical assumptions of HLM were met (e.g., homoscedasticity, normality, independence, and linearity).

The HLM results confirm findings from prior research that most students can solve simple genetic probability problems. Traditionally instructed students and iSTEM-instructed students were not significantly different (posttests of 69% vs. 76% correct, HLM $b = -0.02$, $p = .85$, honors and Teacher Pretest Probability Score as covariates). This null result for simple genetic probability problems held across all of the statistical models that were tested. By contrast, iSTEM-instructed students were significantly more able to calculate genetic probabilities for complex problems (52% vs. 34% correct; see Figure 5; HLM $b = 0.27$, $p = .02$, honors and Teacher Pretest Probability Score as covariates). The condition effect

TABLE 5
Regression Coefficients and Model Fit Statistics for HLM Models Predicting Complex Genetic Probability Student Posttest Scores

	Fixed Effect Regression Coefficients (Standard Error)						
Variable Name	Model 0	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
				Teacher level			
Instructional Condition		.27*	.30*		.32*	.30+	
		(0.094)	(0.104)		(0.137)	(0.142)	
Honors		.41*	.40*	.45*		.76***	.43+
		(0.139)	(0.155)	(0.199)		(0.147)	(0.214)
Teacher Pretest Probability Score		.20*	.19*	.19+	.34***		.18+
		(0.066)	(0.072)	(0.089)	(0.063)		(0.038)
Intercept		−.17	−.19	−.01	.02	−.36*	−.15
		(NS)	(NS)	(NS)	(NS)	(0.120)	(NS)
		(0.098)	(0.109)	(0.119)	(0.106)		(0.123)
				Student level			
Student Pretest Score			.08*				.08*
			(0.038)				(0.038)
	Estimation of Variance Components						
Teacher level	0.23	0.009	0.015	0.034	0.036	0.040	0.043
Student level	0.87	0.87	0.86	0.87	0.87	0.87	0.86
Degrees of freedom (*df*)	11	8	8	9	9	9	9
Chi square	133.56	14.49	16.62	25.95	31.97	32.86	29.13
p value	<.001	.069	.034	.002	<.001	<.001	<.001
Deviance	2,007	1,989	1,992	1,995	1,995	1,994	1,999
Estimated parameters	2	2	2	2	2	2	2

NS > .1, +p < .1, *p < .05, **p < .01, ***p < .001.

on the difference between standardized mean question gains for simple versus complex probability (by teacher) was statistically significant ($F = 5.4$, $p = .03$).

To explore the robustness of these results across statistical assumptions and covariate choices, a number of different statistical models were tested (Table 5). Models are arranged in order of best fit. The best fitting model, Model 1, includes the covariates of ability grouping (honors) and the classes' prior understanding of mathematical probability (Teacher Pretest Probability Score). It shows that implementation of the unit has an effect size of 0.27. This effect size of approximately 0.3 is maintained in the other models.

Mean pretest score for simple genetic probability problems is significantly greater for the iSTEM-instructed group as compared to the traditionally instructed group, which may better position them to learn complex genetic probability. Therefore, prior understanding of simple genetic probability (mean of student scores for each teacher, due to the matrix sample approach) was added as a covariate. However, it was not found to be a significant predictor of complex genetic probability scores in any of the models. This finding is supported by the literature, which has shown that students have difficulty transferring their understanding of simple genetic probability problems to more complex problems (J. Stewart, 1983).

Specificity of Benefits for a Qualitative Understanding of Genetic Processes. The test items for qualitative understanding of genetic processes included both those processes

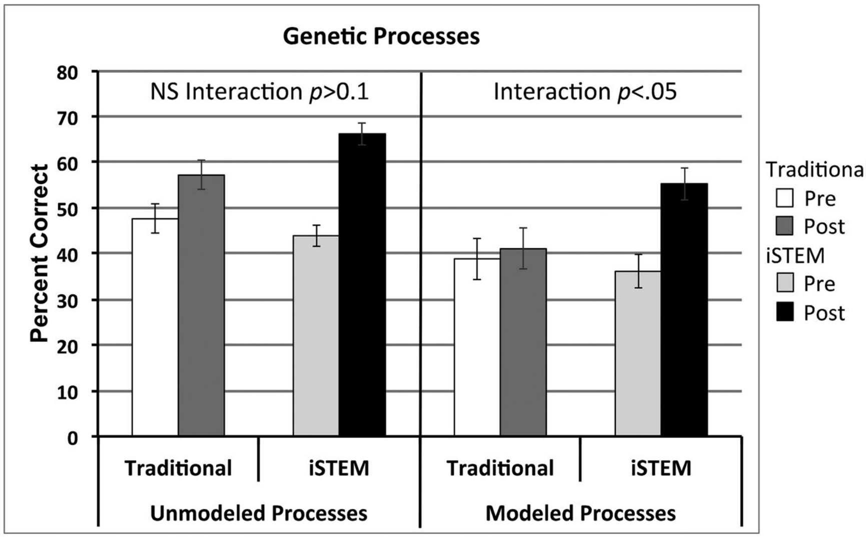


Figure 6. Mean (and *SE* bars) for pre- and posttest scores within each condition for modeled and unmodeled genetic processes.

that were modeled in the mathematical equations of the unit and those that were not. To distinguish between the effect of mathematical process modeling versus a general effect of the methods of iSTEM instruction (e.g., via improvements in overall student engagement or quality of classroom/group discussion), the effect of iSTEM instruction versus traditional methods was examined separately on modeled versus unmodeled genetic processes. If explicitly linking mathematical variables and processes with scientific entities and processes promotes student understanding of those processes, then there should be a differential effect of iSTEM instruction on modeled versus unmodeled processes. Again, given the more refined focus of analysis, significance testing was performed using a two-level HLM analysis on student means across questions with 975 students nested in 17 teachers, and controlling for various other student or contextual factors.

Both traditional and iSTEM instruction showed improvement in student understanding of unmodeled processes (Figure 6). However, the HLM results show that the adjusted posttest scores for traditionally instructed students and iSTEM-instructed students were not significantly different (69% vs. 76%, $b = 0.10$, $p = .55$, honors and Student Pretest Score as covariates). This null result for unmodeled process questions held across all of the statistical models that were tested.

By contrast, only iSTEM-instructed students showed a gain in their ability to answer questions about the mathematically modeled genetic processes (Figure 6). Moreover, HLM results show that the adjusted posttest scores for iSTEM-instructed students were significantly different from traditionally instructed students ($b = 0.34$, $p = .025$, honors and Student Pretest Score as covariates). To explore the robustness of results across statistical assumptions and covariate choices, a number of different statistical models were tested (Table 6). Models are arranged in order of best fit.

Model 1, which is the simplest model that best explains both teacher- and student-level variance, shows that implementation of the unit has an effect size of 0.34. Across models, iSTEM instruction continues to be a significant predictor of modeled genetic process posttest scores, with an effect size of approximately 0.3 or greater across all models tested; removing the variable of iSTEM instruction from the model produces a worse fit

TABLE 6
Regression Coefficients and Model Fit Statistics for HLM Models for Predicting Modeled Genetic Process Posttest Scores

	Fixed Effect Regression Coefficients (Standard Error)						
Variable Name	Model 0	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
				Teacher level			
Instructional Condition		.34* (0.137)	.31* (0.138)	.38* (0.141)	.40* (0.144)		
Honors		.51** (0.145)	.43* (0.160)	.34 (NS) (0.229)		.42 (NS) (0.267)	
Teacher Pretest Probability Score			.07 (NS) (0.066)				
Teacher Pretest Score				.31 (NS) (.317)	.68** (0.205)	.10 (NS) (0.362)	.54* (0.235)
Intercept		−.31* (0.124)	−.25* (0.095)	−.27+ (0.128)	−.19 (NS) (0.117)	−.03 (NS) (0.108)	−.19 (NS) (0.117)
				Student level			
Student Pretest Score		.21*** (0.037)	.21*** (0.037)	.20*** (0.037)	.20*** (0.037)	.20*** (0.037)	.20*** (0.037)
				Estimation of Variance Components			
Teacher level	0.14	0.045	0.043	0.045	0.048	0.070	0.077
Student level	0.91	0.882	0.883	0.882	0.883	0.883	0.884
Degrees of freedom	16	14	13	13	14	14	15
Chi square	129.86	59.05	51.75	53.95	59.87	68.47	76.75
p value	<.001	<.001	<.001	<.001	<.001	<.001	<.001
Deviance	2,712	2,673	2,678	2,675	2,674	2,677	2,680
Estimated parameters	2	2	2	2	2	2	2

Teacher pretest score is the mean student pretest score for each teacher.
NS > .1, +p < .1, *p < .05, **p < .01, ***p < .001.

(e.g., Models 5 and 6). Other explored covariates that did not have a consistent significant effect for either genetic process or genetic probability analyses included: teacher means of genetic process or genetic probability score and school measures such as American College Testing (ACT) scores and state test scores, and school socioeconomic status (SES).

Student Perception of Mathematics in iSTEM Unit. One hundred and forty-five students distributed across all teachers who implemented the iSTEM unit were asked if they thought mathematics was useful in designing a plan to breed a rare gecko and to give an example of how it was useful. Seventy-nine percent of students thought mathematics was useful in the design challenge and gave an example of its use. The examples these students provided were content coded into biology-connected mathematics versus unconnected mathematics (see Table 7 for definitions and example statements). On average, 40% of

TABLE 7
Codes for Student Examples of How Mathematics Was Used to Design a Breeding Plan for a Rare Gecko

Code	Definition	Example Statements
Unconnected mathematics	Statements mention the calculations that would be performed (multiply, divide) or that math is used for calculating financial profit. No biological terms are used.	"We added and subtracted the cost of the geckos to the budget." "Probability, multiplication, fractions."
Biology-connected mathematics	Statements about the use of mathematics make reference to biological entities or processes.	"We use egg type x sperm type to get the number of offspring." "We used a math equation to find out different possible ways that the genes could move (or combinations) when offspring was produced."

these examples involved a biological connection, and this rate was no lower than 30% for any teacher. Thus, we have evidence that many, although perhaps not all, of these students made connections between the mathematics they used and the biological phenomenon of inheritance.

STUDY 2

Methods

Participants. After receiving data on the effect of iSTEM instruction on student learning, two of the teachers in the traditional group volunteered to undertake 20 hours of professional development during the summer and used the iSTEM unit with their classes the subsequent year. In both years, the classes were nonhonors classes. Only students who took both the pretest and the posttest were included in the analysis (Year 1: Teacher 4, $N = 55$; Teacher 9, $N = 45$. Year 2: Teacher 4, $N = 39$; Teacher 9, $N = 29$).

Assessments. The assessments used were the same as described for Study 1, except that the genetics terminology category was eliminated in Study 2. Performance in each category or subcategory was calculated by obtaining a percent correct score for each question for each teacher and averaging.

Results

There were too few students total to conduct generalizability analyses across questions given the matrix sample approach, and therefore we focus on generalizability across students. Because there were only two teachers and the instructional contrast was within teacher, we conducted simple ANCOVAs (rather than HLMs) of the effect of instructional condition on student posttest scores in each subcategory. Only those variables that were shown to be significant in the larger sample were used as covariates with this smaller sample (class mean of mathematics probability score, student composite pretest score). There were

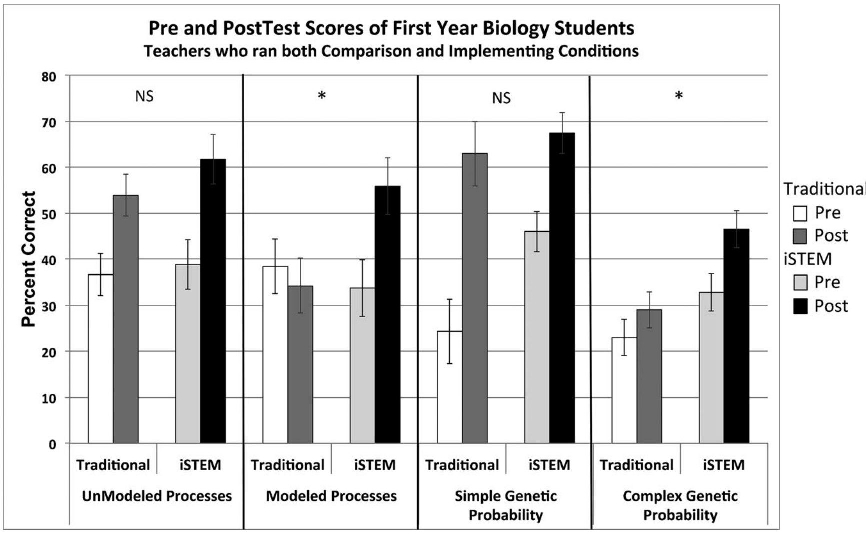


Figure 7. Pre–post means (and *SE* bars) within each instructional condition for student problem solving and understanding of different forms of biology content knowledge for traditional teachers who subsequently adopted iSTEM instruction. NS > 0.1, $^*p < .05$.

no honors classes in the second study. Assumptions for ANCOVA were met (i.e., normality, homoscedasticity, and independence of variables).

Genetics process results were consistent with our between-teacher findings from Study 1 (Figure 7). Students showed gains for unmodeled processes with both traditional and iSTEM instruction (traditional gain = 17, $SE = 5$; iSTEM gain = 22, $SE = 5$). However, only iSTEM instruction produced gains in student understanding of modeled processes (traditional gain = –4, $SE = 6$; iSTEM gain = 22, $SE = 6$). With student composite pretest score as a covariate, adjusted student standardized posttest scores for iSTEM instruction were significantly different from traditional instruction for modeled (traditional $M = 34$, iSTEM $M = 56$, $F(1, 165) = 6.3$, $\eta^2 = 0.04$, $p = .01$), but not unmodeled processes (traditional $M = 54$, iSTEM $M = 62$, $F(1, 165) = 2.20$, $\eta^2 = 0.01$, $p = .14$).

Similarly, both traditional and iSTEM instruction produced gains in students' ability to solve simple genetic probability problems (traditional gain = 39, $SE = 7$; iSTEM gain = 21, $SE = 4$). For complex genetic probability problems, only iSTEM instruction showed significant gains based on standard error of gain (traditional gain = 6, $SE = 4$; iSTEM gain = 14, $SE = 4$). When the mean for math pretest scores for each teacher by instructional condition was included as a covariate, adjusted posttest scores for complex genetic probability problems were significantly greater after iSTEM instruction (traditional $M = 32$, iSTEM $M = 45$, $F(1, 193) = 4.7$, $\eta^2 = 0.02$, $p = .03$). Posttest scores for simple genetic probability were similar for both types of instruction (traditional $M = 63$, iSTEM $M = 67$, $F(1, 95) = 0.79$, $\eta^2 = 0.008$, $p = .38$, with math pretest score included as a covariate).

GENERAL DISCUSSION

We examined a curriculum that included many critical iSTEM practices that are typically absent from science instruction: It was organized around an engineering design problem, students had to develop explanations from data, and iteratively develop and elaborate various models. Most intensely, the curriculum focused on mathematical modeling of processes in biology. Using both between teacher comparison (Study 1) and within teacher comparisons (Study 2), students who were taught inheritance using this curriculum performed at higher levels on assessments than did traditionally instructed students. Differences were found on measures of solving quantitative inheritance problems (particularly more complex problems) and of answering qualitative questions about genetics process (particularly related to the processes that were modeled in the unit).

Because there was not random assignment to condition, one might argue that prior differences in instructional ability (e.g., experience in teaching biology) or student characteristics (e.g., prior performance in mathematics and science) accounted for the results in Study 1. However, a number of factors argue against such possible confounds as the source of the performance differences: (1) Teachers were closely matched in a number of categories, including teaching experience and level of education; (2) traditional teachers came from five of the same schools as teachers implementing the iSTEM condition; (3) school characteristics were not significant covariates in any of the HLM analyses; (4) The effect of iSTEM instruction on quantitative problem solving and qualitative understanding of genetics processes were robust even with the addition of prior ability covariates in the analytic models; and (5) Teachers that switched from traditional to iSTEM instruction showed an increase in student performance after the switch.

A different concern might relate to possible differences in time on task. Often inquiry-based instruction requires more time than does traditional instruction. However, from the teacher logs, the traditional-instruction teachers reported spending a mean of 890 minutes (approximately 22 days) on inheritance; in contrast, the iSTEM instruction only involved 820 minutes (approximately 20 days). By focusing on a major instructional target in the traditional curriculum, it was possible to engage students in many practices of science with the core science content without extending the length of instruction.

Thus, we have good evidence that instructional reform in high school science using the reform practices can significantly improve student understanding and problem-solving ability. These improved instructional outcomes occurred in a range of instructional contexts and appeared on relatively traditional measures of student performance (i.e., multiple choice), similar to ones used for accountability purposes in many settings. Although rich instruction is likely to produce even stronger results on rich performance assessments, the results on simpler multiple-choice assessments are practically important for influencing the reform movement in the United States and beyond.

Theoretical Implications

This iSTEM intervention in inheritance was designed around mathematical modeling of genetic processes, based on the theory that asking students to develop a mathematical model of genetic processes and subsequently refine and use that model would cause them to connect mathematical variables and processes with scientific entities and processes, leading to a better understanding of the modeled scientific processes (Hestenes, 2010). In support of this theory, we demonstrate that a plurality of students who have been asked to develop a mathematical model of a biological phenomenon do indeed connect the use of mathematics with that biological phenomenon. Prior research in physics and chemistry also found that students who are able to link their mathematical equations with scientific concepts are better able to solve more complex problems (Bing & Redish, 2008; Taasoobshirazi & Glynn, 2009).

The current findings extend the prior research on quantitative problem solving in science by showing that deliberate instruction in modeled process mathematics can improve student problem solving as problems increase in complexity. That is, even with quantitative problem solving, there are benefits to linking equations to scientific concepts that are revealed on more complex problems. In inheritance, traditionally instructed students typically can solve simple genetic probability problems with ease, but struggle with more complex problems (J. Stewart, 1983). J. Stewart (1983) argued that students could not solve complex problems because they lacked an understanding of the underlying genetic processes and were using an algorithmic approach to solving the single gene problems that did not transfer well. The currently obtained results show a qualitative interaction between method of instruction and change in student scores for simple and complex genetic probability problems. Both traditionally instructed and iSTEM-instructed students show a comparable and significant change pre to post in their ability to solve simple genetic probability problems, which if anything is slightly smaller for iSTEM compared to traditionally instructed students. However, traditionally instructed students show little to no change in their ability to solve complex genetic probability problems, whereas iSTEM-instructed students show a significant increase pre- to postinstruction. The finding of a qualitative interaction between simple and complex genetic probability gains for students in the two conditions means that the difference in gains is significant. Indeed, the condition effect on the difference between standardized mean question gains for simple versus complex probability (by teacher) was statistically significant ($F = 5.4$, $p = .03$). This interaction suggests a deeper explanation than that proposed by Stewart (1983): Mathematical procedures that are directly connected to processes provide a method for students to generalize a learned procedure to more complex problems. In other words, it is not that understanding of scientific processes turns an algorithm into something that is generalizable; rather, we suggest that understanding must be connected to the mathematical procedures themselves to obtain generalizable performance. The mechanism of action is not fully resolved. Perhaps by framing the mathematical equation as rooted in and derived from the scientific phenomenon, students are more likely to engage in more productive problem-solving procedures such as blended processing, by mapping meaning to the mathematical equation itself (Bing & Redish, 2008; Kuo, Hull, Gupta, & Elby, 2012; Tuminaro & Redish, 2007). Alternatively, the modeling cycle used to develop and modify the mathematical equation may foster a better understanding of the connections between mathematics and the scientific phenomenon allowing for a "working forwards" approach to problem solving where students can represent and solve the problem in different ways and check their answers (Chi et al., 1981; Taasoobshirazi & Glynn, 2009)."

We postulated that inclusion of modeled process mathematics would not only increase student quantitative problem solving ability but also increase their understanding of the mathematically modeled scientific processes. Curriculum and instructional units that ask students to mathematically model scientific concepts have previously shown improved understanding of the modeled concepts (Lehrer & Schauble, 2004; Liang, Fulmer, Majerich, Clevenstine, & Howanski, 2012; Wells et al., 1995). Our study extends these findings in two ways. First, instead of embedding mathematical equations within a rich scientific context, this intervention specifically asks students to model scientific processes within the mathematical equation. Second, the study shows that within the same unit, processes that were modeled mathematically were better understood than those that were not modeled mathematically. This specificity of which qualitative understandings showed improvements suggests that benefits are unlikely to be due to a generalized effect of iSTEM instruction (e.g., increased student discussion, increased use of scientific practices such as analyzing data or developing an argument from evidence). However, we should note that the gains from pre- to postinstruction for nonmodeled processes were directionally larger for the iSTEM instruction (an effect size of 0.1 *SD*). Given the sample size of the current studies, we cannot rule out that a larger sample size of teachers might also reveal a generalized, if perhaps smaller, effect of the iSTEM instruction.

The current studies did not directly address how the mathematical modeling of scientific processes increases student understanding of those processes. One possible explanation is that by embedding scientific processes within the mathematical model for solving quantitative problem solving, teachers and students are forced to spend more time on those scientific processes. Indeed, in the iSTEM unit, more time is spent on the modeled processes than in traditional instruction. Unlike in traditional instruction where teachers report only briefly presenting in PowerPoint or lecture format these key processes for understanding inheritance, in the iSTEM unit, students are forced to discuss these processes each time they engage in quantitative problem solving. Another possible explanation is that by asking students in the iSTEM unit to develop, and later refine, a mathematical model that is connected to entities and processes in the phenomenon of inheritance, students have to engage in deeper thinking about which entities and processes within the phenomenon are important and how they are linked to one another. Then, in the process of refining the model, students are asked to confront misconceptions about the processes. Thus, students work to construct and refine their understanding of the mathematically modeled processes. Other model-centric approaches could similarly have such benefits through deeper reflection. For example, Cartier (2000) used a model-evaluation approach to provide students with opportunities to develop a better understanding of how knowledge claims are structured in genetics.

Practical Concerns

Instructional approaches to science education that use mathematics raise questions about whether students' mathematics ability then serves as a barrier to accessing science (Maerten-Rivera, Meyers, Lee, & Penfield, 2010). Indeed, physics was historically placed last in the high school sequence because of concerns that the required mathematics was beyond the abilities of many ninth graders (Sheppard & Robbins, 2005). We argue that some forms of mathematics are well within the reach of most ninth graders and can serve a productive basis of science instruction, especially when treated in a modeling approach (i.e., not relying on previously memorized complex mathematical algorithms). The unit was effective in classrooms with relatively low prior ability in solving probability problems, and prior mathematical ability was not a strong predictor of performance, especially not qualitative understanding.

Furthermore, the improved outcomes did not require large increases in instruction on mathematical techniques. Traditional instruction teachers reported spending on average 260 minutes on genetics probability instruction, as compared to approximately 270 minutes in the iSTEM unit. It was the nature of the quantitative instruction that was the larger difference. Traditional teachers report teaching only calculated procedures methods for problem solving versus the scientifically connected modeled process used in the iSTEM unit.

Others have designed instructional interventions that have increased student quantitative problem solving ability and/or understanding of the inheritance processes modeled mathematically in the iSTEM unit. One approach asked students to pictorially represent the processes (Moll & Allen, 1987). While students showed an increase in understanding of the pictorially represented processes, half of the students chose not to pursue the drawing method when engaging in quantitative problem solving. Moreover, those who used a calculated procedure method were more successful at solving complex problems. The authors speculated that this was because representing the processes pictorially became more cumbersome as problem complexity increased.

Two other groups have shown that students increase their understanding of genetics processes (Buckley et al., 2004; Tsui & Treagust, 2003), and one has shown that students also increase their ability to solve genetics probability problems (Buckley et al., 2004), following instruction using a computer simulation that models genetics processes (described in Horwitz, Gobert, Buckley, and O'Dwyer, 2010). However, many science classrooms do not have regular access to computers. Thus, the mathematical modeling of processes in the iSTEM inheritance unit described here provides a low-tech alternative, at least for the biology concepts that could be modeled with relatively simple mathematics. Other aspects of biology, involving more complex mathematics, might be best supported with computer simulation methods.

CONCLUSIONS

We have provided evidence that mathematical modeling of inheritance processes can increase students' ability to solve quantitative genetic probability problems and to answer qualitative questions about the modeled genetics processes. Thus, we have generalized prior findings (Bing & Redish, 2008; Taasoobshirazi & Glynn, 2009), which have suggested that making connections between a mathematical equation and the underlying scientific processes increases the ability to solve mathematical problems in a scientific context. Furthermore, we have provided support for a theoretical idea that modeling scientific processes and entities mathematically through explicit connections between mathematical variables and processes and the entities and processes within a scientific phenomenon increases understanding of the scientific phenomenon. While further research needs to be done into how including modeled process mathematics increases problem-solving ability and student understanding of science, the unit on inheritance presented here provides a successful model of iSTEM instruction that integrates mathematics and biology in an engineering context.

APPENDIX

TABLE A1
THREE WAYS MATHEMATICS CAN BE USED IN INHERITANCE INSTRUCTION

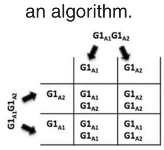
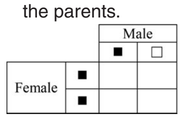
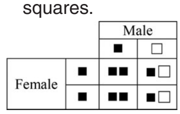
	Embodiment OF Mathematics in Inheritance Instruction		
Name	Punnett Square	Probability Rules Method	Modeled Process
Type of embodiment	Calculated procedure	Calculated procedure	Modeled process
Summary	Depiction of the rule that each parent will give one gene from each trait to offspring combined with an algorithm.	Treat the problem as purely a mathematical probability problem.	Apply a modeled process equation that makes explicit connections between the biology and the mathematical process.
Representation		$P_1 * P_2$ where P_n is the probability of getting genotype of gene *n*	$\frac{W1 * W2}{(\#\text{egg types})(\#\text{sperm types})}$ where W_n is the ways of getting genotype combination of gene *n*
Worked example	Calculate the probability of producing an offspring with the genes ■□ from breeding a male with ■□ genes and a female with ■■ genes.		
Step 1	Separate the 2 genes in the parents. 	The male contains two types of alleles so the probability of passing on one of them is ½.	Determine the number of types of eggs and the number of types of sperm that can be produced (each parent can only contribute 1 gene for each trait): 2 sperm types and 1 egg type.
Step 2	Combine the genes from the parents in the inner squares. 	The female contains one type of allele so the probability of passing on one of them is 1.	Because each egg type can join with each sperm type, multiply the number of egg types times the number of sperm types to obtain 2 possible offspring types.
Step 3	Count how many of the inner squares contain the gene combination of interest: ■□ = 2	Apply the appropriate probability rule: If both events are required then multiply the probability of the two events together.	Refer to the rule that each parent can only contribute one gene for each trait to determine the number of ways that each offspring genotype can be obtained.
Step 4	Count the number of total cells: = 4	½*1 = 1/2	For ■□ offspring, the female can only contribute ■, the male must contribute □: 1 way to get ■□ offspring.
Step 5	Place the number found in step 3 over the number found in step 4 = 2/4 and reduce the fraction = ½.		The probability of a desired event equals the number of desired outcomes as a proportion of the total number of possible outcomes. Place the answer from step 3 over the answer from step 4 = ½.

TABLE A2
TEACHER AND SCHOOL CHARACTERISTICS

Instruction Condition	School ID	Teacher ID	Biology Level	Hours of Professional Development	Exposure Workshop Attendee/ Recruited	Number of Students	Years of Teaching Biology	Masters Degree	Biology Masters Degree	Biology Undergraduate Degree	School ACT	School ACT Sci-ence	School State Sci-ence	School State Read-ing	School State Math	Percent Minority Enrollment	Percent Free and Reduced Lunch
Traditional	1	1	Honors	0	Recruited	24	11+	Yes	No	Yes	19.2	19.2	23	51	22	68	57
	2	3	Regular	5	Recruited	42	0-1	No	No	No	21.6	21.2	35	66	42	26	16
	3	4*	Regular	0	Recruited	55	11+	Yes	Yes	No	17.5	18.2	11	39	13	26	68
	4	9*	Undesignated	0	Recruited	70	11+	Yes	No	Yes	19.4	19.9	16	47	37	15	43
	4	6	Undesignated	0	Recruited	71	NA	NA	NA	NA	19.4	19.9	16	47	37	15	43
	6	13	Honors	0	Recruited	59	11+	Yes	No	Yes	21.5	21.5	38	64	38	14	19
iSTEM	1	2	Honors	25	Exp Wkshp	19	11+	No	No	Yes	19.2	19.2	23	51	22	68	57
	2	3	Honors	5	Recruited	88	0-1	No	No	No	21.6	21.2	35	66	42	26	16
	3	5	Regular	22.5	Exp Wkshp	133	11+	No	No	Yes	17.5	18.2	11	39	13	26	68
	4	7	Undesignated	25	Exp Wkshp	49	6-10	No	No	Yes	19.4	19.9	16	47	37	15	43
	5	10	Honors	25	Exp Wkshp	15	11+	Yes	Yes	Yes	18.1	18.2	17	41	12	47	55
	6	11	Regular	5	Exp Wkshp	82	2-5	No	No	Yes	21.5	21.5	38	64	38	14	19
	6	12	Regular	25	Exp Wkshp	119	11+	Yes	No	Yes	21.5	21.5	38	64	38	14	19
	7	14	Undesignated	10	Recruited	39	11+	No	No	Yes	23.3	23.1	57	63	74	33	8
	7	15	Undesignated	25	Exp Wkshp	46	6-10	No	No	Yes	23.3	23.1	57	63	74	33	8
	7	16	Undesignated	25	Exp Wkshp	17	11+	No	No	Yes	23.3	23.1	57	63	74	33	8
	7	17	Undesignated	10	Recruited	24	NA	NA	NA	NA	23.3	23.1	57	63	74	33	8
	7	18	Undesignated	10	Recruited	114	NA	NA	NA	NA	23.3	23.1	57	63	74	33	8

Asterisk means the teacher taught using both the iSTEM unit and traditional instruction. Teachers 4 and 9 used traditional instruction in Year 1 and then received professional development and used the iSTEM unit in Year 2. Only the data from their traditionally instructed classes is considered in Study 1. NA means that teacher's data was not available. Undesignated means that the school did not designate honors and regular biology classes. These classes were considered as nonhonors classes.

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