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8. HOW COGNITIVE SCIENCE CAN PROMOTE CONCEPTUAL UNDERSTANDING IN PHYSICS CLASSROOMS

ABSTRACT

Cognitive science research focuses on how the mind works, including topics such as thinking, problem solving, learning and transfer. Much of this research remains unknown in science education circles, yet is relevant for the design of instructional strategies in the sciences. We outline some difficulties in learning science, along with a discussion of some relevant cognitive science research. We then present a cognitive science-based intervention in physics education aimed at promoting conceptual understanding within a problem solving context. In addition, we present assessments of problem solving and conceptual understanding to better examine the differences between knowledge learned from this approach compared to traditional instruction. Finally, we present some pilot data on an initial implementation of the approach

INTRODUCTION

Knowing any discipline in the sciences well requires considerable knowledge and skill. Every science has its array of major principles and concepts, which in turn subdivide and expand into a wider array of related sub-principles and sub-concepts. All of this conceptual knowledge needs to be stored in memory in ways that are efficient for recall and deployment, which means having a richly interconnected network that both relates relevant knowledge and discriminates unrelated knowledge. Also needed are techniques and procedures for applying conceptual knowledge to solve problems, especially in the physical sciences. This in turn requires mathematical and reasoning skills in order to build coherent solutions and/or arguments. Further, in order to be able to apply conceptual and procedural knowledge flexibly to solve problems, one also needs to have this knowledge richly linked to the multitude of contexts in which it can be applied. Even this cursory analysis lends credence to the common perception that science is a difficult subject to both teach and learn.

Physics instructors at all levels value their deep conceptual understanding of physics, and indeed consider this to be the beauty of physics—that a few major principles govern the behavior of the physical world. What has been apparent for a number of years is that too many students are emerging from introductory physics courses taught by very competent instructors at both the high school and university

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levels with the same major flaws in their understanding of *basic* physics concepts that they entered the course with (Bowden, Dall'Alba, Martin, Laurillard, Marton, Masters, Ramsden, Stephanou and Walsh, 1992; Mazur, 1997; McDermott, 1993). For example, tests like the Force Concept Inventory (Hestenes, Wells & Swackhamer, 1992) and the Conceptual Survey of Electricity and Magnetism (Maloney, O'Kuma, Hieggelke & Van Heuvelen, 2001) have documented students' poor conceptual understanding immediately following the completion of introductory courses. We seem to be doing a much better job at teaching problem solving than conceptual understanding, but problem solving skills without the underlying conceptual base to guide and support them are fragile skills soon forgotten after physics courses are over. Physics instructors' goals in teaching physics are not just conceptual understanding or problem solving skills, but the integration of the two, and traditional approaches to teaching physics fall short in two of these goals.

From the teaching side, traditional strategies for teaching and assessing science at the secondary and post-secondary levels are successful in some regards, but also unintentionally send mixed messages that work against learning conceptual knowledge. For example, in teaching physics, most instructors mention the major principles/concepts that they are applying when they model problem solving for students, but most often it is only the equations used to implement the principles that are written down for students to view. This inadvertently conveys to students that it is the equations, not the concepts that are important. Further, traditional instruction does little to relate and synthesize major ideas, especially within problem solving contexts—there is, in fact, no natural mechanism by which students can see clearly and often the role of conceptual knowledge in problem solving. Assessments used to evaluate students' knowledge don't help alleviate the situation since they typically focus on problems requiring numerical solutions; students, therefore, quickly realize that in order to perform well they need to be able to find and manipulate equations to get answers, and finding and manipulating equations that match the variables/quantities given in problems is something they do reasonably well. What is needed are instructional strategies that directly promote the learning of conceptual knowledge while continuing to help students develop problem solving skills.

Findings from cognitive science can provide guidance on ways to structure instructional activities that promote and integrate conceptual knowledge within problem solving contexts. We begin with an overview of findings from three areas of cognitive science, *specialized systems*, *specificity of knowledge*, and *transfer appropriate processing*, related to the nature of knowledge in memory, learning, problem solving, and transfer. We continue with a more specific overview of learning research into problem solving and conceptual understanding in the domain of physics, and suggest how those findings can be translated to instructional practice in science teaching. We then describe a specific application in physics that draws from cognitive science in designing problem solving instruction that highlights the role of conceptual knowledge. A pilot study is discussed that implements the approach with students at risk of failing an introductory course and compare their performance on conceptual and problem solving measures to that of regular students. We conclude with a brief discussion of the future role of cognitive science in science teaching and learning.

COGNITIVE SCIENCE

Cognitive Science examines how the mind works. Given the importance of the mind in learning, some research and ideas in Cognitive Science may be helpful in thinking about instruction in the sciences. We outline here three general types of problems in learning and problem solving in science, discuss some relevant research in Cognitive Science, and the implications of this research for instructional practice. Although there are a wide variety of possible topics one could look at, we have chosen three that seem particularly apt given our examination of physics learning: specialized systems, content-dependence, and transfer-appropriate processing.

Specialized Systems

Problem: As mentioned earlier, in science learning there is often a focus on how to solve a problem with little emphasis on the underlying conceptual knowledge. This may lead to a situation where a student is able to solve a difficult problem but be unable to explain the underlying conceptual knowledge. Instructors often find this frustrating in that it seems that if someone is able to solve these problems, we assume they must have some understanding. Although such a disconnect may be caused by a variety of issues, one possibility is that the part of the mind that guides the problem solving is separate, in some important sense, from the part that might have a conceptual understanding of this type of problem.

Some relevant cognitive science: An important idea that has emerged from work in Cognitive Science and neuroscience is that we have evolved specialized systems for addressing the myriad of tasks we need to survive. Although science learning is not usually considered a necessary survival task we make use of these systems when solving such tasks. These systems have different purposes and somewhat different means of representing and retrieving knowledge. A useful distinction for examining science learning is that between declarative and procedural memory (e.g., Anderson, 1976; Cohen & Squire, 1980). The evidence for this distinction includes behavioral studies showing different patterns of performance, neuroscience research showing differences in activated brain regions, and amnesia work in which the amnesiacs can show little declarative memory but show no diminished procedural memory.

The term *declarative memory* refers to factual, articulable knowledge, "knowing that" information. When someone asks "Do you remember that…" that person is querying declarative memory. Examples of declarative memory in science include a definition, a formula, or the answer to a problem. The hallmarks of such memories are that they are learned quickly and can be flexibly accessed and applied. You can learn $F = ma$, access it by the cues "force," "mass," or "acceleration," and solve it for any one of the variables given the other two.

The term *procedural memory* refers to "knowing how" to do something (a procedure). However, it is not the recipe for doing it, such as in a cookbook (that is declarative information), but rather the processes that accomplish the procedure. For example, those of us who have learned to drive a stick shift car many years ago can still do it, but often have trouble explaining it to others. A common tactic in such cases is to perform the action and observe how you are doing it, then communicate that observed set of actions to the learner. As that person practices, she will learn to operate the manual transmission too. Eventually, as she becomes skilled and hones her technique, the declarative information from which she began will become much less accessible.

Implications: We need to understand that different types of knowledge may be largely separable so that evidence that a student knows how to solve a problem is not evidence she understands the underlying conceptual knowledge. If we want students to understand the underlying principles in a domain, we need to not just mention those when showing how to solve problems, but also to give students practice in accessing and applying their conceptual knowledge and we need to make this part of the assessment for the course. Suggestions on how to do both of these are provided later.

Of course, some types of knowledge might be highly interrelated, but the general message is that we need to decide what type(s) of knowledge we want the students to learn and make sure that they practice acquiring that knowledge. Although it might seem reasonable that giving someone practice at solving problems of a particular type should give them a better understanding of that type, it is not always true.

Specificity of Knowledge

Problem: A common occurrence among students is that they often decide on the type of problem they are facing based on the superficial aspects of the problem, such as the objects, rather than a conceptual analysis of what is going on. Given a physics problem with blocks, pulleys and an inclined plane, they are likely to try to use Newton's Second Law for solving it when work/energy concepts may be a much more efficient, or perhaps the only appropriate approach for solving the problem. Thus, very specific formally-irrelevant material may be used as a cue to the type of problem (either correctly or not).

The effects of this specificity go much further than simply cuing problem types. We have all had cases with students (or even ourselves) who look back to earlier problems in a chapter section and try to use those problems to solve the current one by figuring out which aspects of the earlier problems correspond to aspects of the current problem. That is, rather than understanding the concepts and the meaning of the variables for instantiating a principle, learners are using a much less conceptual mapping between similar aspects of the problems.

Some relevant cognitive science: People have difficulty thinking abstractly. Perhaps this should not be surprising given that almost everything people think about is fairly concrete. We could try to solve the everyday task of how to most efficiently accomplish errands we have in our city or town by using an analytic solution based on the distances and routes, but we don't. In this case perhaps there are additional constraints that make such solutions hard (ice cream cartons should not be bought right before one has a haircut appointment), but mainly people think about specific objects and events, not abstract ones.

Formal education, and particularly science education, often asks learners to think quite abstractly. Consider a student who gets a physics problem about a particular situation. She has to characterize the aspects more abstractly (e.g., are there non-conservative forces doing work on an object), assign abstract variables to the various aspects, and then describe the relations mathematically (abstractly) in terms of these variables. What are the (often invisible) forces that operate on the object, how are they quantified, and which one(s) are conservative and which are non-conservative, and how does one deal with both types of forces in applying work-energy principles in equation form to attack the problem? Determining all this is very abstract and difficult. Instructors often do not appreciate the difficulty well, due to their expertise and, most likely, their self-selection to work in an abstract domain. What do we know about this problem from Cognitive Science research?

The anecdotal characterizations of students using superficial aspects of problems is well supported by a variety of experimental research. Students will use the content of a word problem, often available a few words into reading it, to decide on the likely problem type and bring knowledge about that problem type to bear on solving the problem (Blessing & Ross, 1996; Hinsley, Hayes, & Simon, 1977). Experts are more likely to do a conceptual analysis of the underlying principles (e.g., Chi, Feltovitch, & Glaser, 1981; Hardiman, Dufrense, & Mestre, 1989), but even experienced problem solvers rely on usually-predictive content to access potentially relevant knowledge earlier (Blessing & Ross, 1996). Even in formal domains there are often correlations between content and structure, partly due to the constraints of the domains (e.g., physics which is obliged to obey the physical laws) and perhaps partly due to lack of imagination of the textbook authors (though it is certainly true that some problem types lend themselves to particular contents much more easily than to others).

However, the specificity of the knowledge goes further than using the content of the problem to access relevant categorical problem knowledge. In fact, it can influence the specifics of how the problem is solved and this effect has implications for our instruction. When we teach a new concept, it is common to present the concept and then give an example to illustrate that concept. The illustrative example is presented because we recognize (at least at some level) that the abstract characterization of the principle is inadequate for instruction. It is often difficult to understand exactly what is meant and exactly how the principle would be applied without seeing it applied to a particular case. Thus, we might go over the example in much detail, pointing out the correspondence to the parts of the principle. We then assume that the student understands the principle, but this is not the full story. Instead, what often happens is that students' understanding of the principle becomes bound up with the example used to illustrate it. When solving a later problem, even if that principle is presented, solvers often end up relating the new problem to the illustrative example (Ross, 1987, 1989).

An example may help make this abstract idea more clear (sorry, we could not resist). In a short teaching experiment, physics students learned about the principle of refraction. We recruited 48 undergraduate engineering majors in a calculus-based physics course from the University of Illinois at Urbana/Champaign. Participants were randomly assigned to one of two training conditions that each used a different illustrative example, a triangle prism (T-trained, n=25) or a rectangle prism (R-trained, n=23). In each of the training conditions, the participants were given a lecture about the principle of refraction. The contents of the lectures were identical across the conditions except for the example that was used to instantiate the principle of refraction near the end of the lecture. (See Figure 1 below) The lecture lasted about 20 minutes.

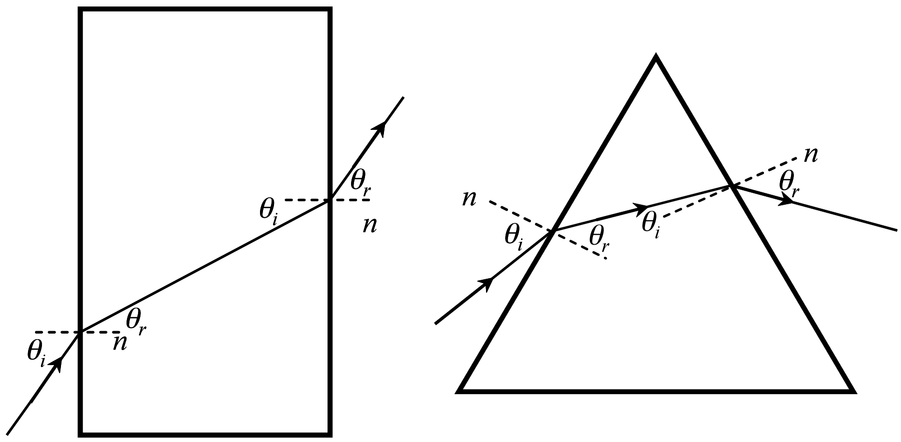

Figure 1. The two illustrative examples used to instantiate the principle of refraction during the refraction lecture.

In a transfer test that immediately followed the lecture, we presented subjects with diagrams of glass prisms with an incident ray (one diagram per page) and asked them to complete the ray diagram in each case. Some of the test questions were designed to share superficial features with one or other of the training examples. Figure 2 shows the effect of training example on subjects' ability to transfer their learning to new contexts. The first line shows the responses of a R-trained subject to two of the test questions. Notice how he is able to transfer his learning successfully in the case of the triangular prism, but when presented with a rectangular prism scenario (bearing superficial similarity to his training example) he bends the light ray exiting the prism in the wrong direction. The second line shows the responses of a T-trained subject to the same two test questions. Note how he is able to successfully transfer his knowledge of refraction in a new context (the rectangular prism) but incorrectly bends the light ray exiting the triangular prism slightly below the normal. Both subjects were able to correctly apply the principle of refraction in a third question that bore no superficial similarity to either training example. This specificity effect was substantial, affecting a total of 20 out of the 48 subjects.

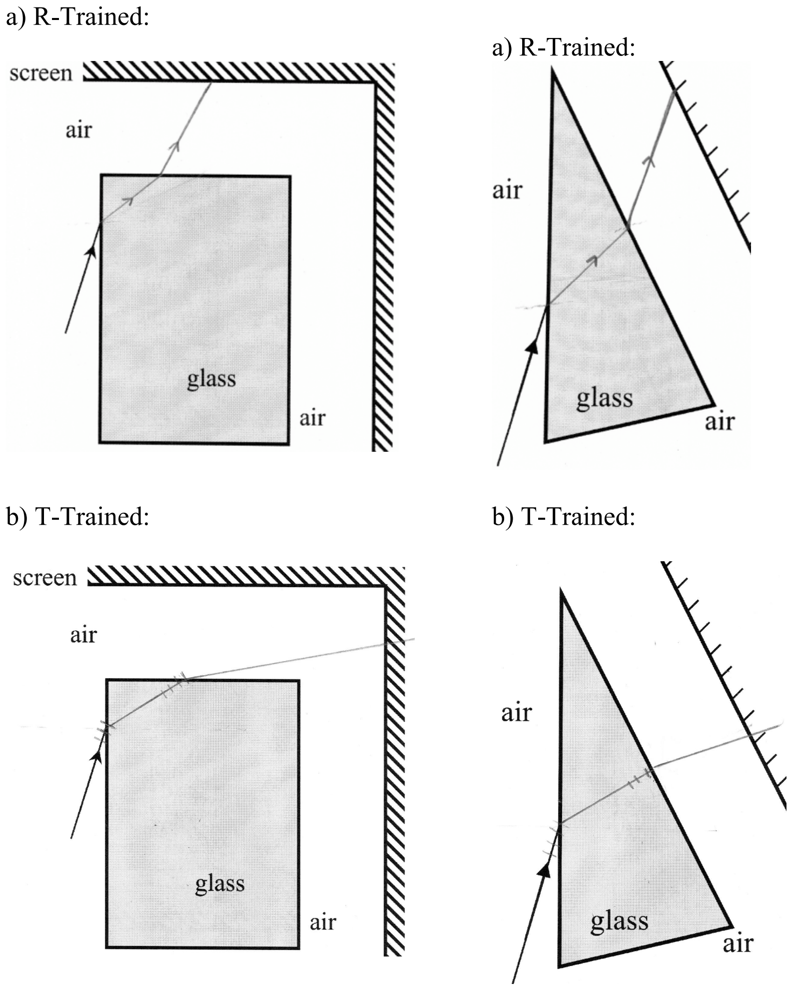

Figure 2. Example of responses of two subjects, R-trained (first line) and T-trained (second line) to two of the test questions.

A common practice that might ameliorate this difficulty somewhat is to give students multiple examples to illustrate the instantiation of important principles in a domain. Even if they learn specifically about one, perhaps the combination of multiple examples will lead them to generalize and leave them with a more abstract representation of the examples. However, there is good reason to believe that this representation will still be less abstract than we desire. First, it is not at all clear that people generalize across problems unless they have some task that forces them to, such as using one problem to solve another (Gentner, 1983; Ross, 1984). Second, even when multiple examples are given and there is some evidence of generalization, it is often very conservative generalization (Gick & Holyoak, 1983; Ross & Kilbane, 1997). People learning a concept often are not sure exactly what is relevant and what is not so without explicit predication (to themselves) of the relevant abstraction, the mind keeps much of the surrounding content.

One practice, not often conducted in classes or homework, that appears to help people to better generalize is to require a comparison between problems (e.g., Rittle-Johnson & Starr, 2007; Ross & Kennedy, 1990). By explicitly requiring students to make a comparison between the current problem and some earlier one, the irrelevant aspects of each problem are better recognized, as is the underlying commonality, such as the principle needed for solution.

Implications. It is important to be clear about what we see as the implications of this research in Cognitive Science. We do not want people thinking that we believe illustrative examples are not important in learning. Obviously, to the extent we want students to learn how to solve problems, giving them problems is necessary. Rather we see this research as pointing out some non-obvious unintended consequences of common instructional practices that need to be considered more thoroughly by instructors. We mention two here.

First, illustrative examples are probably necessary in many cases. Principles are simply too abstract to be fully understood without examples. However, it is important to appreciate the extent to which some examples may become intertwined with the understanding of the principle. Although research is lagging a bit behind in terms of what might be better ways to teach, we mention two practices that seem likely to help given our understanding of the cognition involved. One, when going over the illustrative example be clear to the students how the parts of the principle are corresponding to the parts of the example. This may not lead the learners to generalize as much as you'd like, but it at least makes clear to them the illustrative nature of the example and clarifies explicitly the abstract parts of the principle. Two, use at least one other example soon and draw the same explicit connection to the principle parts.

Second, when using these multiple examples, it is important to force explicit comparisons of problems and give feedback on these comparisons (Rittle-Johnson & Starr, 2007). Providing multiple examples may give learners multiple separate sources from which to try analogically to solve later problems, but forcing comparisons may lead to the desired goal of generalization.

Transfer-appropriate Processing

Problem: A common problem is that students seem to understand the material when examined in some ways and do not understand it when examined in other ways. Similarly, an instructor will go over material or have a student do it in one way and then find the student cannot express that knowledge when asked about it another way. More generally, the ability to show they understand what is being taught seems difficult to predict and understand, so it is very difficult to instruct so that this does not happen.

One can think of this as a more subtle, even less-obvious, version of the procedural versus declarative distinction mentioned earlier, which also touches upon the just-discussed content-dependence. Here, the problem seems to be not that the knowledge is in a form for a different system, nor that the knowledge is shrouded in a different content that hides its relevance. Rather, the relevance of the knowledge would be clear IF it were accessed, but the learner never accesses it. Bransford et al. (1989) referred to this as "inert" knowledge.

Some relevant cognitive science: Retrieving relevant knowledge is a central function of our memory system and underlies our abilities to accomplish many goals. Although most people have many misconceptions about memory, we do know that different cues to memory will result in different probabilities of accessing the desired information. We also know that different ways of learning some material are likely to result in different probabilities of being able to remember the information later. If one reads over a chapter on mechanics painstakingly looking for typographical and grammatical errors, it is less likely that one will remember the concepts and formulae than if one reads it carefully going over the worked-out examples. What is less obvious is that even if you process the material in a conceptual way, the particulars of that processing will influence the probability that you will be able to access the relevant knowledge later. It will depend upon how you are processing the material later.

So, how you process something during study will influence what you can recall of it later. In addition, the process you use in trying to retrieve the information will influence what you can recall. The final point is that the match between processing at study and retrieval has an additional influence on what you can recall—this is the concept of *transfer appropriate processing* (Morris, Bransford, & Franks, 1977).

We give a simple example and then relate it to science learning. Suppose you are shown a list of words, one word at a time. For each word, you are asked one of two questions, either about the sound of the word (e.g., "Does it rhyme with *weight?"*) or about the meaning of the word, such as whether it fits in a sentence (e.g., "He met a ___ in the street?"). The well-known (and intuited) result is that when asked to recall words later, recall is higher for words for which sentence questions were asked than ones for which rhyme questions were asked (e.g., Craik & Lockhart, 1972). Most people can see why this is--one requires semantic (meaning) processing and one does not. However, if at test one is given cues for recall that either involve the rhyme given at study or the sentence given at study, an interaction occurs. Words encoded as rhymes at study are better remembered when cued with rhymes at test and words encoded as sentences are better remembered when cued with sentences at test (Fisher & Craik, 1977). (For those interested, words encoded with sentences are overall still remembered much better.)

So, what does this have to do with science learning? Whenever we learn something, we are processing it in a particular way. Our ability to retrieve that information at a later time will depend upon the processing we are likely to be doing at that later time. Applying this idea is not as easy as it sounds since it requires analyzing what might be learned from the learning task and how it relates to the later task. The results can be rather surprising.

For example, if the goal is to have students be able to solve problems of a particular type, which activity is likely to be better for study: having them solve problems or having them study worked-out examples? Most people, we believe, would think that solving problems would be better. However, a variety of research suggests that solving problems is often not a good strategy for later problem solving and that studying worked-out examples is better (Cooper & Sweller, 1987; Paas 1992; Paas & van Merrienboer, 1994). Paas (1992) taught students basic statistical concepts (mode, median, mean) and included two training conditions of interest here. In one group the learner solved each problem whereas in the other group they were shown worked-out examples and only solved every third problem. The latter group transferred better to later test problems. Perhaps it is having two different learning tasks that helps? No. Paas and van Merrienboer (1994) contrasted this worked-out plus problem solving condition to a pure worked-out example condition. The former condition took longer and was more effortful (as determined by a measure of cognitive load), but the pure worked-out example condition led to substantially higher transfer performance.

Why are worked-out examples more effective for learning than problem solving? Most of us find this counterintuitive—it seems that if one wants to improve problem solving that giving people practice in it should be most helpful. The answer has to do with the difficulties of learning complex cognitive tasks. As tasks get complex, they require more processing capacity, more things to keep track of, and this is a limited capacity (as elaborated in the well-known Cognitive Load Theory of Sweller, 1988). The idea is that problem solving has two particularly negative effects on learning. First, it is a very high-processing task as learners have to decide upon plans, keep track of various subgoals, check whether they appear to be going in a useful directions, plus perform the various calculations and manipulations to solve the problem. Second, learners often make mistakes in solving problems and those mistakes both take processing capacity and may be retrieved when performing later problems, leading them astray again. Because we have limited processing capacity, focusing on irrelevant or incorrect aspects and procedures of the task, means that less processing capacity is available for the relevant aspects.

Worked-out examples address both of these problems. First, many of the processing-limited tasks are taken care of in the worked-out solution. The problem solver does not have to plan, keep track of subgoals, or perform algebraic manipulations. Second, erroneous solution paths are avoided. Although one could argue that learners also lose practice at determining how to plan the problem solving, the results are quite clear: the benefits of focusing on the correct solution and using their capacity to understand how the steps follow is much more effective a way of mastering routine problem solving.

Implications: These ideas and results strongly suggest two changes to instruction. First, most practically, worked-out examples should be used much more than they currently are for most of us. They take less time and effort during study than conventional problem solving and lead to greater transfer. This seems like a modest change in instructional practices that could have major learning implications. We do think instructors need to also take into account the active learning of the student—the worked-out examples will not be helpful if the learner cursorily reads them over. Rather, they must be directed to relevant aspects and probably some activity (e.g., occasional problem solving or completion of worked-out solutions) is needed both to keep them alert and to check that they are.

Second, more generally, we need to think more about what is being learned from each task and how it relates to what we want students to learn. This examination of worked-out examples shows that the link between the task and the learning can be subtle, so it is likely that we need to rely less on intuition and more on careful experimental research.

SUMMARY

In this section, we have tried to take some common problems in science learning, analyze them from a Cognitive Science perspective, and suggest particular instructional changes that might be implemented. First, we need to recognize the separation of problem solving skill and conceptual understanding. Instruction needs to provide a way of rapprochement, perhaps by having the problem solving more integrated into the conceptual understanding. One possibility is to require conceptual analysis and only allow problem solving to be done once the problem has been broken down into its conceptual parts.

Second, we need to realize that illustrative examples both illustrate and constrain learners' understanding of new concepts. Although examples are important, we need to try to overcome the dependence on particular examples by having multiple examples and requiring comparisons between examples to promote generalization.

Third, the particular way that problems are processed will influence later problem solving performance. Results suggest that focusing more on worked-out examples and less on problem solving will be beneficial.

Research in Physics Problem Solving and Conceptual Understanding

In the previous section we reviewed findings from cognitive science that have a bearing on developing problem solving skills with conceptual understanding. In this section we review studies exploring similar issues but specific to the domain of physics.

A series of studies (Dufresne, Gerace, Hardiman & Mestre, 1992; Mestre, Dufresne, Gerace, Hardiman, & Touger, 1993) suggests that students benefit from practicing concept-driven approaches to solving physics problems. Following practice with a rudimentary menu-driven computer tool that constrained its users to follow a conceptual analysis prior to solving mechanics problems, physics novices displayed expert-like attributes following a few hours of practice. The computer-based tool, termed the "Hierarchical Analysis Tool" or HAT, was modeled after research findings from Chi, Feltovich and Glaser (1981) indicating that, when asked to state an approach for solving a physics problem, experts began by identifying a principle, then justifying why the principle applied to the specific problem context, then describing a procedure for instantiating the principle. In this 'working forwards' problem-solving approach (Simon & Simon, 1978), one reasons from the principles or deep structure of the problem to the solution. Using the deep structure to guide expert problem solving has been shown across many different domains including chess (Chase & Simon, 1973; Gobet & Simon, 2000), computer programming (Adelson, 1981) geometry (Koedinger & Anderson, 1990), and mathematics (Silver, 1981), among others. The purpose of the HAT was to create an environment that helped novices perform (and hopefully learn) expert-like conceptual analyses.

The HAT used a hierarchical framework to structure qualitative analyses of mechanics problems on the basis of principles and procedures. To analyze a problem, the user answered a series of well-defined qualitative questions about the problem under consideration by making selections from menus. When the analysis was completed, the HAT provided a set of equations that were consistent with the menu selections made during the analysis. If the analysis was carried out appropriately, then those equations could be used to generate a solution to the problem. Note that the HAT was a "blind" tool in that it contained no feedback or tutorial features since no information was programmed in the HAT about the problem being analyzed.

Despite the HAT having no feedback or tutorial features to indicate whether or not the analysis was appropriate, moderate success in using the tool to solve problems resulted in improved ability both in categorizing problems according to the underlying principles needed for constructing a solution (Chi, et al., 1981; Hardiman, Dufresne & Mestre, 1989; Schoenfeld & Hermann, 1982) and in problem solving, compared to control treatments (e.g., using a searchable equation data base, or the textbook) where students solved the problems using more traditional, equation-based approaches.

Another study (Leonard, Dufresne & Mestre, 1996) conducted in a large introductory college physics course focused on the effect of a different implementation of the Chi et al. (1981) findings that experts verbalized the principle/justification/procedure when asked for the approach they would use to solve a problem. Leonard et al. compared two large introductory physics classes—one taught traditionally, the other requiring students to write *strategies* to accompany problem solutions. A "strategy," as defined in that study, consisted of a conceptual analysis of a problem containing three pieces: the major principle(s), a justification for why the principle applied to the specific context, and a procedure for applying the principle. In short, strategies were prose paragraphs that discussed qualitatively an approach that could be applied to solve a problem. Strategies were modeled in lecture when problems were worked out: First a strategy was presented followed by an algebraic solution showing how the strategy was instantiated. Figure 3 below displays a moderately difficult problem from the third hour exam, together with two student strategies, one very good, one poor. Note how the good strategy implements the three components of a strategy, and how the poor strategy is tantamount to a "fishing expedition" consisting of physics terminology.

When compared to students in a traditional course, students in the course requiring strategy writing were significantly better at categorizing problems according to the principles needed for solution. In addition, when the two classes were divided into quartiles according to performance on the final exam, students in the lowest quartile from the strategy class were equivalent in problem categorization performance as students in the first quartile from the traditional class. Perhaps most interesting, students from the strategy writing class displayed better long-term retention months after the course was over as measured by ability to identify the most important physics ideas used to solve mechanics problems.

Strategy writing is effective for several reasons. First, it focuses students on the critical conceptual aspects of the problem. By having to identify the appropriate principles students learn to look for the deep structure of the problem. Determining the principle can then guide further problem solving, such as determining the appropriate procedures to solve the problem. Second, students have to generate justifications or explanations as to why the principle applies to a given problem. Much work in Cognitive Science has shown that explanation is a powerful learning activity (see Chi, 2000; Roy & Chi, 2005; Siegler, 2002 for reviews). By explaining why a given principle applies the student must generate a series of inferences as to the application conditions for the principle. Explanations facilitate learning through the generation of inferences, or causal connections, that help to link concepts together. In addition, explanation provides the learner an opportunity to identify misunderstandings and correct them (Chi, 2000). Finally, in strategy writing the students must explicitly relate the conceptual knowledge to the problem features. It is likely that this process of building relations through explanation helps the learner understand the relevant connections between their conceptual and procedural knowledge. Integrating conceptual and procedural knowledge is often described as a marker of deep understanding (Rittle-Johnson, Siegler, & Alibali, 2001; Silver, 1986). In the next section we describe how we build upon research from physics education research and from Cognitive Science in designing an instructional intervention.

In this section we provide an example of curricular materials based on the research literature reviewed in the previous sections and specifically designed to promote conceptual understanding within the context of problem solving in physics. The example is not meant to illustrate *the* way to design conceptual learning materials based on cognitive science but rather *a* way. In the major section that follows, we will provide some preliminary findings on the effect of implementing the materials in a class of students that were identified to be at risk of doing poorly in a calculus-based introductory mechanics course.

A disk of mass M=2 kg and radius R=0.4 m, has string wound around its rim and is free to rotate about an axle through its center. A block of mass m=1 kg is attached to the end of the string and the system is released from rest with no slack in the string. What is the speed of the block after it has fallen a distance d=0.5 m.

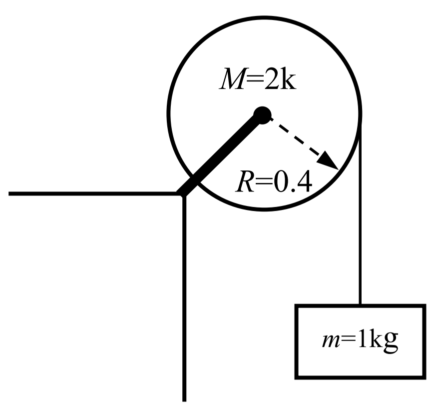

Figure 3. Sample strategies written by two students

Good Strategy Written by a Student

Use the Conservation of Energy since the only non-conservative force in the system is the tension in the rope attached to the mass M and wound around the disk, (assuming there is no friction between the axle and the disk, and the mass M and the air), and the work done by the tension to the disk and the mass cancell [sic] each other out. First set up a coord. syst. so the potential energy of the system at the start can be determined. There will be no kinetic energy at the start since it starts at rest. Therefore the potential energy is all the initial energy. Now set the Initial energy equal to the final energy that is made up of the kinetic energy of the disk plus the mass m and any potential energy left in the system with respect to the choosen [sic] coord. system.

Poor Strategy Written by a Student

Using d=0.5 m, we could find θ and from there we could figure out the time using θ, α, and ω_0. After we found this, we could use that, along with the other given information to determine the angular speed. Once we know this, we can relate the angular information to the block.

Design of Physics Material Informed by
Cognitive Science and Physics Education Research

Description of Instructional Materials

There are theoretical and practical design constraints that need to be met in order to construct materials that both help develop a more conceptual problem solving approach, and that would be considered for adoption within an established and tradition-laden physics education community:

– Ease of implementation. Since the way physics is taught at the undergraduate level is not likely to change dramatically in the short term, any instructional materials that have a hope of being adopted need to be easy to implement.
– Integration within problem solving. Buy-in from students is important, and given that students' grades are largely driven by problem solving performance, it is important to demonstrate the usefulness of a conceptual approach to solving physics problems.
– Transparency of conceptual knowledge in problem solving. The materials need to make the tacit conceptual knowledge used by experts in solving problems visible for novices.
– Reduction of memory load. In order for students to learn concepts while solving problems, it is not productive for them to devote inordinate memory capacity to crafting the actual solution. Therefore, problems within which our approach is embedded should not be extremely challenging so that students are be able to focus on the conceptual and strategic aspects of the solution.
– Making conceptual knowledge generalizable. To avoid specificity of conceptual knowledge it is important to provide a variety of examples for applying principles/concepts across a wide variety of contexts.

The Appendix contains an example of part of the materials meeting these constraints. We call our approach c*onceptual problem solving*. The approach assumes that students have been introduced to the concepts in class and are ready to begin applying them to solve problems. For each major concept (e.g., conservation of energy) the materials begin with an invitation for students to take a strategic approach to problem solving. Students are not used to, and are often clueless about, how to analyze a problem strategically, so we begin by giving them a worked out problem that illustrates the approach; this reduces memory load and allows students to internalize what we mean by a strategic approach. The strategic approach is based on the physics studies reviewed earlier. The solution to each problem begins by indentifying three strategic components needed for a solution which often remain tacit in problem solving instruction. These components are: 1) The major *principle* (i.e., declarative knowledge), 2) A *justification* for why that principle applies to this particular problem (i.e., linking declarative knowledge to problem attributes), and 3) A *plan* for applying the principle to generate a solution (i.e., procedural knowledge). Following the strategic analysis comes a *two-column solution* that shows how the principle, concepts, and procedures are implemented in equation form to generate an answer; the two-column solution links the plan and the equations used. Thus the approach separates the equation-manipulation aspects of problem solving from the conceptual analysis that precedes it.

After the initial illustrative worked out example, students are asked to work a parallel problem on their own using the same approach. Note how the second problem is chosen to show how a problem that looks very different from the previous one in surface similarity can be solved with almost exactly the same strategy and solution. In cases where students are asked to generate their own strategic analysis and two-column solution, they are provided with feedback so that they may compare their work to the ideal. Although we do not show more of the details here, following the second problem students are asked a series of three questions comparing and contrasting the two problems, the strategic analyses, and two-column solutions so that they can begin both to generalize how principle/ concepts are applied across various contexts and to realize that surface attributes of problems are not always the best clue for determining solution similarity. As reviewed earlier, this latter point is important since it is well known that students tend to rely on surface similarity of problems when deciding on solution similarity—a finding that has also been shown in physics and mathematics problem solving (Chi, et al., 1981; Hardiman, et al., 1989; Schoenfeld and Hermann, 1982). Feedback is again provided after students answer the three compare/contrast questions. Near the end of the course when students have covered all the major principles/concepts, some problem pairs are juxtaposed to demonstrate the converse of the above, namely that two problems can look extremely similar in surface attributes and story-line but are solved with entirely different principles and procedures.

Pilot Study of Materials with At-Risk Students

We developed conceptual problem solving materials modeled after the examples in the Appendix, and piloted them using a supplemental course offered concurrently with the calculus-based introductory mechanics course for scientists and engineers at the University of Illinois at Urbana-Champaign. In addition to their normal lecture, laboratory, and discussion sessions associated with the mechanics course, the at-risk students spent one additional two-hour session each week working collaboratively in small groups on the conceptual problem solving and compare/contrast activities described above and in the appendices. The students recruited for this supplemental course were previously identified as being at-risk for poor course performance based on their incoming standardized test scores, high school physics experience, and performance on a mathematics and physics diagnostic test.

To explore the effects of these activities, we gave participants a problem categorization task at the end of the semester. We presented four physics multiple choice problems with a list of five major principle combinations learned in the mechanics course (see Figure 4 for a sample problem). We asked participants to choose the single-principle or two-principle combination required to solve the problem without actually first solving it. We compared their responses to baseline responses collected at the final examination from a previous semester of the non-supplemental course where none of the students used the conceptual problem solving materials described in the appendices, but where some web-based homework activities were conceptual in nature. While there was some variation among the four questions, overall the at-risk students in the supplemental course who used the conceptual problem solving materials scored 65%, while the baseline results for "not-at-risk" students was 60%, and 57% for at-risk students who chose not to participate in the supplemental course.

To assess participant buy-in and motivation for the materials, we surveyed participants in the supplemental course at the end of the semester on their perceptions of the conceptual problem solving materials. Of the total responses, 48% reported that the conceptual problem solving materials either contributed "a great deal" or be "essential" to their conceptual understanding (4 or 5 on a 5 point scale). In addition, 46% reported the same contributions to their overall performance in the parent mechanics course. This is significant, as the supplemental course participants' mean examination scores (73%) were identical to other at-risk students who did not take the supplement (73%). All the at-risk mean exam scores were below the rest of the course (78%) irrespective of participation in the supplemental course. Participants perceived the activities as contributing strongly to their understanding and course performance despite having mean examination performance identical to other at-risk students. This represents significant perceived value attached to the conceptual problem solving activities. While their problem-solving ability still lags that of the rest of the course, at-risk participants' ability to categorize problems is as good as or better than that of both other at-risk students and the rest of the not-at-risk students in the course. Our interpretation of the at-risk students' positive perception of the conceptual problem solving materials is that they recognize that the materials sharpen their somewhat initial "fuzzy" understanding of the principles/concepts covered in the main course, and they begin to see how those principles/concepts can be used to analyze and solve problems.

Please answer the following multiple choice questions. Choose one answer for each question.

A 1 kg stick stick of length 2 m is placed on a frictionless surface and is free to rotate about a vertical pivot through one end. A 50 g lump of putty is attached 80 cm from the pivot. Which of the following major concepts should be applied to determine the magnitude of the force between the stick and the clay when the angular velocity of the system is 3 rad/s in the most efficient manner?

a. Newton's Second Law

b. Work-Energy Theorem of Conservation of Mechanical Energy

c. Linear Momentum or Conservation of Linear Momentum

d. Conservation of Linear Momentum followed by Conservation of Mechanical Energy

e. Angular Momentum or Conservation of Angular Momentum

Figure 4: Sample Problem Categorization Task

The question depicted above is an example of the tasks used to assess problem categorization ability. This particular item was the most difficult on the assessment. Novices often associate its terminology and the presence of a rotating object with angular momentum. However, the question asks for force, necessitating instead the use of Newton's second law.

CONCLUSION

We began by identifying an important problem in physics education (and likely in other STEM disciplines), namely that students seem to emerge from introductory courses with reasonable problem solving skills but without the accompanying understanding of the concepts that underpin their problem solutions. We then reviewed cognitive findings from three areas that can provide insights for addressing this important problem. An approach was presented, *conceptual problem solving*, that is informed by both cognitive research findings and problem solving research in physics education intended to integrate conceptual analyses of problems within a problem solving context. The conceptual problem solving approach attempts i) to integrate an analysis of concepts within the activity of solving the problem itself, ii) to make use of worked examples in order to reduce the cognitive load that the conceptual approach places on students, and iii) to make use of contrasting cases to encourage students to generalize across examples and develop a deeper conceptual understanding. The conceptual problem solving approach is easily implemented in traditional instructional settings. A pilot study using the conceptual problem solving approach with students at risk of failing an introductory physics course for engineering students suggests beneficial outcomes, but more research is needed to ascertain how the approach can be “taken to scale” in large introductory courses and how it benefits regular students.

REFERENCES

Adelson, B. (1981). Problem solving and the development of abstract categories in programming languages. *Memory and Cognition*, *9*, 422–433.

Anderson, J. R. (1976). *Language, memory, and thought*. Hillsdale, NJ: Erlbaum.

Bowden, J., Dall’Alba, G., Martin, E., Laurillard, D., Marton, F., Masters, G., et al. (1992). Displacement, velocity, and frames of reference: Phonomenographic studies of students’ understanding and some implications for teaching and assessment. *American Journal of Physics*, *60*(3), 262–269.

Blessing, S. B., & Ross, B. H. (1996). Content effects in problem categorization and problem solving. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *22*, 792–810.

Bransford, J. D., Franks, J. J., Vye, N. J., & Sherwood, R. D. (1989). New approaches to instruction: Because wisdom can’t be told. In S. Vosniadou & A. Ortony (Eds.), *Similarity and analogical reasoning*. Cambridge: Cambridge University Press.

Chase, W. G., & Simon, H. A. (1973). Perception in chess. *Cognitive Psychology*, *4*, 55–81.

Chi, M. T. H. (2000). Self-explaining expository texts: The dual processes of generating inferences and repairing mental models. In R. Glaser (Ed.), *Advances in instructional psychology*. Mahwah, NJ: Erlbaum.

Chi, M. T. H., Feltovich, P. J., & Glaser, R. (1981). Categorization and representation of physics problems by experts and novices. *Cognitive Science*, *5*, 121–152.

Cohen, N. J., & Squire, L. R. (1980). Preserved learning and retention of pattern-analyzing skill in amnesia: Dissociation of knowing how and knowing that. *Science*, *210*, 207–210.

Cooper, G., & Sweller, J. (1987). Effects of schema acquisition and rule automation on mathematical problem-solving transfer. *Journal of Educational Psychology*, *79*, 347–362.

Craik, F. I. M., & Lockhart, R. S. (1972). Levels of processing: A framework for memory research. *Journal of Verbal Learning and Verbal Behavior*, *11*, 671–684.

Dufresne, R., Gerace, W. J., Hardiman, P. T., & Mestre, J. P. (1992). Constraining novices to perform expert-like problem analyses: Effects on schema acquisition. *Journal of the Learning Sciences*, *2*, 307–331.

Fisher, R. P., & Craik, F. I. M. (1977). Interaction between encoding and retrieval operations in cued recall. *Journal of Experimental Psychology: Human Learning and Memory*, *3*, 701–711.

Gentner, D. (1983). Structure-mapping: A theoretical framework for analogy. *Cognitive Science*, *1*, 155–170.

Gick, M. L., & Holyoak, K. J. (1983). Schema induction and analogical transfer. *Cognitive Psychology*, *15*, 1–38.

Gobet, F., & Simon, H. A. (2000). Five seconds or sixty? Presentation time in expert memory. *Cognitive Science*, *24*, 651–682.

Hardiman, P. T., Dufresne, R., & Mestre, J. P. (1989). The relation between problem categorization and problem solving among experts and novices. *Memory & Cognition*, *17*, 627–638.

Hestenes, D., Wells, M., & Swackhamer, G. (1992). Force concept inventory. *The Physics Teacher*, *30*, 141–158.

Hinsley, D. A., Hayes, J. R., & Simon, H. A. (1977). From words to equations meaning and representation in algebra word problems. In M. A. Just & P. A. Carpenter (Eds.), *Cognitive processes in comprehension*. Hillsdale, NJ: Erlbaum.

Koedinger, K. R., & Anderson, J. R. (1990). Abstract planning and perceptual chunks: Elements of expertise in geometry. *Cognitive Science*, *14*, 511–550.

Leonard, W. J., Dufresne, R. J., & Mestre, J. P. (1996). Using qualitative problem-solving strategies to highlight the role of conceptual knowledge in solving problems. *American Journal of Physics*, *64*, 1495–1503.

Maloney, D. P., O'Kuma, T. L., Hieggelke, C. J., & Van Heuvelen, A. (2001). Surveying students' conceptual knowledge of electricity and magnetism. *American Journal of Physics Supplement, Physics Education Research*, *69*, S12–S23.

Mazur, E. (1997). *Peer instruction: A user's manual*. Upper Saddle River, NJ: Prentice Hall.

McDermott, L. C. (1993). Guest comment: How we teach and how students learn – a mismatch? *American Journal of Physics*, *61*, 295–298.

Mestre, J. P., Dufresne, R., Gerace, W. J., Hardiman, P. T., & Touger, J. S. (1993). Promoting skilled problem solving behavior among beginning physics students. *Journal of Research in Science Teaching*, *30*, 303–317.

Morris, C. P., Bransford, J. D., & Franks, J. J. (1977). Levels of processing versus transfer appropriate processing. *Journal of Verbal Learning &Verbal Behavior*, *16*, 519–533.

Paas, F. G. W. C., & Van Merrienboer, J. J. G. (1994). Variability of worked examples and transfer of geometrical problem solving skills: A cognitive-load approach. *Journal of Educational Psychology*, *86*, 122–133.

Paas, F. (1992). Training strategies for attaining transfer of problem-solving skill in statistics: A cognitive-load approach. *Journal of Educational Psychology*, *84*, 429–434.

Rittle-Johnson, B., Siegler, R. S., & Alibali, M. W. (2001). Developing conceptual undestanding and procedural skill in mathematics: An iterative process. *Journal of Educational Psychology*, *93*, 346–362.

Rittle-Johnson, B., & Star, J. R. (2007). Does comparing solution methods facilitate conceptual and procedural knowledge? An experimental study on learning to solve equations. *Journal of Educational Psychology*, *99*, 561–574.

Ross, B. H., & Kennedy, P. T. (1990). Generalizing from the use of earlier examples in problem solving. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *16*, 42–55.

Ross, B. H., & Kilbane, M. C. (1997). Analogical mapping views and the effects of principle explanations. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *23*, 427–440.

Ross, B. H. (1989). Distinguishing types of superficial similarity: Different effects on the access and use of earlier problems. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *15*, 456–468.

Ross, B. H. (1984). Remindings and their effects in learning a cognitive skill. *Cognitive Psychology*, *16*, 371–416.

Ross, B. H. (1987). This is like that: The use of earlier problems and the separation of similarity effects. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *13*, 629–639.

Roy, M., & Chi, M. T. H. (2005). The self-explanation principle. In R. E. Mayer (Ed.), *Cambridge handbook of multimedia learning* (pp. 271–286). New York: Cambridge University Press.

Schoenfeld, A. H., & Herrmann, D. J. (1982). Problem perception and knowledge structure in expert and novice mathematical problem solvers. *Journal of Experimental Psychology: Learning, Memory & Cognition*, *8*, 484–494.

Siegler, R. S. (2002). Microgenetic studies of self-explanation. In N. Garnott & J. Parziale (Eds.), *Microdevelopment: Transition processes in development and learning* (pp. 31–58). Cambridge, MA: Cambridge University Press.

Silver, E. A. (1986). Using conceptual and procedural knowledge: A focus on relationships. In J. Hiebert (Ed.), *Conceptual and procedural knowledge: The case of mathematics* (pp. 181–198). Hillsdale, NJ: Erlbaum.

Silver, E. A. (1981). Recall of mathematical problem information: Solving related problems. *Journal of Research in Mathematics Education*, *12*, 54–64.

Simon, D. P., & Simon, H. A. (1978). Individual differences in solving physics problems. In R. Siegler (Ed.), *Thinking: What develops?* Hillsdale, NJ: Erlbaum.

Sweller, J. (1988). Cognitive load during problem solving: Effects on learning. *Cognitive Science*, *12*, 257–285.

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Appendix Sample Conceptual Problem Solving Materials. The two examples begin with a worked out example to reduce memory load and to demonstrate the approach to problem solving desired, and then ask the students to practice the approach on their own. Students are provided with feedback after they attempt to generate strategic analyses and two-column solutions. Following this (though not shown here), compare-contrast questions ask students to reflect on similarities and differences in the problems, the strategic analyses and two-column solutions. The two problems here look different in surface features/story-line but are solved with nearly identical strategic analyses and two-column solutions.

Physics Problem Solving

Instructions to student:

We'd like you to try a new way of approaching and solving physics problems. It will take a bit more work at first, but it will greatly help you to better understand the problems and how to solve them. You first write a *strategy* for how the problem should be approached and you then write a *two-column solution* to the problem. In some cases we will ask you to first solve the problem and then study a worked solution. In other cases you'll just study the worked solution without solving the problem yourself.

By "strategy" for solving a problem, we mean a written description for solving a physics problem that has no equations and contains three components:

1) The major physics principle or concept being applied,
2) A justification for why that principle applies to the specific problem, and
3) A plan for applying the principle to the problem.

A "two-column solution" helps the solver to implement the plan for solving the problem.

The left-hand column provides a step-by-step explanation of what the conditions are that are relevant to solving the problem at this point. You can think of this as the left-hand side explaining what the equations are going to do.

The right-hand column contains the equations that meet those conditions. If the left-hand side is done correctly, those equations should allow you to solve the problem.

Please do the problems in the order presented and do not look ahead. However, you may look back at any pages you have already seen whenever you wish.

You will be given a formula sheet, and feel free to use a calculator. Study the strategy and two-column solution below so that you can generate strategies and two-column solutions

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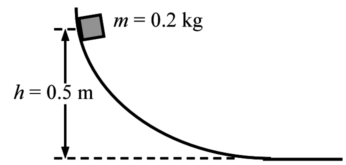
A mass, m=0.2 kg, is released from rest at a height h=0.5 m on a frictionless curved track. What is the speed of the mass when it reaches the horizontal portion of the track?



Strategy

1. Major physics principle that can be applied to solve:

   Conservation of mechanical energy can be applied to solve this problem (mechanical energy is the sum of kinetic and potential energies).

2. Justification for why this principle can be applied to this problem:

   To apply conservation of mechanical energy, non-conservative forces cannot do net work on the system. The normal force exerted on the mass by the curved track is a non-conservative force (the gravitational force is conservative), but the work that the normal force does is 0 because the force is always perpendicular to the displacement of the mass; hence mechanical energy must be conserved.

3. Plan for applying principle.

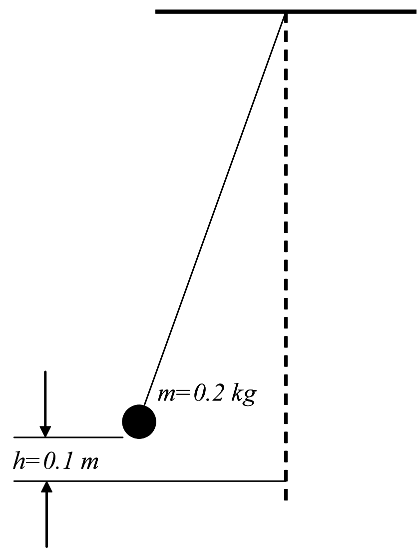
   Select the initial state to be the block immediately upon release at the top of the ramp with all potential energy and no kinetic energy. The final state is the block at the bottom of the ramp where it possesses only kinetic energy and no potential energy. Set the initial and final mechanical energies equal to each other since it is conserved (does not change). Solve for the speed.

Two-Column Solution

Description of strategic step	Equation(s) used in solving
Apply conservation of mechanical energy since non-conservative forces do no work.	$E_i = E_f$ $(KE)_i + (PE)_i = (KE)_f + (PE)_f$
Pick coordinate system with origin at bottom of ramp. Initial kinetic energy is 0, and final potential energy is 0.	$(KE)_i = 0$ $(PE)_f = 0$ thus $(PE)_i = (KE)_f$
Substitute for kinetic energy and gravitational potential energy.	$(PE)_i = (KE)_f$ $mgh = \frac{1}{2}mv^2$
Solve for *v*.	$v = \sqrt{2gh}$ $= \sqrt{2(9.8 \text{ m/s}^2)(0.5 \text{ m})}$ $= 3.13 \text{ m/s}$

For the problem below, first write a strategy, then write a two-column solution.

A pendulum is formed by attaching a mass, m=0.2 kg, to the end of a string, with the other end attached to a ceiling. The mass is pulled with the string taut, and released from rest at a height, h=0.1 m, from the lowest point. What is the speed of the mass when it reaches the lowest point?



Strategy

1. Major physics principle that can be applied to solve:

2. Justification for why this principle can be applied to this problem:

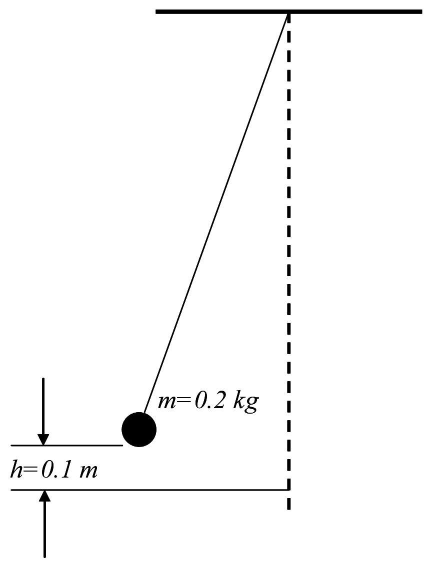
3. Plan for applying principle.

Two-Column Solution
(You can use more or fewer than the 4 steps in the grid below)

Description of strategic step	Equation(s) used in solving

If you need more space, use a blank sheet and insert it here. Study the problem below with its strategy and a two-column solution.

A pendulum is formed by attaching a mass, m=0.2 kg, to the end of a string, with the other end attached to a ceiling. The mass is pulled with the string taut, and released from rest at a height, h=0.1 m, from the lowest point. What is the speed of the mass when it reaches the lowest point?



Strategy

1. Major physics principle that can be applied to solve:

    Conservation of mechanical energy can be applied to solve this problem (mechanical energy is the sum of kinetic and potential energies).

2. Justification for why this principle can be applied to this problem:

    To apply conservation of mechanical energy, non-conservative forces cannot do net work on the system. The tension force exerted on the mass by the string is a non-conservative force (the gravitational force is conservative), but the work that the tension force does is 0 because tension is always perpendicular to the displacement of the mass; hence mechanical energy must be conserved.

3. Plan for applying principle.

    Select the initial state to be the pendulum immediately upon release at height 0.1 above the lowest point, with all potential energy and no kinetic energy. Select the final state to be the pendulum at the bottom of the swing with all kinetic energy and no potential energy. Set the initial and final mechanical energies equal to each other since mechanical energy is conserved (does not change). Solve for the speed.

Two-Column Solution

Description of strategic step	Equation(s) used in solving
Apply conservation of mechanical energy since non-conservative forces do no work.	$E_i = E_f$ $(KE)_i + (PE)_i = (KE)_f + (PE)_f$
Pick coordinate system with origin at bottom of pendulum's swing. Initial kinetic energy is 0, and final potential energy is 0.	$(KE)_i = 0$ $(PE)_f = 0$ thus $(PE)_i = (KE)_f$
Substitute for kinetic energy and gravitational potential energy.	$(PE)_i = (KE)_f$ $mgh = \frac{1}{2}mv^2$
Solve for *v*.	$v = \sqrt{2gh}$ $= \sqrt{2(9.8\text{ m/s}^2)(0.1\text{ m})}$ $= 1.4\text{ m/s}$

[They are then asked to compare and contrast the strategies and two-column solutions of the previous two problems and answer a series of questions (and then given feedback).]