Engaging Diverse Stakeholders in Innovative Curriculum Design and Research: The Case of the Middle-school Mathematics through Applications Project (1990-2002)

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I. Introduction

The purpose of this case study is to analyze the inner workings and trajectory of the Middle-school Mathematics through Applications Project (hereafter MMAP), a research and development project that worked to reconfigure the usual relationships between research and practice, and between researchers and practitioners. The goal of this case study is to understand how this project did this work and the challenges that it faced in doing it in order to provide guidance to those who would like to do similar things.

MMAP represents one general class of such projects known as design-based research (previously design experiments) in which researchers and practitioners collaborate to design innovative classroom interventions while using research in teachers' classrooms to inform the iterative redesign of those interventions (Stein & Coburn, 2003; see also Collins, Joseph, & Bielaczyc, 2004; D'Amico, 2005). In addition to creating interventions that draw on the wisdom of both research and practice, design-based research has the potential for both transforming the practices of participating teachers and providing participating researchers with a school-based laboratory for investigating new models of what schooling could be like. In the middle of the project, MMAP also began to resemble a designing-for-scale project when its funder encouraged it to scale up its curriculum (Stein & Coburn, 2003).

Introducing the Middle-school Mathematics through Applications Project (MMAP)

MMAP began in the early 1990's during an early stage of design-based research in which the focus was on using new theories of learning to design innovative educational interventions. The goal was to provide existence proofs that these theories could be embodied in real classrooms, and that doing so had the potential for addressing some persistent problems of schooling (D'Amico, 2005).

In MMAP's case, its founders were interested in addressing equity issues in mathematics education, especially the tendency of many underrepresented students to become disengaged from mathematics during the middle-school years, leading to poor participation and performance in crucial gate-keeping courses in high school like algebra. MMAP was founded at the Institute for Research on Learning (IRL) in Palo Alto where theories about situated cognition and communities of practice were just emerging, and MMAP's founders thought that these theories might provide guidance for developing new kinds of curricula to address such equity issues. In particular, their hypothesis was that more students, especially traditionally underserved students, would become engaged in learning mathematics if it were presented as a tool necessary for achieving important goals in the working world. The idea was to provide students with mathematically-rich and technology-intensive design problems from fields like architecture and biological modeling as contexts in which students could learn the mathematical concepts and skills that they needed to learn. The hypothesis was that such units would provide compelling answers to students for why mathematics was important to learn. At the same time such units would provide specific contexts in which students could check the accuracy and sensibleness of their mathematical computations and conceptual understandings.

This was a radical idea, representing a new vision for what a mathematics curriculum could be. At the time MMAP proposed it, many key issues were completely open: how to design curriculum units that embodied this vision; whether any teachers would be willing to try them out; how students, parents, and administrators would respond to them; whether students would

actually become engaged in the mathematics they needed to learn and not just the real-world design problems; and finally whether such units would actually engage a much wider range of students in school mathematics as was hoped.

MMAP was funded to investigate all of these issues by the National Science Foundation (NSF), first in a one year planning grant (1991-1992), and then in a full 3 year grant (1992-1995). The goal of these grants, like those for other kinds of design-based research projects occurring at that time, was to see whether MMAP's ideas were viable in real classroom settings (Goldman/Knudsen interview 3/20/03; NSF program officer interview). Current policy concerns about the scaling up of such interventions or providing rigorous evidence for their impact on student learning were still a long way off.

At the same time, the early 1990's represented the dawn of the most recent reform movement in mathematics education. The California Mathematics Framework (1985) and original set of National Council of Teachers of Mathematics (NCTM) Standards (1989) had just been published, engendering great excitement about the possibilities for improvements in mathematics instruction among mathematics educators, enterprising teachers, and funders like the National Science Foundation. At the same time, the fact that U.S. students had not done well on the Second International Mathematics and Science Study (SIMSS, McKnight et al., 1987) as compared to students from other countries made it pressing at NSF that something be done (NSF program officer interview). In fact, a year before NSF funded MMAP, it also began generously funding a suite of projects at the elementary, middle-school, and high-school levels to design comprehensive mathematics curricula that would embody in different ways the various new ideas about mathematics instruction that were being generated at that time (Goldman personal communication 12/28/05; NSF award abstracts; NSF program officer interview). The goal was for each project to use its comprehensive curriculum as leverage for addressing the whole constellation of factors that would need to change to reform mathematics education: what was to be learned, how teachers would teach, how classrooms would be set up, how assessment would be organized, and how all of this would relate to the various institutional structures in which mathematics education occurs (NSF program officer interview). NSF noticed, however, that it did not have any curriculum projects in its middle-school portfolio that focused on the appropriate use of technology, so it decided to fix that problem by including with them MMAP, which had been funded through a different program. As a result, MMAP became accountable not just for enacting its own vision but also for making sure that its curriculum designs were consistent with what the NCTM standards were recommending. Because MMAP was put in with the other NSF curriculum projects, it was later funded to turn its original curriculum units into a comprehensive curriculum that could be published like the other projects (1995-1998). Later, MMAP and the other projects were provided with some additional funding to disseminate their curricula around the country (1998-2002).

At the same time as MMAP sought to create a curriculum that could engage a wider range of students in mathematics, the project also sought to engage a wider range of stakeholders in the process of curriculum design. Specifically, rather than leaving curriculum design to curriculum developers, MMAP proposed that its curriculum be collaboratively designed by teams of teachers, researchers, curriculum developers, and math-using professionals. In so doing, MMAP worked to change the usual relationships between these people, experimenting with several new models for how they could productively interact to create innovative curricula. How does a project like this successfully do the work of moving toward a particular vision of innovative practice while still including many different voices in the conversation? How does it deal with

the strong status differences between different participants? The various forms of wisdom that the project developed for how to do those things is the main legacy of MMAP and therefore the primary focus of this case analysis.

Preview of the Analysis to Come

The analysis of MMAP that follows is organized in a historical manner according to major shifts in the focus of the work on the project. As many of these shifts were associated with changes in NSF funding, most sections correspond to particular grants the project received. In these core sections, I will examine how the project got the work of that phase done as well as how it managed various external contexts. As part of this, I will pay particular attention to how the project engaged a diverse set of stakeholders in making progress with its work, which I will often refer to by the shorthand expression "productive collaboration" to refer to both the progress that was made ("productive") and the fact that people worked together as a team to make that progress ("collaboration").

In seeking to understand how MMAP learned to engage such a wide range of stakeholders in productive work, I kept being reminded of some principles that Faith Conant and I had earlier found were embodied in learning environments in which students were productively engaged with each other (Engle & Conant, 2002; see also Engle, 2004; Engle & Faux, 2006).[1] Despite the fact that the nature and location of the work in MMAP was very different from that of the classrooms I and others had studied earlier, I kept on noticing ways the MMAP project had embodied these same principles. I therefore saw that they might be helpful for understanding collaborative enterprises more generally. That is, they may be helpful for understanding productive collaborations that also involve adults, occur outside of schools, and include representatives from different communities with different goals and values. I will elaborate on these ideas more throughout my account, but basically my hypothesis is that if a collaborative enterprise like MMAP is going to productively engage a diverse group of stakeholders, the following four principles (at least) need to be embodied:

(a) *Problematizing* together: The group finds problems to work on that all members of the collaboration are committed to working on together, problems that are considered to be sufficiently unsolved, important, and accessible to everyone.

(b) Respecting everyone's *authority*: Participants are truly given the *agency* to contribute their own perspective on the joint problems; in doing that they become true *contributors* and in some cases even authors of joint products; and through their ongoing participation they may develop into *local authorities* about particular aspects of the work.

(c) Engendering a dynamic internal *accountability* to others and to shared norms and goals: Specifically, in the process of working together, participants are held responsible for accounting for how they are addressing what others have done as well as agreed-upon norms and goals for the enterprise.[2]

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[1] These principles were originally developed with guidance from Jim Greeno, a major player in the MMAP project, so they are hopefully not theoretically incompatible with the principles that were guiding the project.

[2] Here I am expanding Engle & Conant's (2002) original language about "disciplinary norms" to be broader, encompassing enterprises like MMAP in which disciplinary work is only one small part of what people are doing with each other. What matters is not that "disciplinary" or any other kind of norms are being oriented to, but that participants are holding themselves accountable to some shared norms important for their type of enterprise.

(d) Having access to sufficient *resources* to make all of the above possible: Resources may be as seemingly straightforward as having sufficient time or money to do something to having access to specialists who can share relevant tools and means of addressing problems. Some resources, like these, support productive collaboration directly while others support the embodiment of the other principles as when one assembles a team with different perspectives on a key issue in order to encourage that issue to be problematized.

Throughout my account of MMAP, I will provide evidence of how all four of these conditions were embodied in various ways by the project, and then show how this might help explain the nature of the productive (and at times, not so productive) collaborative engagements that occurred in the project around both curriculum design and research.

In addition to considering how the project shaped its own collaborations, I will also consider how various contexts shaped the project. First, I will consider the many ways that contexts like the funding available to MMAP, its institutional homes, and the kinds of workplace collaborators it worked with shaped the types of collaborative design work the project engaged in and how it went about engaging in that. Second, I will consider how changes in key external contexts like MMAP's funding context, the curriculum publishing industry, and especially the policy context around mathematics instruction influenced when and how MMAP's curriculum was used.

Before turning to the analysis, however, I need to explain what kinds of data I had to draw upon as well as provide some additional background on the origins of the project.

II. Data Collection Methods

This report is based upon interviews with a variety of MMAP participants, documents related to MMAP that I was able to gather from participants and other sources, a few videotapes of MMAP project meetings, and my own experience as a participant helping to write the MMAP paper about coordinating research and reform in the project (Greeno et al., 1999) as well as from collecting data in MMAP classroom during the spring of 1999. I will now provide further detail about the interviews and documents that I drew upon for purposes of this report.

Interviews

Twenty interviews were conducted with eighteen MMAP participants. Most of the interviews were with core members of the project, primarily as those people were easier to locate given that the project has been over for several years. I attempted to track down more peripheral participants, particularly teachers who had only collaborated with the project briefly or who had taught MMAP units but not helped design them, but was unsuccessful. With respect to outside players involved in sharing MMAP units with the rest of the world, I was able to secure interviews with one former NSF program officer from early in the project and the director of one of the reform networks that helped distribute MMAP units. However, my email inquiries to the director of the middle-school mathematics implementation center went unanswered.

Specifics about who was interviewed, the dates they were interviewed, their role or roles in the project, and other relevant details appear in the Appendix. As is documented in the table for the teacher interviews, I purposely selected a set of teachers to interview who varied in when they joined the project, their level of teaching experience, the type of school they taught at, and whether they eventually were paid to play additional roles on the project besides that of a collaborating teacher. Almost everyone was very enthusiastic about being interviewed for the purposes of this report, with many wishing to read and provide comments on it. Everyone but two teachers and the NSF program officer were willing to have their real names used. To protect those people's identities, I have given each participant who began the MMAP project as a teacher a set of initials to serve as their pseudonym and refer to the program officer by his/her role. A lab scientist whom I did not have an opportunity to interview will also remain anonymous.

Most interviews were conducted in the same basic manner, which resulted in extensive dialogical narratives about the project. First, as part of the consent process I told participants that the focus of the study was on projects like MMAP that had worked to reconfigure the usual relationships between research and practice, and between researchers and practitioners. With this framing, I then asked the person I was interviewing to tell me the story of their participation in the project. I encouraged them to start with what they were doing before they joined MMAP and continue through anything they did afterwards that they thought was influenced by their participation in it. When topics especially germane to the focus of this report came up (e.g., the influence of some contextual factor on MMAP, the role of teachers versus researchers on the project, how curriculum design got done, etc.), I would ask additional questions to encourage interviewees to elaborate upon them. Throughout the interviews, I sought to regularly repeat what I thought interviewees were saying back to them in order to give them an opportunity to correct my initial interpretations. Whenever possible, I attempted to use the language that the interviewee had used in my questions and other responses. But above all, I worked to put myself

into the role of a responsive and interested listener who wanted to know more about the project and the interviewee's perspective on it.

Interviews with two participants, Shelley Goldman, the Principal Investigator (PI) for the project, and Jennifer Knudsen, longtime project manager, worked differently. At their request, the first interview with them was done with them together and was more in the form of a project history, rather than their personal histories of participation in the project. There was then a long series of follow-up interviews, informal conversations, and email exchanges with one or both of them focused on a variety of topics both before the case was written and in response to various drafts. These are also documented in the Appendix.

In addition to the interviews I conducted in 2003-2005, I also drew on interview data collected by the MMAP project itself as part of the work that led to the Greeno, McDermott, Cole, Engle, Goldman, Knudsen, Lauman, & Linde (1999) paper about how MMAP coordinated research with reform. In particular, I got permission from Jennifer Knudsen and Shelley Goldman to use rough transcripts of interviews conducted in 1996 that involved four of the teachers that I interviewed again for this study. Goldman also provided me with an eleven minute videotaped interview from 1994 about the beginnings of IRL and MMAP with George Pake, who was a co-founder of IRL, its first director, and a co-PI and content consultant on almost all of MMAP's grants.

Because many of the interviews were organized in the form of extended narratives of personal experience (Labov, 1972; Linde, 1993), this allowed me to learn about certain kinds of issues. First, people almost always provided me with accounts of:

(1) What they had done and already knew prior to MMAP that they might have brought to the project (a narrative's setting);

(2) How they got involved in the project (its initiation);

(3) What activities they participated in during the project (its events);

(4) What they thought about those activities (its evaluation);

(5) Why they stopped participating in the project; and

(6) How, if at all, their experience with the project might have informed their later work.

When I was effective at probing about the activities that they participated in, then I also had opportunities to learn more about what others in addition to themselves might have contributed to those activities, their perspective on whether and how those activities were valuable for achieving their goals and those of others or of the project as a whole, and how various contextual factors might have shaped the conduct and outcomes of those activities. Finally, I could use the evaluative remarks (or "morals of the story") that participants spontaneously included in their narratives to gauge the range of participant perspectives on what the project meant to them.

However, because these were narratives that focused on individuals, I may not have learned as much about less individually-oriented aspects of the project. In particular, the accounts I elicited probably provided less information about issues like:

(1) The perspectives of other participants, except when impacting the person being interviewed;

(2) How subgroups in the project and organizations outside it collectively acted and felt about various issues;

(3) Characteristics of the project that were not tied to particular events or times; and

(4) Anything that the participant did that would not easily fit into a coherent narrative (see Linde, 1993, on this last point).

Archival Documents and Videos

I also drew upon the following historical documents and videos for this report when relevant:[3]

1) Copies of MMAP's first major proposal to NSF, and its final report at the end of its second major grant.
2) Copies of many of the MMAP curriculum and support materials, including five different versions of the Antarctica curriculum and some sample Voyager Pathways units.
3) Evaluations of MMAP by national review panels and external evaluators.
4) Selected papers written by the project for publication and conferences.
5) Selected written and other products of the project for practitioner audiences like teachers' guides, marketing materials, presentations for reform networks, and the like.
6) A file of invoices and spreadsheets detailing MMAP's internal sales of its materials.
7) Videotapes of many MMAP project meetings. For purposes of this report I have drawn upon excerpts of the video of the first MMAP orientation with teachers.
8) Agendas and notes from some MMAP project meetings.
9) NSF award abstracts for MMAP and related projects (available by searching at http://www.nsf.gov/awardsearch/index.jsp).
10) Files displayed on MMAP's final website at WestEd, the last non-profit in which it was housed, which were still up when I began this project.
11) Finally, CV's of key researchers on the project, a publications list from an old MMAP website (http://mmap.wested.org/research/ResearchArticles.html), references to publications in interviews, and some Google Scholar searches were combined in order to compile a Project CV. See Appendix 2 for an abridged version of this CV that only includes MMAP publications (defined as books, dissertations, journal articles, chapters, and conference proceedings papers), but none of the many presentations, technical reports, or working papers that were produced.

[3] Many thanks to Karen Cole, Shelley Goldman, Jim Greeno, Duane Habecker, Rogers Hall, Robert Hurd, Jennifer Knudsen, Ray McDermott, Judit Moschkovich, and Annie Yount for providing many of these materials to me.

III. Project Origins

In this section, I describe the contexts and events that helped lead to the beginning of the Middle-school Mathematics through Applications Project. Information about these contextual conditions is crucial as many changed over the life of the project, which had serious consequences for its trajectory. In particular, the project arose amidst the intellectual and institutional context provided by the Institute for Research on Learning (IRL), the political and economic context of the post-Cold War era in the national weapons labs, and the late 1980's educational research and development funding context. It was significant for the project arising in the way it did that MMAP's founders were influential players in each of these contexts and that aspects of the project resonated with key goals existing in each of them. At the same time, to generate the core idea for MMAP and get it funded, MMAP's founders drew liberally on their existing social networks, their previous similar experiences, and knowledge and practices drawn from each of these contexts.

The Setting and Context in Which MMAP Emerged

IRL was founded in January 1987 with a 5-year, $5 million grant from the Xerox Foundation, which was initiated by John Seely Brown and George Pake, then the incoming and outgoing directors of Xerox PARC (Pake MMAP interview 10/14/96; Hall interview). According to Pake, who became IRL's first director, the idea was that "people around PARC skilled in AI [artificial intelligence] or cognitive science…could form the nucleus for a new kind of institute that could make great strides in pushing back the frontiers of learning" (Pake MMAP interview 10/14/96). In particular, Pake hoped that the institute would develop a "really deep theory of how it is [that] human beings learn," "test" that theory in real learning settings, and especially design products that could improve learning in schools and workplaces (ibid.). As we will see, MMAP can be seen as fitting that agenda: it designed software and curricula that embedded some of the theoretical ideas being explored at IRL, with the work in classrooms being both an opportunity to "test" the ideas as well as make the "products" more usable. Thus doing research and designing products would become integrated in a model of what was called "research-in-action" in which "researchers, designers, and educators" collaborate "to produce immediately useful findings throughout the research process" (Clancey, 1992, p. 572).

What kinds of theoretical ideas was IRL exploring that specifically provided the intellectual context for MMAP? It was the late 80's and early 90's when the notions of communities of practice and situated cognition were first being proposed (see esp. Brown, Collins & Duguid, 1989; Clancey, 1991; Greeno, 1989; Lave & Wenger, 1991), and IRL was considered by others to be at the very center of their development and use (e.g., see Schein, 1996). As eventual MMAP researcher and co-PI Ray McDermott explained, Pake had "started IRL around the very idea that theory and practice could be brought together in learning," an issue which he and primary MMAP PI Shelley Goldman were "really driven by…completely in all our work" (10/27/04 interview). Consistent with this, there was also considerable interest at IRL in issues around how mathematics and "other forms of disciplinary activity happen in the workplace" (Hall CV and interview) and in using technology not to "automate what people do" but to "facilitate conversations for constructing information and meaning" (Clancey, 1992, p. 572). As we will see, the MMAP units themselves ended up embodying both of these ideas; they used workplace scenarios to ground mathematics units in which technology was designed to facilitate productive conversations among students.

Events That Led to the Initiation of the Project

As MMAP PI Shelley Goldman tells it, the seeds of MMAP began when she and MMAP co-PI Ray McDermott were asked for their help by a senior scientist at a nearby federally funded weapons lab (Goldman interview 3/20/03; also Greeno personal communication). With the Cold War recently over, the head of this lab was looking for ways to use the lab's technical savvy to address important social problems, and one of them was education (McDermott interview; Goldman/Knudsen interview 3/20/03). The lab was getting pressure from the head of the Department of Energy, Admiral James D. Watkins, to help education and was even being allowed to use paid work time to do so (Goldman/Knudsen interview 03/20/03; Greeno personal communication). So far the lab's employees had been visiting schools and doing fancy demonstrations of their technology, but this did not seem to be contributing much to the education of the children whose classes they visited (Goldman/Knudsen interview 3/20/03). The lab scientist's question was: what would be better? McDermott and Goldman's response was to hold a series of informal meetings between him and various fellow researchers at Xerox PARC and IRL, where Goldman had just started working and Stanford professor McDermott was also affiliated.

After six months of such meetings, the group, which now also included Stanford professor and IRL senior researcher Jim Greeno and IRL director George Pake, came up with the idea to begin a collaboration between the lab's scientists and IRL researchers to adapt the kinds of computer applications that the scientists used in their work to provide real-world contexts for middle-school students to learn mathematics. The group decided to focus on middle-school mathematics because of mathematics' powerful role as a gate-keeping subject and because so many students begin disengaging from it in middle school (Goldman/Knudsen interview 3/20/03; McDermott interview 10/27/04; Pake interview 10/14/96). To address this problem, the project envisioned instructional units that would "lead with plausibly realistic applications of mathematics so these would be really engaging with kids" (Hall interview; see also Goldman & Greeno, 1998; Pake interview 10/14/96). This idea was popular with the IRL folks because it helped to embody the vision of learning that was emerging there. The lab scientist was enthusiastic about the project both because it helped him address Watkins' push for education-related work, but also because it gave him the opportunity to work with physicist Pake, who just happened to be one of his professional heroes having invented the MRI, won a National Medal of Science, and founded Xerox PARC (Greeno personal communication; Goldman personal communication 12/10/05).

Besides fitting well with the overall IRL vision, the core idea of mathematics through applications also seemed like it would be fundable by both private and federal funders. The group thought the National Science Foundation (NSF) was the most promising possibility, however, as Pake had especially strong connections with them (Pake interview). Besides being a famous NSF-funded physicist, Pake was close to a top NSF education director who had been on the faculty under him when he was a senior administrator at Washington University in St. Louis. Pake and the national lab scientist, also well respected at NSF, went to Washington to pitch MMAP's core idea, and were given much encouragement (Greeno, personal communication). NSF apparently appreciated the boldness of the idea, the team that would be involved in investigating it, and the fact that the project involved a true multi-institution collaboration between a large national laboratory, an R&D organization that had been launched from industry,

university faculty from a top research university,[4] and middle-school teachers (Greeno personal communication). The group was invited to first apply for a one-year planning grant to explore their ideas and then to submit a full proposal.

Work On the Planning Grant

The planning grant proposed to test MMAP's hypothesis about embedding mathematics within applications by creating kid-friendly versions of workplace software and seeing "whether or not we'd ever get some [math] out of that" (Goldman/Knudsen interview 3/20/03). The initial funding, which lists Shelley Goldman as PI, was $50K for one year (7/1/91-7/31/92) through NSF's Instructional Materials Development program (NSF award abstract #9154005). For that grant Goldman, post-doc Rogers Hall, and a programmer created and piloted three versions of a simplified Computer Aided Design (CAD) system that allowed students to design houses and calculate their building costs, a forerunner of what became ARCHITECH, software that provided the foundation for MMAP's Antarctica and Dream Home units (Hall interview; Goldman personal communication 9/1/05). They found students to test out the program in various places and analyzed videotapes of them using it (Cole interview; Hall interview). From that, they learned several lessons that guided the project thereafter, including that: (a) they would need curriculum to guide students to make use of the math embedded in such a program; (b) students would be more likely to engage with mathematics if distracting bells and whistles in the software were removed; (c) adding relevant and challenging adult-like problems seemed to foster increased engagement in the activities and in mathematics; and (d) software like this could allow teachers and students to do more interesting projects by performing the tedious computations for them (Goldman personal communication 9/1/05; Goldman interviews 03/20/03, 10/27/04).

At the same time as this piloting was being done, the group worked on writing the full MMAP proposal, which was a collaboration between Goldman as PI, and McDermott, Greeno, and Pake as co-PI's, with others like post-doc Hall contributing as well. McDermott described writing the proposal as "us dreaming out loud" (interview). Because of the cutting-edge nature of the ideas, they proposed a three-year rather than five-year project so that neither they nor NSF would be stuck with a long-term commitment if the ideas did not end up being viable (Greeno interview). The proposal was funded by the Instructional Materials Development Program at a level that Pake viewed as generous given the funding climate at the time, which amounted to a little less than a million dollars per year (NSF award abstract #9154119; Pake 1996 interview). Around the same time, MMAP also was able to get a grant from Apple Computer that they used to provide collaborating teachers with computers for their classrooms (MMAP orientation video, 5/1/92). In addition, the Department of Energy provided some initial funding for the project (MMAP orientation video, 5/1/92), which included use of the lab's facilities and paid time for lab staff to participate.

[4] However, the fact that the top research university was Stanford was a problem as a well as a benefit as MMAP was submitting its proposal in the midst of a scandal over Stanford misspending federal funding for indirect costs.

IV. Beginning to Realize MMAP's Collaborative Goals (1990-1992)

The core analysis of MMAP begins in this section. Specifically, I discuss two ways the MMAP project laid the groundwork for productive collaborations between teachers, researchers, and math-using professionals through its choices about whom to include in its collaborative team, and how to orient them to the work they would be doing together. Using the Engle and Conant (2002) principles I will show how problematizing, authority, and accountability were embodied in these first sets of actions in ways that would have resonances for the kinds of collaborations that occurred during the rest of the project.

Assembling a Collaborative Team

Once the main project had been funded, project leaders began setting the stage for fostering productive collaborations between the wide variety of stakeholders that it wished to engage in its work. The first way they did that was to carefully assemble one of the most important resources for the project's collaborative work, its initial team of staff and collaborating teachers. Shelley Goldman, Ray McDermott, Jim Greeno, the lab scientist, and the rest of MMAP's founders drew on their ample personal and professional social networks to find and recruit both collaborating teachers and additional staff for the project who would enhance such a collaboration (Hall interview). I will show that MMAP specifically sought people for the project who were likely to serve as resources for realizing the problematizing, authority, and accountability principles (Engle & Conant, 2002).

The project purposely began with just six teachers so it "could feel [its] way…[about] what to do with them and what would work" to support the collaboration (Goldman/Knudsen interview, 3/20/03). This initial set of teachers was recruited and selected both from a school that the lab scientists had connections with in Hayward, and through a reform-oriented mathematics professional development project that IRL staff had connections with in the San Francisco Unified School District (Goldman/Knudsen interview 3/20/03; Teacher FJ interview; Teacher OO interview 5/3/04; Teacher TO interview; MMAP orientation video 5/1/92 for identity and number of teachers).

Staff were primarily found from current or former doctoral students at the Stanford School of Education whom McDermott and Jim Greeno came to know (Berg interview; Cole interview), and from students or alumni of the Stanford undergraduate dorm in which Ray and Shelley served as faculty resident fellows (Goldman/Knudsen interview 10/27/04; McDermott interview; Milanese interview).

Overlap in Goals as a Resource for Problematizing

In attracting and selecting MMAP's initial team of participants, attention was paid to determining whether there was some overlap in goals and interests between the project and them. Having some overlap in place from the start helped make it easier for the project to find relevant problems that participants would want to engage in, a resource for problematizing. Specifically, IRL staff interviewed teachers in order to select those who:

- Had taken calculus, a requirement of the collaborating scientists who were concerned that teachers have sufficient mathematics background (Goldman personal communication 12/10/05, Goldman/Knudsen interview 3/20/03), and

- "Were into doing interesting things in their classroom[s]," something they found out about by asking teachers to tell them about the kinds of math-related projects they did in their classrooms. (Goldman/Knudsen interview 3/20/03; also Teacher OO interview 5/3/04).

Having teachers who were interested in experimenting in their classrooms meant that these teachers would be more open to experimenting with the new ideas that MMAP wanted to be tried out in real classrooms.

These initial MMAP teachers' accounts of their joining also expressed interests in various goals related to MMAP like "looking at math through applications" (Teacher KM interview) or "build[ing] on this interest that children had with technology to understand strands in mathematics" (Teacher OO interview 5/3/04; personal communication 10/17/06). At the same time, curriculum coordinator and later project director Knudsen clarified that "we didn't want to work with the teachers who were already like some researcher's dream of reform; not that any of those may exist but…the teachers we got, were not like that" (Knudsen interview 5/2/05). What mattered was not *where* teachers had gotten to in their instruction, but that they were interested in working toward some of the goals of the project.

Similarly, graduate students at Stanford were invited to consider joining the project when they expressed some overlapping research interests with the project (Berg interview; Cole interview). The overlap between the goals of particular participants and those of the project did not need to be extensive, nor did their goals need to be the same as those of any other participants. Instead each person's goal needed to be large enough to present rich enough problems to engage in with others on the project.

Respect for Teachers by Staff as a Resource for Supporting Teachers' Authority

One shared norm the project specifically selected for was respecting teachers and children (Cole interview; Goldman/Knudsen interview 10/27/04). As Goldman explained,

> "We didn't want anyone on the project ever who wasn't going to be collaborative with the teachers or had put-down attitudes about teachers or kids. You could not be thinking the problem with education was either the teachers or the kids and hope to get a job on the project. That was whether you were in education or not. So I think that was something we were always looking for." (Goldman/Knudsen interview 10/27/04)

For example, Larry Gallagher, who was hired as a programmer on the project, described an interview process in which he felt he was being carefully screened for his ability to be respectful of other people, especially teachers (something confirmed by Knudsen and Goldman, 10/27/04 interview). He was asked to meet many different people involved in the project, and got the sense that he was being watched for the extent to which he could listen as well as talk, to make sure that "I wasn't, for lack of better words, one of those kind of arrogant engineers who think people who understand technology are just kind of genetically superior to other human beings" (Gallagher interview). When he was hired, the project had already had some bad experiences with some undergraduate computer science students from Stanford who appeared to be more interested in implementing interesting programming techniques than doing what the project needed (Goldman interview, Knudsen interview 5/2/04).[5] Thus, in endeavoring to select staff who already seemed to respect teachers, MMAP stacked the deck with people who would be

[5] Similar stories could be told about how staff were hired from Goldman and McDermott's dorm, with them and their young daughter informally identifying those who tended to treat others with respect (McDermott interview 10/27/04; Goldman/Knudsen interview 10/27/04).

more likely to treat teachers as they wished them to be treated, thus using the resource of MMAP's staff to promote the principle of authority vis-à-vis the teachers.

At the same time, several of the initial set of MMAP teachers noticed and were attracted by the fact that this looked to be a project in which they would have a measure of authority over what they would be doing. For example, two teachers noticed that the project was offering opportunities for them to be contributors to the effort, with one seeing it fitting his own practices of designing project-based curricula and the other intrigued by the chance to make something new with a "consortium" of teachers, researchers, and scientists (Teacher TO interview; Teacher FJ interview). One of these teachers also noted that the project fit with his "emerging theories of what [he] was doing" which made it a "good and promising extension" to his work with the reform-oriented network he had been recruited from (Teacher TO interview). However, others did not know what to expect and were pleasantly surprised when they discovered that the project actually respected them and their contributions (e.g., Teacher EH interview; Teacher OO interview 5/3/04). With some teachers, it also did not hurt that they were in the words of one teacher, "lavished with gifts," including a programmable calculator, an email account (unheard of in those days), and access to computers for their classrooms (Teacher FJ interview; Teacher TJ interview).

Diversity in Expertise as a Resource for Accountability and Problematizing

At the same time, however, that the project sought to have a certain level of shared goals and values among its members, it otherwise emphasized the importance of having a diverse pool of expertise, perspectives, and values to draw upon. This was a key resource that it drew upon in its work for embodying both accountability and problematizing. For example, having different types of expertise represented in the project meant that when MMAP's participants held themselves accountable to the ideas of other participants, this in effect held their ideas accountable to the expertise and professional communities from which those people were drawn. In addition, diversity among MMAP's participants often led to productive problematizing of issues as not everyone immediately agreed with each other, so differences would end up getting hashed out.

The Stanford dorm in which Goldman and McDermott served as faculty fellows was a particularly helpful source for recruiting staff with a range of expertise and perspectives around MMAP's work. Students there were pursuing a variety of majors and extracurricular activities, all of which Goldman and McDermott became intimately familiar with while living with them (Goldman interview 10/27/04). For example, the project was able to recruit a gifted physics student who was also an excellent tutor and programmer to work for them during its first summer. In that time, he successfully taught mathematics to the teachers, and in a single weekend programmed the first version of what became the project's population biology modeling program, HABITECH (Hall interview; McDermott interview). Similarly, two students from the dorm who had won awards for their writing later became curriculum writers on the project, with one helping invaluably with organizational issues and the other providing "sparky writing that spoke directly to kids" (Knudsen interview 3/20/03). Because many of these people were not typically focused on mathematics education or even education, they provided expertise to the project that it might not have had were it to have recruited more narrowly.

That there was such a range of skills and interests in the project was also highly relevant for continuing to attract and retain both staff and teachers. In numerous interviews, people told me about how cool, friendly, talented, and creative the other teachers and staff at MMAP were and

how getting to work with these people was a key element that kept them engaged in it over a long period of time (e.g., Berg interview; Cole interview; Milanese interview; Teacher EH interview; Teacher FJ interview; Teacher OO interview 5/3/04; Teacher TO interview; Teacher TJ interview). Participants also especially appreciated their interactions with MMAP's founders, with one of the writers from the dorm commenting, "people were drawn to them—they believed in their ideas, they're open-minded, they're successful, they're interesting, and they make things happen" (Milanese interview). Many people—both teachers and MMAP staff—also found the larger environment at IRL to be incredibly stimulating to them, noting that it was a place in which cutting edge ideas could be explored, again enhancing support for problematizing (e.g., Berg interview; Hall interview; Teacher EH interview; Teacher FJ interview; Teacher TO interview).

With the resource of a group of people interested in creating new ways of organizing mathematics instruction (problematizing) but prepared both institutionally and personally to respect each other's ideas (authority and accountability), the project had some key resources in place to have the potential to make progress on the tough problems the MMAP project would work on together.

Getting the Collaboration Off the Ground: MMAP's First Orientation

When MMAP's directors got the main project underway, they hoped to create a community characterized by respect for different people's authority and the marshalling of diverse expertise that the project as a whole would hold itself accountable to (e.g., Greeno et al., 1999). To do that, MMAP worked to create its own norms that included more respect for teachers' authority than is often emphasized as well as an understanding that everyone would be expected to make substantive contributions to the joint work that they would be doing together. One key example of how the project did that was in how it conducted its first orientation for its initial group of collaborating teachers in the spring of 1992 (MMAP orientation video, 5/1/92), which I will now analyze.

Present at the orientation were all six of the newly recruited MMAP teachers, the director of the reform network who had worked with the San Francisco teachers, two representatives from the national labs (the lead scientist and a programmer), Goldman, McDermott, Greeno, Hall, and six Stanford research assistants, half of whom became longtime staff on the project (ibid.). The primary orientation events included: a quick description of the agenda by Goldman, brief introductions, opening remarks by Goldman and Greeno, a community-building joke by McDermott, five minutes of very active informal chatting, an ice breaking learning activity, presentations about what the project was about from various people, a catered lunch, and then a choice of afternoon brainstorming sessions around ideas for possible MMAP units. In the next paragraphs I will show how the principles of problematizing, authority, and accountability were embodied in remarks from MMAP's PI's Goldman and Greeno as well as in the ice-breaking activity.

Problematizing Around Classroom Problems with Important Practical and Theoretical Implications

In its orientation, MMAP problematized the work everyone would be doing as addressing important practical and theoretical issues. Coincidentally the orientation was held the day after the Los Angeles riots, and Greeno's introductory remarks made reference to them, which he used to vividly illustrate why the problems they would be working on together were important:

> "[The lab scientist's] comment sort of in passing that his work has changed because it's a calmer and safer place in the world today, I guess it probably feels like that in the State Department. I doubt it feels like that in downtown Los Angeles this morning. We've got a moment, we are all aware of it, we care about it, we worry about it. Today particularly, we've got reason to want, very badly, to change the world so that some people who are left out, …who are kept away from a society's resources can get it so that separation, some of those blocks get a little less strong. And that frankly has been the lead paragraph on this project since the first time we started talking about it. There are many people in this society for whom the experiences that happen in school, and the rest of their lives, lead them to a belief that…the understanding of mathematics and science simply are not available to them.… So the experiment that we are engaged in, which from my academic standpoint is an experiment to help us understand the nature of learning better, is also this very important social experiment." (MMAP orientation video, 5/1/92)

So it was implied here and elsewhere that working together to create models to engage more students in mathematics had the prospect of addressing larger equity issues in society like gaps in school achievement and economic standing that result in part because of the gatekeeping role of mathematics. At the same time, Greeno emphasized that these equity goals would be achieved through a "different kind of research" that they would be doing together. He said that his job was to "get a little bit more information, a little bit more insight, a little clearer theories about the processes that you all cause to happen everyday when you're in the classroom, getting kids to learn things," which is an "academic problem, but a lot more than that of course." Thus the idea was that making progress on theories of learning and on equity issues in mathematics instruction would go hand in hand.

How mathematics instruction is usually conducted was then further problematized during the ice-breaking activity in which the group became a class of students learning a new procedure for the first time, except the procedure was learning how to write shorthand with IRL's office manager instructing the group using the kinds of demonstrate-then-practice methods that are typical in U.S. mathematics classrooms (ibid.). In the discussion that followed, the group reflected on their and others' experiences as students in such lessons, which led to ideas about how and why lessons like these might not fully support middle-school students' initial and continued engagement in mathematics. This discussion then helped to support PI Goldman's earlier invitation that the group jointly develop a "new vision and a new practice for kids learning mathematics in our schools." Thus, in its orientation, MMAP's activities were initially problematized around addressing important issues of equity in mathematics by rethinking standard instructional practices in ways that might foster more student engagement with mathematics while leading to better educational theories.

Establishing Expectations About Teachers' Authority

Many of the remarks by the PI's at the orientation were filled with language like "jointly develop" (Goldman) and "our shared activity" (Greeno) that presumed that teachers would function as equal-status participants in the project along with curriculum developers, educational researchers, and the lab's scientists (MMAP orientation video, 5/1/92). Consistent with this, Goldman talked about being eager to "make use of everyone's knowledge, and energy, and good ideas" (ibid.) This statement implies that all participants, including teachers, have some authoritative knowledge that they could contribute and that they were not just authorized, but encouraged, to do that. Later, when a teacher explicitly asked about the relative power of teachers versus researchers and expressed concern that the project could become a heavily top-down thing, Goldman made it clear that "we're trying not to have it be that way" and that "the spirit is of full participation and equal rights to planning something or suggesting something" (ibid.). Later Goldman embodied this by specifically inviting teachers to share their ideas about

real-world applications that could serve as anchors for the mathematics units as "your idea for a project is as good as anybody else's" and therefore just as worth thinking through and considering. So in general, teachers were authorized and encouraged to share their ideas to the same degree that other participants were. And in fact, in the discussion that followed the shorthand lesson, five of the six teachers made extended comments to the whole group about what they had learned. The fact that everyone in the room was new to shorthand may also have helped level the playing field of presumed expertise among the participants, making it easier for the teachers to contribute what they thought. In general, this began to create a pattern in the project in which diverse stakeholders were not just told that they were welcome to contribute to it, but were frequently given opportunities in which they could do just that (Goldman personal communication 12/28/05).

However, with respect to one specific issue, MMAP's PI's made it clear at the orientation that they expected teachers to function as the primary authorities within the project. As Greeno explained to the teachers:

> "This is very definitively NOT a situation where, the science of learning, has a bunch of results, that we are now going to explain to all of you, so that you can go into the classroom and use them…. [Instead we will] work with you…on the problem of figuring out how your teaching can make use of the resources that will get built…. How are you going to make use of that? What will be YOUR interaction with it?… You're gonna find that we are going to treat this as YOUR problem more than ours." (MMAP orientation video, 5/1/92)

Thus, the teachers were positioned as the ultimate authorities for deciding whether and how to use the curriculum materials in their classrooms, something that makes sense given they were the ones who both knew their classroom situations the best and would be affected most directly by how well lessons went. And in fact over the lifetime of the project, MMAP's teachers continued to hold the ultimate authority for making instructional decisions.

Encouraging Accountability to Others and for Changing Teaching Practices

In addition to supporting teachers' authority, during the orientation the project also set the stage for MMAP's participants to hold themselves accountable to each other and for teachers specifically to hold themselves accountable for making changes in their teaching practices.

With respect to supporting accountability to others, on several occasions the PI's mentioned the importance for the later productivity of the project of everyone getting to know each other and "the places that we're working from" (Goldman, MMAP orientation video 5/1/92). McDermott commented that the project consisted of "four chunks of people [from schools, labs, universities, and R&D firms] who don't talk to each other a lot, or get paid off by institutions to talk to each other in particular ways" and that the goal was to get everyone's ideas and concerns "on the table as much as possible" to be able to eventually develop a consensus about some ideas for mathematics units (ibid.). To get started on this process, the schedule for the orientation and the first summer institute included time for various participants to introduce themselves and their contexts in greater depth. This then provided a stronger basis for participants to account for how their suggestions related to what other members of the project cared about or had already contributed.

At the same time, it was made clear during the orientation that although teachers were the ultimate authorities for deciding what would happen in their classrooms, they were expected to somehow change their teaching practices as a result of participating in the project. The clearest statement to this effect was made by Greeno in his initial remarks to the teachers:

> "We're assuming that this is going to involve a change on your part. That you get some new [instructional] resources…it's going to change what you do. We hope. That is we hope they're useful in a way that will make that happen…. Because you have some [new] things to work with, or just because of whatever else goes on. Maybe the conversations we'll have about these things will be interesting and bring about some changes." (MMAP orientation video, 5/1/92)

Later, Goldman talked about how the summer institute would probably result in the creation of some initial materials that teachers could then try out in some way during the fall (ibid.). Thus she helped provide some specific parameters around when and how teachers might begin experimenting with their teaching practices on the project.

Summary of the Analysis So Far

In this section, we saw how MMAP's founders assembled a collaborative team and oriented them to the work ahead in ways that embodied the Engle and Conant (2002) principles. First, the project began fostering problematizing around MMAP's goals by inviting to the project potential participants whose own goals already overlapped with them, and by demonstrating to the project as a whole the importance of core MMAP goals like enhancing equity and developing new models of mathematics instruction. Second, the project began supporting teachers' authority by selecting staff already inclined to respect teachers and their contributions, by both telling and showing teachers that their contributions were desired, and by setting expectations that teachers would have the ultimate authority for how MMAP materials would be used in their classrooms. Finally, MMAP began fostering accountability by telling teachers they would be expected to make changes in their teaching practices; and by selecting a diverse set of participants with various types of expertise and encouraging them to begin getting to know each other and engaging in focused conversations with each other.

V. Embodying MMAP's Vision and Collaborative Goals in Curriculum Design and Research (1992-1995)

The key foundational work of MMAP was done from the summer of 1992 to the summer of 1995. In particular, it was during this phase that MMAP developed its methods of collaborating with teachers and math-using professionals. Through these collaborations, the project designed the five applications-based curriculum units that it became most known for as well as began several innovative programs of research. In this section, I will first present evidence from my interviews that participants valued the collaborations they had in MMAP. Then I will separately consider how collaborations around curriculum design and then research were supported using the Engle and Conant (2002) principles. Finally, I will close by considering how the project and its members coordinated with various external contexts during this phase of the work.

Evidence Participants Valued the Collaborations in MMAP

The MMAP teachers that I interviewed who were involved in this phase of the work were unanimous at expressing the value of the collaborations that they had with MMAP staff and with other teachers involved in the project. Teachers talked about there being "a lot of teacher buy-in" and about "really feeling valued in a professional sense," something most had rarely experienced in the rest of their teaching career before or after (Teacher EH interview; Teacher TO interview; also Teacher OO interview 5/3/04). Some even said MMAP was the best professional development they had ever participated in (Teacher LE interview 3/2/03; Teacher CA interview). For example, one teacher summarized what she got out of her participation in MMAP like this:

> "But again I did learn so much from them. The relationships with those people are so wonderful. I'm glad I know them…. The interaction is always great. Like you're meeting with great people and you're learning and you're absorbing and you're getting the wisdom of other people…. It was the best professional experience I ever had." (Teacher LE interview 3/2/03)

More specifically, teachers spoke of appreciation for:

- Having materials to teach in the ways they wanted to teach (Teacher EH, Teacher CA);
- Being exposed to innovative ideas (Teacher EH, Teacher FJ, Teacher OO, Teacher TO);
- Being able to talk with others about curriculum issues (Teacher EH, Teacher LE, Teacher OO);
- Getting others' perspectives on what was happening in their classrooms (Teacher OO, Teacher CA, maybe others);
- Developing new instructional skills (Teacher LE, Teacher CA); and
- Learning that they could be creative in designing curricula for their own and others' teaching (Teacher EH, Teacher FJ).

These results were also corroborated in a 1998 evaluation report of MMAP, which concluded "teachers felt professionally respected in MMAP in ways some had never before experienced. Those who participated consistently spoke of MMAP not merely enthusiastically, but with a fervor that bordered on the evangelical" (Lichtenstein, Wiessglass & Ercikan-Alper, 1998, p. 50).

Staff members had similar perspectives on the collaborative environment of the project. For instance, one MMAP programmer noted that, "This was truly a place where I felt like the whole

was better than the sum of the parts, like the team worked as a team" (Gallagher interview). Similarly, one graduate student and longtime staff member commented, "For me, I always felt unbelievably fortunate to have stumbled in there, and once I was there I never wanted to leave…. I was in heaven" (Karen Cole interview). The quotes could go on and on. Although to some extent these quotes can be considered to be examples of what Greeno et al. (1999, p. 320) referred to as "how members of one mathematics learning and teaching community sing the community song that enables all those involved to work together," it is important to note that these interviews were conducted long after the project had ended and the interviewer was considered to be either an outsider or at most a peripheral member of the project to most of the people who were being interviewed.

Supporting Productive Collaboration Around Curriculum Design

It was also during this phase of MMAP work that it initially created the five design-oriented curriculum units that MMAP became most known for: Antarctica, Wolves and Caribou, Codes, Dream Home, and Guppies. Design of these units was initiated roughly in the order specified above, with the first unit, Antarctica, providing a concrete model that crucially informed the design of the other units and helped the project become more explicit about what it meant for a unit to be "applications-based" (Cole interview; Goldman interview 10/27/04). Design of each unit involved work on: constructing a workplace-based applications scenario to anchor the unit (e.g., designing a field station in Antarctica or deciding how to manage the Wolves and Caribou populations in Alaska); determining what mathematics could be mined from that scenario; designing software to support each unit;[6] creating specific activities for students to directly address the challenges of the scenario and learn needed mathematical skills and understandings; piloting and revising the evolving units; and developing materials and activities to support other teachers in being able to teach the unit. As with other examples of design-based research, units were designed and redesigned over numerous iterations that involved extensive pilot testing, which according to one teacher-turned-staff member allowed "their most useful form [to] emerge" (Teacher TO interview). Design of units was done by teachers and MMAP staff together during monthly teacher workdays and several week long summer institutes, by MMAP staff between these meetings, and sometimes during the process of piloting a unit as new activities were made up on the fly by teachers and/or MMAP staff to deal with an emergent problem or learning opportunity. However, the exact process through which each unit was designed differed along many important dimensions—the participants who were involved, the pace of the design, how ideas were generated, the ordering of curriculum design tasks, and how curriculum design was coordinated with other project activities (see also Lichtenstein et al., 1998). [7] Thus, there was no such thing as a typical MMAP curriculum design process. However throughout, promoting effective collaborations between teachers, MMAP staff, and math-using professionals was addressed by the project through its embodiment of the four principles, which I now turn to next.

[6] ARCHITECH for Antarctica and Dream Home; HABITECH for Wolves and Caribou and Guppies; and CODING TOOLBOX for Codes.

[7] For example, one extreme was Wolves and Caribou, which was written day-by-day as the unit was being piloted. In contrast, the Guppies unit was written over the 1993-1994 academic year and then fully vetted by teachers during the 1994 summer institute before being piloted in the fall of 1994 (Teachers OO and LE 1994 interview).

Problematizing Through Multi-Purpose Design Tasks

In my prior research (Engle & Conant, 2002), I argued that productive engagement requires something to engage with that is genuinely problematic to everyone involved and that is viewed as sufficiently important so that all members of the group are committed to working on it. In MMAP, such problematizing was anchored by multipurpose design tasks in which teachers, researchers, and technical specialists like programmers and writers began to co-construct curricula and other practical tools to support new kinds of mathematics instruction (Knudsen interview 04/05/04; Greeno et al., 1999). For example, Greeno et al. (1999) analyzed two examples of coordinated research and teaching products that were launched by practical problems or dilemmas in which teachers, researchers, and curriculum developers all had a stake. Similarly, Knudsen (interview 04/05/04) talked about how discussions about pedagogy with teachers created in part for research purposes "fell flat" until teachers were given the design task of writing a teachers' guide for other teachers. As she explained, designing the teachers' guide "was a practical thing to do, and it also was a way for people to make…the things that were really important to them explicit, not because they could get…the floor in the discussion…but because they really needed to tell the other teachers about it," something that fit better within the practices of teachers (ibid.).[8]

Design tasks were a particularly good anchor for collaborations in MMAP because of their relatively open-ended nature. To complete these tasks, the groups needed to design things (curricula, teachers' guides, etc.) that did not already fully exist, providing space for everyone to make contributions. Staff reported that they typically provided at the very most a broad "map" of a proposed unit along with "some sample activities" or "prototypes of the software" that everyone would then try out at meetings (Cole interview; Hall interview), basically presenting them as unfinished "working hypotheses" for others to improve upon (Greeno et al., 1999, p. 302). Teachers would then be asked for their feedback on what changes should be made (Cole, Teacher EH, Teacher FJ, Teacher TJ, Teacher OO, Teacher TO interview, Teacher CA interview). As one teacher described the process, "they would run their ideas by us: 'is this something that would work in your classroom?' 'would you test it in your classroom?' and we would offer some ideas too" (Teacher CA interview). As a result new versions of the design would be created, which would then provide concrete anchors for subsequent revisions both of that unit and of others that were being worked on in parallel or subsequently.

At the same time, the kinds of design tasks MMAP chose simultaneously helped accomplish multiple goals, each of which was important to different MMAP participants, a key aspect of promoting true problematizing (Greeno et al., 1999). As Greeno et al. (1999, p. 318) noted, there was "a pattern in our project of inviting joint work on design followed by separate work on development, teaching, and research." For example, in developing the teachers' guide, the same design activity allowed: (a) the project as a whole to better disseminate its curriculum, (b) teachers to use their professional expertise to engage in the valued work of helping other teachers, and (c) researchers to learn more about what was pedagogically important to participating teachers in the curriculum (Knudsen interview 04/05/04). Similarly, in designing curricula, teachers had new materials to use in their classroom that better embodied their visions of how they would like to teach (Teacher EH interview; Teacher FJ interview; Teacher CA

[8] As Knudsen further explained, "Pedagogical discussion always fell really flat. It was kind of like there was no reason for anybody to do anything… particularly for teachers who really live in a different culture than researchers…. There's really no…[practice of having] a discussion and making an important point…. It was like 'why bother to make a big point out of that?' " (ibid.).

interview; probably others) while researchers were provided with environments that would allow them to study new methods for teaching and learning mathematics (e.g., Berg interview; Greeno et al., 1999; Greeno interview 3/24/03; Hall interview).

Thus, one aspect of problematizing across a diverse group of stakeholders is working on joint problems that simultaneously satisfy the sometimes differing goals of participants from different communities. As Jim Greeno summarized it, "That merging of functions, purposes, and goals was…one of our most enjoyable and perhaps productive outcomes" (Greeno interview).

Continuing to Foster Teachers' Authority

In this section, I first analyze how the project supported teachers' authority, and then explain how the project became committed to supporting their authority in part through its struggles to coordinate teachers' authority with that of math-using professionals.

How the project supported teachers' authority. Building on the work MMAP had done in its selection of staff and its orientation, the project also structured itself in ways that fostered a variety of aspects of teacher authority. Focusing on establishing teachers' authority was especially important as teachers are commonly viewed as having lower-status positions than researchers. Specifically, the project treated its collaborating teachers as *agents*, *contributors*, and *authorities* (or local experts; see Engle and Conant, 2002, p. 404 for definitions) in an effort to productively engage them with authority in the project.

First, with respect to *agency*, MMAP authorized teachers to decide how they wished to participate in the project. This fit with the overall working environment at IRL where "you could pick and choose [and] you could multi-task" (Goldman/Knudsen interview 10/27/04). For example, one teacher described initial MMAP summer institutes this way:

> "Those summer institutes, they were pretty open-ended too. It wasn't like 'you have to do this, this, and this.'… My sense was that they really wanted you to participate at the level that was most meaningful for you and every teacher knows how they can make the most out of that time.… And you definitely felt like you had choices in what you worked on and what your focus was gonna be, and it was just so nice to be able to do that. … So they [the teachers] were able to sort of help formulate their professional development around what they knew they needed in order to go use the stuff." (Teacher EH interview)

This model, used for the first two summer institutes at IRL, needed to be revised, however, as many teachers found so many choices overwhelming and were torn when they could not attend everything. It also conflicted with the working environments teachers were used to from their schools, about which Goldman and Knudsen remembered one teacher saying something to the effect that,

> " 'We come from jobs where people tell us what to do from the minute we are there in the morning until the minute we leave in the afternoon. Every place we go, every person we see, it's mostly a structured kind of thing. Now you are asking us to make choices and we are uncomfortable doing this.' " (ibid.).

In response, MMAP staff reduced the teachers' need to make choices between activities and instead continued to foster the teachers' agency by involving them in the planning process for what would be on the group's agenda for workdays and summer institutes (Goldman/Knudsen interview 10/27/04). Later, teachers were given opportunities to join (or not) long-running working groups like the Assessment Club, the Codes Unit design group, the Graph Theory Group or the Mathematical Functions group that allowed teachers to design and learn about something they cared about with a group of similarly interested colleagues (Berg interview; Cole interview; Moschkovich interview). In this way, teachers were given choices about how to use their time, but in a way that conflicted less with their usual work practices. As one teacher summarized it, "There were a lot of ways of participating and contributing to [the project].…

They didn't come in trying to tell people what to do. That made a huge impact, and a high level of trust was developed quickly out of that" (Teacher TO interview).

Another aspect of teacher authority fostered during this phase was teachers' authority to be true *contributors* to MMAP curriculum design. As mentioned earlier, teachers were presented with partially worked out curricula, and were then asked for their feedback (e.g., Teacher CA interview). But what was key for making teachers actually be contributors was that their feedback and other ideas were regularly reflected in the next iteration of the curriculum, something that many teachers I interviewed noticed and were especially pleased about (Teacher EH interview; Teacher LE in 1994; Teacher OO interviews in 1994 and 5/3/04; Teacher TO interview; Teacher CA interview; also corroborated by Lichtenstein et al., 1998). For example, as one teacher described it:

> "The staff had a propensity to listen and to listen carefully to what teachers had to say. A lot of times teacher suggestions would show up in the next iteration of the curriculum. Teachers had a pretty good sense that they were valued and that their ideas were valued." (Teacher TO interview)

Several teachers also mentioned specific things they had authored[9] that became part of the MMAP curricula, including initial ideas for units, specific activities to add to them, and assessment methods to use during them (Teacher LE interview 3/2/03; Teacher TJ interview; Teacher TO interview), with staff also crediting these teachers with such authorship of particular MMAP products (Goldman/Knudsen interview 4/27/04; Hall interview; Knudsen interview 5/5/04; and many others). A few teachers also noted that this practice of regularly incorporating teachers' feedback and other ideas into the next versions of the curricula was striking as it contrasted positively with standard practices in schools and other curriculum projects of asking for teachers' feedback and then doing nothing with it (e.g., Teacher EH interview; Teacher OO interview 1994). One teacher claimed that this use of teachers' ideas also encouraged teachers to keep using the curricula that they had helped to shape (Teacher TO interview).

Finally, MMAP fostered the principle of authority by treating teachers as having *authority* in their own work (Knudsen interview 3/20/03; Moschkovich interview). But given common assumptions by many teachers and the public at large that people with advanced academic degrees are more knowledgeable than teachers, how did the project help teachers to understand both that they had important expertise to share and that researchers valued that expertise? First, when teachers shared their knowledge, MMAP researchers actively listened, giving teachers the idea that "the researchers were, or appeared to be, genuinely curious about teachers' practices" (Teacher TO interview; also see Knudsen interview 5/2/04). Second, in the few cases in which someone attempted to use an academic degree as a basis for deciding an issue, that person was informed privately that this was unacceptable on the project (Goldman interview 10/27/04). As one MMAP teacher described it:

> "Those people we worked with were so common, so humble, and you know that they had all this education behind them and everything but it was like you wouldn't know it. They almost hid it from you. You know what I mean? I mean you knew they were brilliant and stuff like that but their interaction with us was so on a par." (Teacher LE interview 3/2/03)

This drew from the fact that MMAP's ideal was that everyone has important knowledge to contribute to the task of school reform (Greeno et al., 1999). In particular, the project believed and generally acted on the belief that "teachers and children, in some significant sense, knew more about the circumstances of school than did the researchers and developers" (ibid., p. 307).

[9] This list does not include authoring of content that occurred when some of these people were later hired by MMAP to be regular staff.

And this made sense because, as one of the MMAP researchers and curriculum developers explained:

> "Collaborating with teachers…added so much credibility to our work. We weren't just coming up with stuff out of the air that we thought ought to happen in classrooms. We had these experts that we were working with that could tell us right away, 'oh, that's not going to work,' or 'okay, but can we add this in because my principal needs to see this or that.' " (Cole interview; personal communication, 5/2/06)

So MMAP teachers were treated as authorities whose expertise complemented that of other participants, in what might be characterized as a "distributed expertise" model of collaboration (Brown et al., 1993; Huberman, 1999).

Learning to support teachers' authority via challenges to it by math-using professionals. The general value of supporting teachers' and staff members' authority in the project developed in part due to the project's initial experiences attempting to foster productive collaborations with what it later came to call math-using professionals, people like scientists, architects, and event planners who were not mathematicians but used mathematics in their work. The first set of math-using professionals that the project worked with were the scientists at the federally-funded weapons lab who had helped initiate the project. It was through them that the project became acutely aware of the challenges of drawing on the expertise of such people while not allowing them to use their generally higher overall status to trammel on the teachers' authority (Goldman interview 10/27/04).

Several of the lab scientists' actions showed MMAP staff and directors that they did not sufficiently respect the authority of the project's participating teachers as well as other members of the project. First, activities planned by the scientists for teachers often involved the scientists demonstrating or lecturing at the teachers and putting them into the kinds of recipient roles that provided them with few opportunities to share their own ideas, thus reducing their agency and opportunities to be contributors (Goldman/Knudsen interview 3/20/03; Staff CC's memo 7/31/92). Second, the chief lab scientist had different ideas about what the group should do than everyone else and was generally not willing to compromise on them (McDermott interview). This made it impossible for the project to design curricula that fairly incorporated everyone's ideas and held itself accountable to what other members of the project knew was necessary to be included in any middle-school mathematics curriculum. For example, the scientists apparently felt strongly that proportional reasoning, a key foundation of the middle-school curriculum, should not be included in MMAP's curriculum because students should already have mastered it much earlier. They also insisted that the units be designed around military situations that MMAP teachers and staff did not think were appropriate for this age group (Goldman/Knudsen interview 10/27/04). Finally, there were times when the scientists showed clear disrespect for teachers' expertise. For example, one staff member recalled an especially uncomfortable session in which the main lab scientist brought in his teenage daughter to do a graphing calculator activity with the teachers in which the scientist made a point of demonstrating that his daughter knew more of the relevant mathematics than some of the teachers (Gallagher interview).

After trying explanation, negotiation, mediation, and other ways of addressing these problems, the project eventually decided (in consultation with its program officer, advisory board and director of IRL) to get an official divorce from the lab scientists on the grant (Goldman/Knudsen interview).[10] This action made it clear that teachers' authority must be

[10] The divorce was also facilitated by the fact that money that had been promised to the scientists to work on the project by their bosses had not come through, which meant that their level of effort and ability to make contributions was also decreasing because of this fundamental resource issue (Goldman/Knudsen interview 10/27/04).

respected by everyone in the project no matter who they were, and that anyone who did not do that could not remain in it long-term. "Honestly, our instinct was to protect the teachers from them [the scientists]. They got that strong. So we felt like we were not going to be able to keep teachers in this project if we kept the scientists there, in that way." (Goldman/Knudsen interview 3/20/03). Goldman later speculated that all the effort the project "spent trying to manage the level of how everyone interacted" may have been strongly affected by this initial "shock with the scientists" (interview 10/27/04). The "strong orientations that they had and how they were willing to impose their views…may have really sensitized us" (ibid.).

Thus, overall the attempted collaboration with the lab scientists did not work, which can be understood by applying the principles to it. With respect to problematizing, it was difficult to work toward designing curricula in a collaborative manner when the visions of the curricula that the scientists and the rest of MMAP were aiming for were so antithetical to each other. Even when the project found points of overlap in vision between the lab scientists and everyone else, the fact that the lab scientists wished to be the primary authority to whom others should listen meant that voices of teachers and others could not be heard, making it no longer a collaborative enterprise. Similarly, with respect to accountability, the lab scientists held themselves only minimally accountable to others in the project, with debate between the two groups about conflicting norms to hold themselves accountable to. At the same time, the resource of time that was provided by funding from the government for the scientists to participate dried up, reducing the time available for the scientists to work on the project and potentially to work out their differences with the rest of the MMAP project. For all these reasons, it is not surprising that this ended up being a failed collaboration.

In response to the problems they had with the lab scientists, MMAP devised two other ways to work with math-using professionals that did not have the same risk of them using their status to have undue influence. One way of doing that was to have teachers shadow math-using professionals doing their work, giving them the authority to identify the interesting applied problems and middle-school mathematics that were embedded in it (Goldman/Knudsen interviews 3/20/03, 10/27/04; Hall interview; McDermott interview; Moschkovich interview). The project found this practice particularly effective as middle-school teachers were the ones with the relevant knowledge of middle-school students and of the middle-school curriculum that allowed them to see curriculum topics in the professionals' work better than they could themselves. The most promising of these workplace problems then became the basis for potential MMAP units. So having teachers mediate the participation of math-using professionals was a means of drawing upon those professionals' practices and expertise while avoiding potentially difficult power dynamics had these professionals been included as direct collaborators. It was also a more efficient way of learning about many different workplace contexts (Goldman/Knudsen interview 3/20/03).

Later, the project informally consulted with various math-using professionals in an effort to make sure the units were as authentic as possible. For example, Cole reported calling a whole host of experts on wolf and caribou populations in different Alaskan habitats in order to be able to use realistic population numbers for the Wolves and Caribou population biology unit (Cole interview). Similarly, the types of codes used in the coding units were verified with one of the country's top cryptography experts, who noticed that the types of codes used in the units were the ones he had first found himself becoming fascinated with as a teenager (Goldman/Knudsen interview 10/27/04). These people, who were often found through the social networks of MMAP

staff, were often glad to help out when they found out the project was seeking to improve education (Goldman/Knudsen interview).

As Knudsen summarized it, the project learned what math-using professionals could best contribute versus what was better left to teachers or staff:

> "It…became clear that we [teachers and staff] were going to have to be the ones to develop curriculum, that we had that expertise. So it was really helpful to have these people show us mathematics that they used in great detail, brainstorm ideas with us, and observe their world, and then look at our products and say, 'yeah, that's in line with what I do, but simplified down for a middle school.' "

From this, the project found ways to organize their collaborations to make it more likely that those contributions would be made.[11] In doing so, MMAP addressed the potentially overwhelming authority of such high-status professionals by allowing them to contribute their ideas, but without the power to make decisions for the project.

Fostering Accountability to Each Other Without Impinging on Teachers' Authority

If teachers' and others' engagement in the project was to be truly productive, however, much more had to be done than to find tasks to engage everyone in and make sure that everyone felt comfortable contributing their ideas. The project had a vision to realize and deadlines to meet, and there were different views among teachers and between teachers and staff that needed to get coordinated to some extent for the project to do its work. How did the project manage this? As this section will demonstrate, the project did so by gradually fostering participants' sense of accountability to each other and to crucial project goals, norms, and ideas. At the same time, it also relaxed accountability in some ways to allow teachers to customize the curriculum to their own teaching contexts. Thus, accountability was selective, focusing only on those aspects that were perceived as crucial for meeting the projects' goals and embodying its values. As mentioned earlier, but it is worth repeating here, the kind of accountability that I am discussing is an internal dynamic accountability in which participants in the project learned to 'account for' how they were being responsive to important project goals and what others in the project were doing (see Engle & Conant, 2002; Engle & Faux, 2006. This is very different from an external form of accountability in which outsiders evaluate whether the project or its members are meeting externally imposed standards.

The project directors described themselves as generally "having a supportive…hands-off management style" but one in which they did "hold people accountable to…the principles and the timelines" (Goldman interview 10/27/04) and in which they became good at "leading where it was important to lead" (Knudsen interview 5/2/04).

To do this, however, the project had to establish accountability to shared norms and ideas. To illustrate this, I will discuss how the project gradually fostered accountability to the idea of an "applications-based unit," a crucial element of MMAP's curricular approach (Knudsen interview 5/2/04). Initially, teachers and staff had many different ideas of what an "applications-based unit" might mean (ibid.; Cole interview; Greeno et al., 1999; Teacher TO interview). When the project initially brainstormed possible MMAP units with teachers during the first summer institute, a huge range of possibilities was proposed, with many of them not making sense at least from curriculum coordinator Knudsen's perspective (Knudsen interview 5/2/04). As one then newer staff member described it, MMAP's directors "wanted to involve the teachers in coming

[11] This was also one early example of a move to more asymmetric models of collaboration that I will discuss as part of the next section about MMAP's work from 1995 to 1998.

up with ideas for activities that would be part of the curriculum. But I think there was kind of a disconnect.… The teachers would come up with things, and the MMAP staff had this sort of more fleshed out idea of how activities would relate and how open-ended they would be…. I don't think we could say even to ourselves what we wanted, we just kind of recognized what we didn't want" (Cole interview). Specifically, what they realized they did not want was the lab scientists' proposal for a good applied problem, the Lifeguard problem, which was basically a standard word problem involving a lifeguard plotting the fastest trajectory between him or herself and someone drowning in the ocean, given different speeds for running and swimming (e.g., Cole interview; Goldman/Knudsen interview 3/20/03; Hall interview; Teacher FJ interview, etc.).

Over the course of the year, Knudsen and others became explicitly aware that by "applications-based" they meant starting with a rich, real-world problem context (like being an architect who needs to design a building) and then pulling the math out of that (see also Cole interview). Therefore at the second summer institute, Knudsen started by trying to carefully explain her vision of what an applications-based unit meant, and then asked, " 'okay given that, what do you think a good context would be to use?' " (Knudsen interview 5/2/04). At the same time, those words were backed up by the activities described in the previous section of having teachers shadow math-using professionals in order to identify relevant real-world problem contexts for possible curriculum units (Knudsen interview 5/2/04). In effect, the project worked to "get enough exemplars of the kinds of experiences that we were looking for so that the teachers could be on the same page with the staff and the staff could be on the same page with the teachers" (Cole interview). This eventually developed to the extent that MMAP could explain to new teachers that its applications provided "a slice of the real world that engages young people in mathematics" (MMAP marketing presentation, slide 3) and that the units "plunge students into an extended role-play as employees involved in a real-world problem" (http://mmap.wested.org/pathways/MMAPMaterials.html). Overall, the pattern was one of gradually refining the norm of an applications-based unit, allowing the project's participants to hold themselves accountable to it more and more over time.

More generally, there was a real tension in the project around whether, when, and how MMAP staff should work to push teachers to consider changing aspects of their practice on the basis of what MMAP staff thought made sense, or what might be called fostering a sense of accountability in teachers to the ideas of MMAP staff. On the one hand, many stories were told of MMAP staff noticing that teachers were not using MMAP units in the ways they had imagined them being used and how in general they learned to expect and accept that such adaptations are a normal part of teaching practice and should not be interfered with. As Knudsen explained:

> "I learned to take a different perspective. In a conversation with other curriculum developers, there were complaints of teachers not implementing materials correctly. In one case, when the details came out, it seemed to me that the teacher was doing something very sensible and valuable mathematically, but it was different from what the developers intended. I realized the enacted curriculum is never going to be the same as the idealized vision of curriculum and we had to get off it… It wasn't an easy lesson to learn, but an important one, that I use to this day—though I often still have that initial reaction, 'Hey, that's not what I had in mind.' " (Knudsen personal communication, 11/13/05)

MMAP units were in fact originally designed with flexible adaptation in mind, including many more activities than any one teacher could possibly use, making it clear that teachers would need to choose what would fit best with their students, instructional goals, style of teaching, school-level demands, and other aspects of their local context (Berg interview; Cole interview;

Goldman/Knudsen interview 10/27/04; Teacher LE interview 3/2/03; Moschkovich interview; Teacher TO interview). This also meant that when a teacher had an idea for a potential activity, it could usually be included in the unit without forcing everyone to come to a consensus that it was a critical element to include (Teacher TO interview).[12] This in effect embodied Tyack and Tobin's (1994) recommendation that for reforms that challenge the "grammar of schooling" to take hold, "one might entertain the notion that [teachers' modifications of reforms] are potentially a virtue—reforms might be designed to be hybridized according to local needs and knowledge" (p. 479). To some extent, then, one might say that in some ways the project was organized to relax certain kinds of accountability that teachers might have had to the ideas of other teachers and MMAP staff in order to be able to customize the materials to their own contexts. In turn, this might have kept teachers using the curriculum and participating in the project. One teacher noted that giving teachers this freedom to experiment with activities in the curriculum in different ways also meant that "their most useful form could emerge" (Teacher TO interview). So relaxing accountability initially might have helped foster better designs eventually.

However, MMAP staff would intervene in cases in which particular teachers' practices strongly conflicted with MMAP's core principles, as a way to encourage accountability to the principles. This occurred, for example, when one teacher regularly blamed students for bad educational outcomes or when another was seen to have in effect removed the design challenge when piloting one unit. Staff did not directly challenge teachers when such conflicts with core principles occurred, but instead would ask them to explain why they had made these choices, thus in effect encouraging teachers to "account for" them to others (Goldman/Knudsen interview 10/27/04). Sometimes the process of responding to such inquiries caused teachers to gradually re-examine their practices and change them as in a teacher who "just never thought that kids could do things," something that was "not very MMAP-like [but] who then came a long way" (Teacher TO interview). However, in other cases, such inquiries elicited an understandable reason that the staff member then felt responsible for accepting, as in the case of the teacher who had removed the design challenge (Goldman interview 10/27/05). No matter the initial outcome, through this process MMAP staff would learn what underlay teachers' skepticism with MMAP ideas and then would be "in a great position to deal with people's worries and to do it [persuade them] indirectly" by gently "suggesting things or giving [teachers] something to try, [and] just showing up to help" (Teacher TO interview).

It was also common for teachers to be encouraged to "account for" their ideas and choices during field-tests of the curriculum in teachers' classrooms. The presence of researchers in the classroom during the field-tests would sometimes spur informal planning sessions that would shape ongoing instruction and lead to the immediate revision of the materials. Such conversations were an especially valued source of professional development for participating teachers (Cole interview; Goldman personal communication 12/29/05; Teacher LE interview 3/2/03; Teacher OO interview 5/3/04) as is made clear by the following teacher's account:

> "We could go over what had happened and plan for the next day. And that was exciting. [When] you hadn't noticed something somebody else had, you went, 'oh, I'll look for that.'…Then you'd think, 'well tomorrow I'll

[12] Later, however, as newer teachers became uncomfortable with the non-traditional open-ended, mix and match nature of the curriculum materials, curricula were rewritten to provide a model sequence so that teachers new to the materials had some idea of how one might start (Cole interview 5/5/05; Knudsen interview about changes over time to the Antarctica curriculum). This was also influenced by the expectations of publishers (ibid.; Teacher TO interview).

question him on such-and-such or I'll help him process or work it through.' … The interaction with them at the end of my class was the most beneficial." (Teacher LE interview 3/2/03)

In this case, these planning sessions helped this teacher notice things happening in her classroom that she had not noticed and to come up with ways of addressing them. More generally, such conversations encouraged teachers and researchers to account for how their different observations of the same events could be made sensible to each other, building much deeper understanding of each other's perspectives.

In still other cases, however, MMAP staff talked about how they explicitly pushed teachers to think differently. So for example, Goldman recalled that the teachers' first reaction to an initial draft of the Antarctica unit was that it was not worth teaching as there was not enough math in it (Goldman interview 10/27/04). Rather than accepting the teachers' initial judgments, however, Goldman reported acknowledging the teachers' reactions but then asking them to try out the unit anyway[13] (ibid.). The group did a partial run-through of the curriculum, with MMAP staff querying teachers on what math, if any, they were working on (ibid.). As a result of giving it a chance, the teachers ended up leaving the session amazed at all the mathematics that could be drawn out of a unit like Antarctica (ibid.). Notice here that even in this case, MMAP staff did not try to directly convince teachers to change their minds, but instead presented teachers with some new ideas, encouraged them to "experiment" with them, and then saw "if they took to them or not" (ibid.).[14] In the end, the authority to use or not use the ideas and materials presented by MMAP staff remained with the teachers. However, there were many opportunities during this process in which teachers were asked to react to and thus 'account for' their decisions about what aspects of the curriculum to use and how to use it. Thus, the teachers' accountability was fostered without impinging on their authority, a key challenge in promoting productive engagement (Engle & Faux, 2006).

Goldman summarized MMAP's practices of engendering accountability as follows:

"They [the teachers] actually saw us as people that they were coming there [to IRL] to learn things with…. They came there to experiment a little bit themselves. So it wasn't all just comfort zone. It wasn't comfort zone on our side either… So they pushed us. We pushed them. They pushed us. So I think it was this mutual pushing around of everyone that was kind of agreed to…. We didn't ever have anybody…not come back because they were being pushed too hard…. I think really the time we spent trying to manage the level of how everyone interacted and where the push was, where the push wasn't, and where the fun was, where the fun wasn't, kind of made it easier to do everything else." (Goldman interview 10/27/04)

As teacher and later staff member TO explained in MMAP there was:

"The strength of this warm and open invitation to play, I think people took it seriously, even some who might have been skeptical at first….We just created a climate where people's intellect and their feelings were taken care of. I mean it wasn't like a huge therapy session or anything like that….But we definitely…did, what I would call, deep work with people, at least part of the time." (interview)

And several MMAP teachers made a point of noting the kinds of learning that was afforded by this level of mutual accountability in MMAP (Teacher FJ interview; Teacher OO interview 5/3/04; Teacher TO interview; Teacher CA interview; probably others). For example one teacher who was on the project from the very beginning commented that:

"The real positive thing of having outside researchers and all that in, is the fact they could see some of our ideas that we'd have, and they'd make suggestions and I think they were able to…think out of the box. I…really

[13] Here is the gist of what Goldman remembers saying: " 'Listen, we are going to experiment with something and then see how it goes….Bear with us….We know you think it's not going to work, but let's try it anyway.'"

[14] These methods also appear consistent with the kinds of activities that are often recommended for promoting deep conceptual change in science.

found that it was interesting…as a teacher to have other people… say, 'well, wait a minute, you know maybe if you just changed it a little bit it would make more sense.'… That was helpful because a teacher may use the same approach for years without realizing that students are not fully learning a concept." (Teacher OO interview 5/3/04; personal communication 10/17/06)

So MMAP's teachers also appreciated the push that the project's methods of fostering accountability provided to them for improving their own practice.

Resources Supporting the Collaboration Around Curriculum Design

Along with support the project provided for problematizing, authority and accountability, several resources were key to the success of the collaborations between teachers and researchers around curriculum design. The extensive time that the project provided to nurture collaborations during this phase was crucial (e.g., Goldman EMST/SESAME colloquium at Berkeley, 10/24/05). Teachers and researchers had time to work with each other during 3-6 week summer institutes, at least once a month during the school year, over email, and much more intensively if a teacher was piloting a unit as part of a field-test (Lichtenstein et al., 1998). Having that time together was supported by money from the NSF grants and other sources that paid teachers $100/day for their participation (the maximum rate allowed by NSF), paid for substitute teachers for them during 9 workdays during the school year, gave teachers email accounts, and often provided computers when they field-tested MMAP units. MMAP participants also highlighted the importance of time to socialize and talk informally in the hallways, over meals, while carpooling, and in shared offices as being important for supporting the collaboration by keeping everyone aware of what everyone else was doing, facilitating the sharing of new ideas, providing non-threatening ways to get feedback, and helping to develop the kinds of close personal relationships among staff and between staff and teachers that helped keep everyone working well together on challenging tasks (Cole interview; Gallagher interview; Goldman/Knudsen interviews 3/20/03, 10/27/04; Moschkovich interview; Teacher TO interview). In fact, the project "learned from the teachers that the more informal it felt, the more friendly and collegial, the better things went. And so we used to have our workshops in the kitchen. And you know, actually we were doing workshops, but you'd see the sink and there were snacks and we'd eat together. Things that really are able to bring people together with each other" (Goldman/Knudsen interview 3/20/03). It might seem to the reader that these resources—time, money, food, and a relaxed tone—are so basic as to be trivial. However, it was emphasized repeatedly by MMAP participants both (a) how rarely these basic resources had been provided to them by the other projects and institutions with which they had worked, and (b) the negative impact on the nature of their collaborations when they were not provided.

Supporting Productive Collaboration Around Research

While all the work was being done to design curricula within MMAP's collaborative community, the project was also simultaneously doing research deeply connected to the curriculum development work, in what the project referred to as an "interactive research and design" model (Goldman personal communication 12/29/95). [15] The goal was to create materials

[15] The project purposely did not refer to itself as doing "design experiments," the then-current term for design-based research as it did not wish to imply that it was doing experimental studies, but rather was doing primarily ethnographic work focused on discovering new phenomena. The phrase "interactive design and research" was meant to suggest both that design and research interacted, and that the design and research work was done in interaction with real world settings including the people in them (Goldman personal communication 12/29/05).

that worked while also keeping an "eye out to generalizability" and academic "resume building" (Goldman personal communication 12/29/05). There were two kinds of research done on the project: research to directly inform the design of the materials and research to make a general contribution to the research literature (Bushéy, 1997; Greeno interview). Both types of research had their origins in the ongoing design work that occurred in the project, but different products would be created out of them to serve these different purposes (Greeno interview; Goldman personal communication 12/28/05). Here I will focus on the research that did not just feed back into the design, but that also turned into products for the research community. Again I will use the Engle and Conant (2002) principles to help explain how productive engagement, but now in research, was fostered.

Problematizing Research: Emergence and Later Decoupling of Studies From Practical Problems

The process of problematizing research in MMAP often involved a complex interaction between the project's or a researcher's interests and the emergent opportunities to address them in the context of the larger collaboration with teachers to design usable, innovative curriculum units. Typically, during ongoing work with teachers, a specific practical problem or compelling phenomenon would be recognized as being relevant to a general research question that one or more researchers were interested in. At that point, work would proceed on the problem or phenomenon not just for purposes of informing the redesign of the curriculum, but also for making a product for the research community. For a fairly long time, progress on both the practical and research aspects of the issue could occur simultaneously. However, eventually problematizing around research would need to become decoupled from problematizing around curriculum materials because the nature of the final products needed to be so different. Because of that, extra efforts would be needed to make sure research was completed.

For example, as MMAP began, both the project as a whole and specific researchers within it had general research interests they wanted to pursue. Among many others, there was interest in issues like explaining how to support more equitable participation in mathematics classrooms, finding out what kinds of mathematical thinking and reasoning would be supported by embedding mathematics within workplace applications, understanding what is involved in teaching in the new ways being recommended by the reforms, and creating better theories of mathematics learning as a social practice (Cole interview; MMAP I proposal; Teacher CA interview). These interests were not as specific as research questions, but they influenced what researchers paid attention to in MMAP classrooms and in the rest of their ongoing work with teachers.

In the course of co-designing curricula, field-testing units in teachers' classrooms, or watching videotapes of those field-tests, practical problems or compelling phenomena would emerge that would capture the attention of both teachers and researchers, compelling them to start to address the problem or explain what had happened (Cole interview). At some point in the course of that process, someone would recognize that the issue they had been captured by was not just a practical problem that needed to be addressed or some classroom achievement to celebrate, but also spoke to an important issue in the research literature. Because of this, many of the empirical studies that MMAP pursued were ones that emerged out of practical problems or salient phenomena that occurred in MMAP classrooms, but that also could be understood as specific instances of broader research questions (Bushéy, 1997; Cole interview; Greeno & MMAP, 1997).

For example, one teacher's concerns during a field-test about boys dominating small group conversations around computers and her hypothesis that this was because they tended to take over use of the mouse was an instance of the general MMAP question "how can we have a more equitable situation that draws in traditionally underserved students?" (Cole interview). This then led to an empirical investigation led by Karen Cole in that teacher's and other teachers' MMAP classrooms to see what control of the mouse meant for the nature of the contributions that each student was likely to make to their groups. Contrary to many people's expectations, students who controlled the mouse were not more likely to dominate a small group's discussions; instead, in many cases they were less likely to do so as mouse users were often asked to simply implement the suggestions of other group members (Cole, 1994, 1995a). This research both contributed to the literature on how to promote equitable small group interactions around technology while also influencing how MMAP staff worked with teachers around supporting more equitable groupwork (Cole interview; Teacher TJ interview).

Similarly, MMAP's use of a curriculum that embedded mathematics within workplace design activities raised the issue of understanding what kinds of mathematics learning is afforded when instruction is organized in this way (Greeno interview; Greeno et al., 1999). This was a practical problem for teachers who needed to be able to identify for themselves, students, parents, and administrators what aspects of middle-school mathematics students were learning during MMAP units. However, it also provided a chance to see, as Bushéy (1997) did in her dissertation, the opportunities that the curriculum provided for students to engage not just in mathematical problem-solving but in the kinds of problem-finding and problem-shaping that Dewey's theory advocates. As Karen Cole summarized it, "the questions really emerged out of what was happening in the classroom and what we felt we were seeing on the videotape….we were looking for examples of something deep and when we found that we [would] explore how it happened, what were the antecedents and what were the results" (Cole interview). Or as Greeno and MMAP (1997) described it:

> "While we had general expectations about the character of activity and learning that we hoped would occur when we planned MMAP, most of the results of our research have an emergent quality. We observe and record interactive social behavior that occurs in classrooms, and take the challenge of making sense of it, trying out many possible conceptual frames, schemata, and theoretical ideas…. The aspects of theoretical issues on which we make progress, including the conceptual categories with which we formulate our interpretations and conclusions, grow out of an interaction of what is observed and the researchers' ideas, hunches, and areas of interest in school change, as well as the knowledge and practices that we bring from our various research disciplines."

Noticing researchable questions within practical issues did not happen automatically or right away, however (Bushéy, 1997; Goldman interview 5/12/06).[16] Such problematizing of research questions occurred when someone would see the research questions embedded in the practical issues that everyone had been engrossed in and would then draw the group's attention to them. As Shelley Goldman recounted, Jim Greeno had a particular knack for doing this on the project (interview 5/12/06). When someone made an interesting claim during a meeting, rather than agreeing or disagreeing with it, Jim would often say "that's an empirical question" or ask the group whether they knew if the claim was true (e.g., video of discussion around designing tasks to promote equity, 8/05/92), prompting additional discussion on how the research team might be able to study it with new or existing data. In addition, before each field-test of a unit, the

[16] In addition, before a researchable question could emerge, many months—and in some cases years—of curriculum development work needed to be completed until usable data could be collected from classrooms around a given unit (Cole interview; Hall interview).

research team would gather and brainstorm about ways that the data could be collected to inform each person's particular research questions (Goldman personal communication 1/4/06). Once data had been collected from field-tests, researchers would identify segments from the videotapes that were simultaneously relevant to their research interests and to teachers' concerns and then ask everyone to comment on what they saw in them, with some of these comments becoming the germ of what would later become a focused study (Cole interview; Teacher TO interview; MMAP II final report, 1998). In all these ways, research issues were problematized while progress was being made on the work of redesigning the MMAP curricula and supporting teachers in using it.

However, although the same practical issues were the origins of both MMAP's research studies and its curriculum revisions, in later stages of the work, problematizing around each became necessarily decoupled (Greeno interview; Greeno et al., 1999). The problems that needed to be solved to go from an analysis of what was happening in a field-test to a successful research presentation or paper were different from (and in some cases at cross-purposes with) the problems that needed to be solved to go from that same analysis to a new version of the instructional materials for teachers to use. Specifically, to create a successful research paper, one needs to fit one's work into this genre, which includes specifying research questions, making it clear how they arise from and contribute to the literature, and working out the analyses and other arguments in enough detail so other researchers can evaluate them (Cole interview; Greeno interview). In contrast, successful materials for teachers might aim instead for clarity, brevity, attractiveness, accessibility to a wide audience, and ease in finding key information—characteristics not typical of most research papers (Greeno interview; Knudsen interview 5/5/04; Milanese interview). Thus the problems involved in completing research and in completing curriculum revisions needed to be worked on separately, which meant that they also competed for time with each other.

Because of this decoupling of the processes of completing research and reform activities, it was very easy for the day-to-day tasks of supporting reform to lure MMAP staff away from getting research papers completed:

> "If your choice is, do you sit home and write a paper or do you go in and help this teacher get through the unit, your choice is you go help the teacher; you don't sit home and write papers. So the timelines for research just stretch out incredibly long, until you get to the point, you say, 'Well, I don't know if it's relevant to write about that anymore.' " (Knudsen interview 3/20/03; also Goldman personal communication 12/29/05)

Seeing other MMAP staff regularly making such choices led those who were externally accountable for frequently getting out academic papers to have the impression that MMAP was not focused as much on research as it was on reform:

> "The culture at IRL was not an academic culture, it was kind of like something else. I'm not sure how to describe it.… People were trying to do neat stuff, and they had their activist commitments… and then I think trying to communicate with others about it was, in some ways an afterthought." (Hall interview)

However, the explicit goal of the project and its directors was that it was a priority that research products get completed (Goldman personal communication 1/5/06). But, even during this first grant, NSF cared much more that the project made progress on curriculum development than academic papers (NSF program officer interview).

One way research completion was supported in this environment was by having university-affiliated researchers in the project whose main priority would be to make sure that academic papers got written:

> "I will tell you one thing that we had in our structure that I think worked well; we had people like Jim [Greeno], who are much more interested in research products than…he was in curricular products. And so he kept his eye

> pretty much on the target of writing research [products]….And it was good that we had someone like Jim on the project, who was focused in that direction, because he was able to fulfill our project's obligation to publish research." (Goldman/Knudsen interview 3/20/03)

In addition to senior researchers like Greeno, the push to publish was led by junior researchers who needed to get papers published and dissertations completed to advance their academic careers, and who could often carve out additional time for doing that because of their simultaneous affiliations with universities (Hall interview). In fact, this strategy of having some dedicated researchers attached to a project has been suggested elsewhere as being especially promising for promoting successful research on design-based research projects (Collins, Bielaczyc, & Joseph, 2001). A second strategy MMAP used to promote research was to arrange for short or long-term "sabbaticals" away from the day-to-day work of the project for staff to get research written up. For instance, Knudsen regularly took off a week or two each year to make progress on her dissertation. Finally, there is some indication that conference deadlines helped drive the completion of research, by providing shorter-term external accountabilities that competed with the day-to-day external accountability involved in working with teachers and helping them successfully use MMAP units (Berg interview).

Supporting Everyone's Authority During Research Activities

Just as MMAP worked to foster the authority of teachers as equal-status players in curriculum design as well as research, it also worked to foster the authority of graduate students and other newer researchers on the project during research activities (Berg interview; Milanese interview). As Goldman and Knudsen explained, this was part of a general value of giving people the agency to contribute from a position of strength:

> "Inside the project, it was about letting people kind of break out and play at their strengths. Very much, it became looking for what people's strengths were inside the tasks we had to do, and then letting them devote their time and attention to where they really seemed to take to it, or really have a strength…. We really provided an opportunity to take the reins and do what they thought was important." (Goldman/Knudsen interview 10/27/04)

This general idea of valuing everyone's contributions and building on diverse expertise was also consistent with IRL, which was run as a "flat organization" with a purposeful effort to reduce hierarchical power relations. One example of this that Knudsen highlighted was IRL's practice of having regular retreat-like intellectual gatherings that involved everyone at IRL from secretaries to project leads to look more deeply at the research contributions of one IRL project or a cross-cutting issue like assessment that several IRL projects were grappling with. Knudsen said this involvement of everyone in the intellectual life of the whole organization gave people the message that " 'I am an individual with the same kind of say in this as everyone else,' " something that she thought "helped in letting people sort of find their own strengths." (Goldman/Knudsen interview 10/27/04).[17] Within MMAP itself, Berg highlighted how authorship over research was encouraged in that other members of the group would "read your work, and you could create a meeting and have people come and reflect on things" (Berg interview; personal communication). Similarly, in video-analysis sessions, it was not just senior researchers, but also teachers and graduate students who were encouraged to comment on what they saw in the videotapes that was of interest to them (Cole interview; Teacher TO interview).

[17] Though some MMAP staff, especially those involved less in research, felt less connected to the larger institution of IRL (Milanese interview; Gallagher interview).

In general, as Greeno et al. (1999, pp. 307-308) summarized it, "we tried to create a community in which everyone could find the good sense of everyone else."

Supporting Accountability to Local and Broader Research Communities

If MMAP research was to be productive, it was important not just that the project found research problems to focus on, and that participants in the research had agency, authorship, and other forms of authority to pursue them, but also that they regularly accounted for how they were being responsive to the research community both locally within the project, and more broadly in the rest of the various fields from which MMAP researchers were drawn.

Locally, it was crucial for such accountability that the project as a whole viewed itself as pursuing both research and reform simultaneously (Goldman personal communication 12/29/05; Greeno et al., 1999; MMAP I grant proposal). During meetings, consideration of both research and reform issues was facilitated because no one would object if the discussion veered off toward one or the other consideration. Either or both kinds of topics were welcomed, with no one getting "impatient and say[ing] 'this isn't what we're here for. Do your theoretical work somewhere else' " (Greeno interview). In addition, accounting for how MMAP's ongoing work contributed to research was facilitated by particular individuals who felt institutionally and/or personally accountable for making sure such progress was made (Goldman/Knudsen interview 3/20/03). Such people would regularly ask the group to consider how the ongoing curriculum redesign work informed important research questions or could be used to support additional empirical studies. For example, Goldman recounted that Greeno regularly encouraged the group to account for the implications of what they were doing during design for fundamental research issues (Goldman interview 5/12/06). The group would be in the midst of grappling with some practical problem or celebrating a success at solving one, and Greeno would step back and say something like "'so what does that say about our definition of learning?'" or "'what have we learned about teaching practices?'" and "that would have the…outcome of pushing the conversation into some territory that we really had to think hard about."

However, in order to make a research contribution, such local accountability to doing research along with reform was necessary but not sufficient. The project needed to hold itself accountable to broader research communities in order to successfully do the work that was needed to turn its analyses into presentations and reports that would be recognized as good research (Cole interview; Greeno interview; Greeno et al., 1999). Analyses that had originally arisen in the context of joint design work had to be situated within some specific research literature (Cole interview), shown to have used sound methodological practices, worked out in enough detail to convince other researchers (Greeno interview), and refashioned into the genre of a typical research report in which a review of the literature leads to a specific research question that is then empirically examined (Cole interview). In addition, MMAP research tended to draw on several different disciplines. That made all these tasks additionally complex, because even knowing where to begin situating a study required extensive familiarity with research lines in several disciplines as well as practical knowledge of likely publication avenues. For newer researchers with less of this knowledge, the path to publication could be a very long, time-consuming, and in some cases overwhelming process (Cole interview; personal communication).

Connecting MMAP's work to the literature was supported during this phase by weekly meetings that were divided between talking about current project work for an hour and about theory for an hour, often based on something everyone had read together (Berg interview). Accountability to the broader research community was supported at IRL more generally by a

constant stream of visiting researchers who would share their own ideas while also providing feedback on the ongoing research that MMAP was doing (Berg interview; Hall interview). Furthermore IRL featured whole-institute activities like the weekly Interaction Analysis Lab in which researchers from IRL and elsewhere brought their videotapes for group analysis, modeling methods that MMAP then adapted in its own video analyses (Berg interview; Bushéy, 1997; Hall interview). Finally, senior MMAP researchers were part of international research networks that allowed them to learn about community standards and current research findings that they then brought back to the project to inform its ongoing research (Hall interview).

Resources Supporting Research Progress

Numerous resources, some of which have already been discussed, supported research progress during this phase. First, NSF's first grant included a subcontract to Stanford that was to explicitly support research on teacher practices (MMAP orientation video, Goldman personal communication 5/27/06). Although funds from both the Stanford and IRL parts of the grant were used to support both research and reform given that the same people often did both, having some funds be devoted to research did help keep accountability to research progress much more salient than it might been otherwise (Greeno interview). Second, time devoted to research during meetings was important as was the informal environment and open office plan of IRL that facilitated the informal sharing of research ideas (Cole interview; Moschkovich interview). Third, teachers were important resources for the research that was conducted on the project, providing inside perspectives of classroom events, identifying research issues that were more likely to have an impact on practice, and being the subjects of research on teaching and teachers' practices, although they often did not appreciate the extent of the contributions they made (Greeno et al., 1999).

Finally, the extensive corpus of data that was collected from field-tests of MMAP units was a major resource that MMAP researchers could mine (and are still mining) for many different research purposes (Berg interview; MMAP II final report, 1998). As longtime staff member Rick Berg explained:

> "There was a lot of data being collected and it could be anybody's data…you have a team of maybe ten people all collecting data. So you can get a pretty good coverage of a couple of classrooms anyway for the course of maybe a six week period and you can see a lot of what went on." (Berg interview)

Typically, MMAP staff would make some baseline observations in the classroom and then would document as much as possible that occurred while a particular unit was being piloted, which usually included videotaping almost every class session, taking field notes, interviewing the teacher, and collecting student work, teaching materials, and other relevant artifacts (Berg interview; Cole interview; Moschkovich interview). During this phase, such data was collected approximately two to three times each year (Berg interview). The corpus of data that was collected, especially the videotapes (see Goldman & McDermott, in press), then informed both the curriculum's redesign and empirical studies that provided researchers the opportunity to study types of mathematics learning that were rare in other classrooms at the time (e.g., Bushéy, 1997; Greeno et al., 1999; Hall interview; Moschkovich interview).

MMAP and Its Participants Negotiating with Their External Contexts

So far, this section has treated the work of the project and the collaboration between teachers and researchers in a manner that has made only brief mention of the originating contexts—that is, schools and academia—to which many members of each group were responsible. To some

extent, this speaks to the degree to which the project and IRL helped provide a "third space" (Wenger, 1998) that was partially sheltered from MMAP participants' originating contexts, allowing them to experiment with ideas and practices in ways that they might not have been able to do otherwise. However, when MMAP teachers went back to their schools and MMAP researchers to their academic colleagues and funders, these contexts and their various constraints and affordances could not be ignored. How did MMAP participants and MMAP as a project negotiate their participation in MMAP with these home contexts in ways that shaped MMAP's ongoing work? That is the focus of the rest of this section.

Negotiating With Teachers' School Contexts

Teachers using the MMAP units needed to find ways to make sure that what they were doing was sufficiently supported by students, their parents, other teachers, and school administrators so that they could continue using them. At this time in the project, which was before later controversies over mathematics instruction (now known as the "math wars") as well as current high-stakes accountability pressures, there was less monitoring of teachers' classroom instruction. Compared to today's climate, teachers had a lot more freedom to experiment with methods of instruction in mathematics (e.g., Teacher FJ interview; Teacher LE interview 3/2/03; also see Wilson, 2003). In addition, the fact that MMAP used computers often made its units popular with school administrators:

> "At that time technology was really coming into its own as something that administrators wanted to be able to crow about. I think the fact that we used technology in such an integrated way gave administrators something they could show off. They could say, 'Look at this classroom. They are designing houses on computers.' " (Cole interview)

Administrators also liked MMAP for other reasons as in the example of one principal who "liked the way the kids were involved" and would bring visitors in to see it in action (Teacher OO interview 5/3/04).

At the same time, teachers and staff noted that most MMAP teachers were careful to combine MMAP units with traditional forms of instruction that helped reassure students, parents, and administrators that they were still teaching mathematics in ways that were recognizable to these important stakeholders. For example, some of what teachers did included:

- Having a separate homework track running in parallel to MMAP units that included drill exercises to reassure parents while making sure students would be familiar with standardized test items, a strategy that was in fact later recommended by MMAP's directors (Cole interview; Goldman personal communication 7/25/05; Teacher OO interview 5/3/04);
- Generously supplementing the units with relevant skill-based exercises, many of which were eventually incorporated into the units that were published (e.g., Cole interview; Teacher LE interview 3/2/03; Teacher TO interview);
- Guiding students through the curriculum in less open-ended ways to make sure students would get correct answers and that specific math topics would be covered (e.g., Teacher LE interview 3/2/03; Goldman/Knudsen interview 10/27/04);
- Starting the year with a single MMAP unit that helped students appreciate the real-world importance of mathematics, thus enhancing their engagement with the conventional curricula that teacher used for the rest of the year (e.g., Teacher FJ interview; Teacher TO interview; cf. Schwartz & Bransford, 1998).

Teachers were able to make these adaptations in part because at this time, as mentioned earlier, MMAP units were being presented to them as supplemental curricula that they could freely modify in coordination with any other curricula or instructional methods that they might like. As we will see in the next phase of MMAP's work, this is very different from being asked to teach a full comprehensive curriculum that teachers must adopt, as-is, for a whole year's worth of instruction.

MMAP staff also supported teachers in engaging positively with their school contexts (Cole interview; Hall interview). Activities would be added to MMAP units when teachers suggested they might help them deal with the concerns of students, parents, or principals (Cole interview). In addition, "Shelley really did a lot to get the administration on board too before we went into a school. She would go and do a dog-and-pony show at the PTA meeting. She did a lot…to try to lay that groundwork for a positive relationship" (ibid.). MMAP would also continue to foster goodwill with teachers' schools by doing favors like allowing one teacher's math department to use IRL for its retreat in exchange for asking those teachers to give the project some quick feedback on something they were developing (Teacher LE interview 3/2/03).

Negotiating with University Contexts

MMAP staff who were also university researchers or had ambitions to become them also had to find ways to coordinate their MMAP work with university contexts. Different people found it more or less easy to manage this divide. Judit Moschkovich, who was hired to work on assessment issues on MMAP as a post-doc after she got her PhD at Berkeley and who now is an associate professor at UC Santa Cruz, greatly credited her time with MMAP and at IRL for helping to launch her academic career. Through the help of informal conversations with sociolinguists and discourse analysts at IRL she was able to learn what she needed to begin a new line of research on bilingual students' mathematics learning (e.g., see Moschkovich, 1996, 2000), noting that at IRL "people are interdisciplinary… so… were tolerant of people who knew little about an area but that wanted to connect" (Moschkovich interview). In addition, she credited mentoring by MMAP's PI's for getting her first two grants (ibid.). On the other hand, Rogers Hall found coordinating what he was doing on MMAP and his academic career more difficult. A few years into his participation in MMAP, he took a faculty position at Berkeley while continuing to work at MMAP as an IRL research scientist. For yearly reviews he would package curriculum units he helped to write as "technical reports," but learned quickly that "the only thing that counted in that environment was conference papers or journal articles or book chapters" (Hall interview). The curriculum units "were always seen as 'oh well good, the guy's active in curriculum development, that's neat.'…But it never counted for anything" (ibid.). As a result, he ended up structuring his work on the project to focus less on curriculum design with the group as a whole and more on getting grants and writing papers around his own unique slant on the work (e.g., Hall, 1995, 1998; Hall & Stevens, 1995), which allowed him to do what he needed to do to be successful at Berkeley.

In comparing the effects of the MMAP project on the academic careers of Moschkovich versus Hall, it might be important to note that Moschkovich followed up her time with MMAP by working on a research project funded by her own research grant at another R&D institute, TERC, before becoming a professor while Hall went directly into academia. Others who worked at MMAP and went to graduate school at Stanford ended up deciding not to go into academic careers, so did not need to negotiate university expectations after they were done. Senior researchers McDermott and Greeno had different types and levels of involvement in research in

the project (Greeno interview; McDermott interview), but as long-tenured professors, the degree to which they got publications out about MMAP in a timely fashion was presumably not as crucial for their career success.

Negotiating With MMAP's Funding Context

Finally, the project and especially its principal investigators and director needed to regularly negotiate MMAP's relationship with its primary funder, the National Science Foundation. In particular, as soon as the main project was funded, MMAP was told that it would be included with NSF's portfolio of middle-school mathematics curriculum projects that had been working for at least a year to develop curricula that embodied the NCTM standards (Greeno interview; Goldman/Knudsen interviews 3/20/03, 4/27/04; Hall interview). This ended up crucially shaping the project's goals while helping it learn things it could not have learned any other way.

Because of key differences in goals, values, assumptions, and backgrounds, however, neither MMAP nor many of the other NSF-funded curriculum projects were fully comfortable with having MMAP in this group (Goldman/Knudsen interviews 3/20/03, 10/27/04). As Knudsen described it:

> "We were surprised to be included in the conferences and community of NSF curriculum developers. And I would guess they were also surprised to see us. We weren't in the business of making a comprehensive curriculum for broad adoption closely aligned with the NCTM standards. And that was the charge of each of the other curriculum development groups…. We did work hard to align with the NCTM standards, but we weren't in that community really either." (Knudsen personal communication 11/13/05)

However, NSF insisted on having MMAP participate in the group in order to provide a contrasting "model of what people could do if they took the risks that you needed to take…with technology and with applied approaches to math" (Goldman interview 3/20/03; see also Greeno interview; Goldman interview 4/27/04; Knudsen interview 5/2/04; Knudsen personal communication 10/13/04).[18] "NSF took a courageous risk by including our project and supporting it as they did. The groups had different priorities, which was an important strength of the portfolio" (Greeno personal communication). Participation in the mathematics curriculum project group primarily involved regular attendance (and in one case taking a lead on organizing) annual conferences with the other projects in which issues of curriculum—especially its development, testing, evaluation, and implementation—were extensively discussed (NSF program officer interview; NSF award abstracts #9255857, #9354099, and #9452864). MMAP was encouraged by its program officers to share its alternative views, which it did, sometimes resulting in "disagreements or healthy debate" about what kinds of mathematics curricula were best (Knudsen personal communication 10/13/05; see also Goldman/Knudsen interview 3/20/03). "I am sure we seemed naïve in our assumptions about teachers and curriculum. And yet, those views were central and important to our project" (ibid.).

MMAP's directors were very clear that being involved with the other NSF projects had a significant impact on them (Goldman/Knudsen interview 3/20/03; Goldman/Knudsen interview

[18] One program officer (not the one I interviewed) apparently defended MMAP's inclusion in the group to such an extent that MMAP's director was worried that the other projects would think that the Foundation was playing favorites (Goldman interview 4/27/04). If so, this wasn't reflected in the funding that the projects received. Using available NSF award abstracts to compare MMAP's funding with that of three of the other projects, I found that all four projects were funded at roughly comparable levels. Specifically, both Math in Context and Connected Math received slightly more funding (both $980K/year) than MMAP ($946K/year) while Seeing and Thinking Mathematically received somewhat less funding ($840K/year). I was not able to locate the award abstract for the fifth project.

10/27/04; Knudsen personal communication 11/13/04). By being involved in this group, MMAP learned from the similar experiences of the other projects, refined their ability to communicate what they were uniquely doing, and were pushed toward more mainstream and potentially more accessible designs. As Knudsen recently described it:

> "Being in that community had an important influence on our project. We were influenced to seek a broader audience.… We learned more about how others worked with standards, [and] with the constraints of the local policy world of schools. We also got a chance to develop our own views about our work as we discussed it with others in those meetings. I think it's safe to say that while no one, least of all us, would ever say we took a mainstream approach, we were pulled more toward the mainstream by inclusion in this group." (Knudsen personal communication 11/13/05)

Other MMAP participants who spent less time at those conferences echoed some of these sentiments, especially the idea that those meetings tended to "help us get better at talking about [what we were doing], and explaining what it was and how it was different" (Teacher TO interview). Including MMAP with the other curriculum projects also began shifting the emphasis of the project to focus less on research around the feasibility and impact of using an applications-based approach (as written in the original full grant proposal), and more on getting more MMAP-style curricula developed and into classrooms (NSF program officer interview). And as we will see, MMAP became increasingly tracked with these other curriculum projects, which increasingly moved the project in just this direction.

Summary of This Section's Analysis

In this section we have seen how the MMAP project created a collaborative community in which teachers, researchers, and curriculum developers designed innovative curriculum units together, using their joint designs as the basis of products for each of their originating communities. This was done by problematizing through jointly valued design tasks; by nurturing the authority of all participants, especially teachers; by embodying an internal form of accountability that did not sacrifice teachers' authority; and by providing resources to support the collaboration. We also saw how research was advanced on the project by supporting everyone's authority to contribute to it, by encouraging the investigation of issues that were simultaneously problematic for research and reform, by addressing the necessary decoupling of research products from curricular ones, by fostering accountability to local and broader research communities, and by making use of resources to support research activities. Finally, there were numerous ways MMAP participants negotiated with their originating contexts to address the fact that what they were doing on MMAP was not standard practice in any of these places.

VI. Turning Replacement Units into a Comprehensive Curriculum While Keeping Research Going (1995-1998)

Once funding for the first stage of MMAP's work was running out, it was made clear to the project by NSF that the next step that they would consider funding would be if MMAP proposed to use what it had done so far to build a comprehensive curriculum embodying the NCTM standards (Goldman/Knudsen interviews 3/20/03, 4/27/04; Greeno interview). This was basically what all the other NSF curriculum projects were already responsible for doing. NSF's desire was that MMAP and all the other comprehensive curricula be widely distributed across the country (NSF program officer interview) so they required each project to submit a plan for getting published (Goldman personal communication 12/22/02). More broadly, this desire by NSF for commercially viable curricula can be thought of as signaling the beginning of what came to be a big push during this time from both private and public funders for design-based educational researchers to scale up their interventions.

In response, MMAP proposed to create the comprehensive curriculum that NSF asked for, and was generously funded to do exactly that ($2,794,224 from 3/2/95 to 9/30/98; NSF award abstract #9452771). MMAP and its program officers were pleased with how well the curriculum was engaging students in mathematics in its collaborating teachers' classrooms, so viewed this as an opportunity to get the materials to more teachers and students (Goldman personal communication 12/29/05; Knudsen personal communication 11/14/05; NSF program officer interview). Rather than being limited to those teachers who had the time and inclination to creatively put together their curriculum by coordinating various replacement units and activities, the project "wanted MMAP II to work for the much broader group of teachers who were more likely to pick up and use a curriculum as a whole" (Knudsen personal communication 12/29/05). All of the work for which the project was responsible to NSF with this grant involved building instructional materials and engaging with teachers around them. Unlike the first grant, no funds were explicitly provided for research on the effectiveness of the materials or about any other topics, something that Greeno, for example, was disturbed about, but understood as being necessary for the project to receive continued funding from NSF (Greeno interview). Thus, at this point, MMAP was firmly placed in the category of the other NSF curriculum projects, and it was therefore treated even more like them than it had been before.

The change in focus to building a comprehensive curriculum had a dramatic impact on the nature of MMAP's work. In the next several subsections I consider how the principles for fostering productive collaboration that were discussed earlier were embodied differently during this phase while working on the comprehensive curriculum and its associated products. Specifically, problematizing around curriculum design was significantly narrowed while new problems around assessment opened up to engage the group as a whole. At the same time, authority became increasingly more asymmetric during curriculum design tasks while accountability to deadlines was increasingly fostered, given limits in the resource of time. These changes were in part a result of the new focus and in part a natural outgrowth of what the project and its participants had learned from working together during the first phase. I then consider how the project managed to get so much research done during this phase given that the resources to support it had been reduced. Finally, I conclude the section by considering a crucial contextual change that the project had to deal with, namely the "math wars" that were beginning to heat up both locally and nationally during this time (Schoenfeld, 2004; Wilson, 2003).

Changes in the Nature of Problematizing During This Phase

The nature of problematizing changed when MMAP began being responsible for producing a comprehensive curriculum rather than creating, testing, and refining innovative replacement units. The problem space for curriculum design narrowed while problems around assessing student learning became increasingly opened up for investigation on the project.

With respect to curriculum design, new units needed to be designed primarily with an eye to how they coordinated with MMAP's already existing units in order to ensure that: (a) across the 3 years each NCTM standard was covered, and (b) the sequence of units would be coherent to students, teachers, and especially school decision-makers who might consider purchasing MMAP's curriculum (Goldman/Knudsen interview 10/27/04; Moschkovich interview). This greatly narrowed the problem space for design. "Covering all of the math in the standards meant we were not free to develop units for the topics that were easiest to think of" or develop. (Goldman personal communication 12/28/05). Problematizing was further narrowed by the fact that the project sought to have at least two units be anchored by each technology application to increase usability, efficiency, effectiveness, and the coherence of the curriculum as a whole (Goldman personal communication 12/22/05). In addition, the project had learned a lot about what was needed to make applications-based units work from the first grant, so did not have to figure nearly as much out during the design process, reducing problematizing further (ibid.). Finally, to cover topics that could not be easily covered with the applications-based units and to give teachers a break from teaching these demanding, technology-intensive units,[19] the project also decided to design two new kinds of mini-units:

- Extensions, which are two to three week units that "start with an applied context (some of which drew directly from the larger, applied, technology units), move students through that, then take them to more standard forms and expressions…so the kids would see the kind of math they might encounter on a standardized test" (ibid.; also http://mmap.wested.org/pathways/MMAPMaterials.html), and
- Investigations, which are one or two week units that introduce standard mathematical concepts and practices "either from a pure mathematics point of view or within fanciful contexts" (http://mmap.wested.org/pathways/MMAPMaterials.html).

Thus, much of the overall content and structure of the comprehensive curriculum and its component units was set, reducing the problematizing of curriculum design tasks significantly.

As Knudsen summarized it, then, the new curriculum design problem was "to create a structure that would contain and take advantage of the original MMAP units and their approach, while creating a curriculum recognizable and usable by others as a comprehensive curriculum, and 'covering' all the NCTM standards" (personal communication 12/29/05). This overall curriculum design problem was one that according to post-doc Moschkovich engaged "a lot of people doing a lot of serious thinking" (interview). However, the day-to-day work of designing particular units was more straightforward as it followed on much more closely with what had

[19] The project had originally envisioned teachers and students doing applications-based units all year long, but learned from their collaborating teachers that this was unrealistic as teachers either could not get access to computers for this long, found the applications-based units too tiring to sustain, or wished to devote time to preparing their students for tests (Goldman personal communications 12/22/05, 12/28/05; Knudsen personal communication 12/29/05).

been done before. In addition, there was less time to field-test the new units, which meant that later units could not undergo the same number of iterations as earlier ones had. Collectively, this might have reduced the extent of people's engagement in designing the new units.[20] Unfortunately, we know little about the exact nature of the collaborations that did occur to create the newer units as interviewees tended to talk much more about the earlier units.

However, at the same time, there was increased problematizing around issues of assessment during this phase, which ended up engaging almost everyone in the project at some point (Goldman personal communication 12/29/05). IRL researchers had a history of working on alternative assessment, including a project Hall, Knudsen, and Greeno had worked on prior to MMAP (Hall, Knudsen & Greeno, 1995/1996; Knudsen interview 5/5/04). At the same time, MMAP's collaborating teachers were becoming increasingly concerned that they were not getting sufficient information about students' progress during the units in order to support students' learning, be confident that a unit had successfully taught students what they needed to know, and justify what they were doing to parents, administrators, and other key stakeholders (Goldman interview 10/27/04; Goldman personal communication 12/29/05; McDermott interview; Teacher OO interview 5/3/04). For these and other reasons, phase two of MMAP included progress on developing new forms of assessment as a second major goal for its work, something that was easy to get NSF support for given that assessment was a hot topic at the time (Goldman personal communication 12/29/05). And as more and more teachers were facing high stakes accountability pressures, the problem ended up engaging more and more of them as the grant went on (ibid.). As a result, the project experimented with many different kinds of alternative assessment methodologies, which it then embodied in a series of research papers, assessment suggestions in the comprehensive curriculum, and a professional development CD that illustrated suggested assessment methods (Cole interview; Teacher CA interview; FJ interview; LE interview 3/2/03; OO interview 5/3/04; TO interview; Goldman/Knudsen interview 10/27/04; Greeno interview; Hall interview; McDermott interview; Moschkovich interview; also see Lichtenstein et al.,1998). Later, Cole appropriated the assessment framework she had helped develop during this process to the Challenge 2000 project. This collaboration between IRL, the Stanford Research Institute (SRI), and Joint Venture Silicon Valley supported teachers in designing and assessing multimedia curriculum across subject areas, and resulted in an award-winning CD about assessment (Cole interview; Knudsen personal communication 12/29/04).

Finally, a third issue became increasingly problematized on the project during this phase, which was how to write MMAP materials to make them usable by a wide range of teachers who had not been involved in designing them (Knudsen interview 5/5/05). To make progress on this, the project shared some almost completed units with a whole department of non-MMAP math teachers in one school in order to observe how they reacted to them. This helped the project rewrite the curriculum materials in ways that would be more attractive to and accessible to new teachers, which included using simpler, more recognizable headings, attending more carefully to

[20] It's hard to know whether or not that actually occurred. Regarding breadth of engagement, I do know that fewer people were involved in designing each unit, but as will be seen below, that was as much a result of how much needed to get done and what was possible to do than anything else. Regarding depth of engagement, I noticed that those who were involved in designing the later units as well as the earlier ones tended to talk much more about the earlier ones in their interviews, but again that may be an artifact of other things, primarily the fact that people's engagement with the earlier units set up patterns for their engagement with the project as a whole.

graphical conventions, and limiting discussion of educational philosophy (Goldman & Knudsen, 2000; Knudsen personal communication 12/22/05; Knudsen interview 5/5/05).

Movement Toward More Asymmetric and Distributed Models of Authority

In part because of the constraints of building a comprehensive curriculum, MMAP increased the speed of an already existing trend in the project from relatively symmetrical models of authority, in which everyone seemingly did everything together at the same time, to more asymmetrical and distributed models, in which the full plate of work was divided up among smaller teams, with individuals specializing in particular functions and with their contributions sometimes being made in sequence rather than simultaneously. Examples of the increasingly asymmetric and distributed nature of the collaboration included: (a) the division of the project into many more, smaller design teams (Teacher EH interview; Goldman/Knudsen interview 10/27/04; Knudsen personal communication 11/14/05); (b) the previously-documented changes in how the expertise of math-using professionals was used from having them being full collaborative partners to observing their practice or having them work as consultants on limited tasks; (c) the hiring of selected teachers with particular skills to be members of MMAP's staff (Teacher EH interview; OO interview 5/3/04; Hall interview; TJ interview; TO interview), and (d) the hiring of professional writers to implement curricular decisions made by design teams (Goldman/Knudsen interview 10/27/04; Knudsen interview 5/2/04).

However, although "teams worked simultaneously on different products and processes, the whole group came together at least once a week and on teacher workdays as well," which helped to keep different subprojects coordinated and facilitated the sharing of ideas among them (Goldman personal communication 12/29/05). In the next paragraphs I explore several factors that may have led to this shift toward more asymmetric and distributed authority: learning how to better coordinate the world of research and practice, making use of expertise that had developed in the project over time, and organizing the work to make sure that everything would get done in time.

First, the shift toward a more asymmetric distribution of authority resulted from the project's realization that fully symmetric collaborations between researchers and teachers did not sufficiently take into account the different contexts in which each operated. For example one staff member talked about what happened early in the project when she tried to continue symmetrically co-designing a unit with two teachers once the school year had begun:

> "I would send them emails about what I had written and stuff like that and they would never answer my emails because they were just too busy. Once school starts, teachers have just zero time to think about writing curriculum…. It was much harder during the school year to keep them involved, except at the meetings. It was really frustrating to me at first and then I said, 'ok, I have to change how I relate to them because I'm asking them for something that they really can't give.' " (Cole interview)

The project then focused on getting feedback and other contributions from teachers when the time had been set aside to do that at monthly workdays. Similarly, despite common assumptions among many projects that teachers or graduate students should be curriculum writers, MMAP found that most teachers and graduate students were inexperienced in this kind of writing, not that interested in it, had trouble fitting it into their schedules, and often needed a lot of support to get writing tasks done (Goldman interview 10/27/04). The project therefore shifted from having almost everyone write curriculum to having a select few who had shown interest and expertise in it do some of this work, while hiring professional writers to do the rest (Knudsen interview 5/2/03). This allowed teachers, for example, to focus on what MMAP most crucially needed

them for: "giving us the initial ideas, setting parameters for things, trying them out in their classrooms for us, and having lots…to suggest in the way of changes or features they want or specific kinds of math they want to see" (Goldman/Knudsen interview 10/27/04).

Second, another important reason for the shift to more asymmetric uses of authority occurred because of expertise that the project and its participants had developed from working collaboratively in larger group during the first phase. The project as a whole had developed enough shared understanding of goals, definitions, and processes to make it feasible for it to be organized into parallel design teams (Goldman personal communication 12/29/05). Also, over time particular people developed expertise in particular areas and became recognized as authorities about them, so it made sense to distribute the workload so people worked on those tasks for which they were most suited (Berg interview). In addition, from a management standpoint, it also made sense for longtime participants to be given "some authority over [the project] themselves" at this point in the project, which was facilitated by putting them in charge of workgroups (Goldman personal communication 12/29/05; also Teacher TO interview).

Finally, this shift to a coordinated set of distributed work teams made it easier to get the work of the comprehensive curriculum phase done in time as it allowed simultaneous but coordinated work on many subprojects at once. In discussing accountability, we will now in fact turn to additional strategies that the project used to get the many tasks of this phase done on time.

Fostering Accountability to Deadlines

The key resource of time was in shorter supply once MMAP became responsible for putting together a full comprehensive curriculum (Berg interview; Cole interview; Goldman/Knudsen interviews 3/20/03, 10/27/04).[21] More units needed to be created than had been created in the first phase of MMAP's work, and the project was also externally accountable for getting more done around these units: coordinating them with each other, developing innovative assessments, providing a whole host of materials to help new teachers be able to teach them while continuing to support its collaborating teachers. By necessity, these later units could not go through the same amount of field-testing and redesign as the original ones had (Knudsen personal communication 12/29/05). One might expect that achieving this rate of production would be challenging to any project, but it might have been especially so for MMAP because it was situated at IRL where one staff member noted that "the challenge for people was to stay productive in that environment, because it was so social and so free" (Cole interview). Interestingly, despite that, the project did a good job of meeting deadlines throughout the life of the project. For example, it even completed its comprehensive curriculum package ahead of schedule (Goldman/Knudsen interview 10/27/04). How did it manage that? Below I share some strategies the project used throughout its lifetime to foster accountability to deadlines, but which definitely bore fruit during this critical phase.

First, a few key, experienced staff members—most notably Karen Cole who came from a private sector educational technology curriculum development background—provided models for everyone else of "pump[ing] out the deliverables in a way that was good" (Goldman/Knudsen

[21] Upon further reflection, Goldman disagrees with this statement. She believes the project's time was as tight in the first grant as it was during this one. However, as will be discussed later, several other MMAP participants appeared to have felt the time pressures much more in this grant than the earlier one. Perhaps this was because some of them were now more responsible for making sure work got done as leaders of work groups? In any case, it is clear that the issue of "fostering accountability" to deadlines was one the project needed to address throughout its history. It was just put into particularly high relief during this phase and for that reason is being discussed here.

interview 3/20/03; Goldman personal communication 12/22/05). This gave newcomers like writer Milanese the message that:

> "MMAP was actually producing results… they produced somewhere around six or eight guides. I mean that's pretty significant with so few people involved, and there was something to show for the effort." (Milanese interview)

Because of this, she viewed MMAP as being a lot more productive than the rest of IRL (ibid.).

Second, since there was a norm on the project that people could choose to work on anything else that interested them about the project once their assigned tasks had been completed, this meant that additional work got done on the side that ended up helping the project meet its deadlines much faster than it might have had staff been only permitted to work on assigned tasks (Teacher EH interview; Goldman/Knudsen interview 10/27/04; Lichtenstein et al., 1998). For example, earlier versions of a coding program and possible units based on it had been rejected by the group as not being worth pursuing on several occasions, but whenever Rick Berg had extra time he tinkered with various ways of redesigning the program. Almost a year later, to everyone's surprise, he came out with a new version that really worked, and this became the technology that anchored three units eventually included in MMAP's comprehensive curriculum (e.g., Goldman/Knudsen 10/27/04, but many others talked about this). Thus, by having selected people to participate in the project who were really interested in certain aspects of it, this meant that when they were given the agency to use extra time to pursue their own interests, they often did so in ways eventually benefiting the productivity of the project as whole.

Third, MMAP strategically made use of already existing materials that were developed during its replacement unit phase to transform them into parts of the comprehensive curriculum. For example, at least two of the new units (Dream Home and Plan an Event) emerged in part out of introductory activities that teachers or researchers had designed to prepare students for the Antarctica unit (Berg interview; Goldman/Knudsen interviews 3/20/03, 10/27/04; Teacher TJ interview; Teacher CA interview). Similarly, two new units were created when the original, larger Codes unit was divided into three different units, each focused on different kinds of codes and coding applications.

Fourth, when someone was being seriously unproductive, there were consequences. People were talked to privately, models for how teachers could work as consultants were revised, and difficult decisions were made to let a few staff members go (Cole interview; Goldman/Knudsen interview 10/27/04; Hall interview).

Last but not least, the project could be more efficient because of all the work that was done during the first phase to develop shared solutions to difficult problems. The project had a clear plan for what it was trying to accomplish, and in many cases it could simply apply the lessons learned from designing and testing earlier units to the new ones, thus increasing its efficiency (Goldman personal communication 12/22/05; Knudsen personal communication 11/15/05). For all of these reasons it is not surprising, then, that MMAP made its comprehensive curriculum deadline from NSF.

Other Accountabilities Beginning to Diverge

However, while all this was going on, there gradually began to be some divergence between what the project as a whole was required to hold itself accountable to and what individual participants in the project wished to hold themselves accountable to. This occurred both with teachers and with some key staff members. In particular, MMAP's collaborating teachers had no need or particular interest in having a comprehensive curriculum from MMAP as they continued

to use MMAP as replacement units (e.g., Lichtenstein et al. 1998; Knudsen personal communication 12/29/05; Moschkovich interview; Teacher TO interview). Specifically, most core MMAP teachers never used the whole host of Extensions and Investigations mini-units that the project designed to make sure the NCTM standards were covered (Knudsen personal communication 12/29/05; Lichtenstein et al., 1998). They had joined the project to teach and help develop the Applications Units and had already found ways to coordinate them with other materials to create their own hybrid curricula for their students (Goldman personal communications 12/28/05, 12/29/05; Knudsen personal communication 12/29/05; Moschkovich interview; Teacher TO interview). So although the project needed to develop the Extensions and Investigations, its collaborating teachers had no compelling interest in them. In fact the project hired a teacher to help write many of these, but when she returned to teaching she did not use what she had written, instead teaching with the applications-based units (Teacher EH interview; Knudsen personal communication 12/29/05).

At the same time, MMAP staff became of increasingly mixed minds about the comprehensive curriculum strategy (Knudsen personal communications 11/14/05, 12/29/05). On the one hand, MMAP directors and other staff saw the potential of a comprehensive curriculum for eventually allowing more students, especially those in poorer districts, to be able to use MMAP materials:

> "Poor urban districts lack funds to purchase 'supplemental' materials such as ours and are often under intense pressure to have students meet basic standards. By embedding our units in a comprehensive program, we hope to change the image of a core curriculum to one that emphasized problem solving and conceptual learning as well as building fluency with procedures." (Goldman & Knudsen, 2000, pp. 4-5)

On the other hand, some staff became increasingly uncomfortable with some of the compromises on design principles that were necessary to put a sellable package together, the reduction in teacher agency that can occur when comprehensive curricula are imposed on teachers by districts, and the implicit expectation in comprehensive curricula that teachers should adopt the whole thing at once rather than gradually incorporating it into their practice (e.g., Goldman/Knudsen interview 3/20/03; Knudsen personal communication 11/14/05; Moschkovich interview). MMAP's directors and staff continued to work on the task of creating a comprehensive curriculum for NSF, but increasingly began to feel that this was not the best approach given their other commitments. In addition, as we will see in the next section, this also caused difficulties for MMAP in representing itself and its curriculum to others as it worked to scale itself up.

Resources Supporting Research During the Comprehensive Curriculum Phase

During this phase, much of my previous analysis of how research was supported on the project during the replacement unit phase continued to hold true. Problematizing of research was supported through research studies that emerged out of practical problems that occurred during ongoing work with MMAP teachers, with a major focus during this period on assessment as mentioned above. In addition, the authority and accountability of both junior and senior MMAP researchers continued to be supported in similar ways. And researchers affiliated with universities continued to push getting research products done, being involved in 69% of MMAP publications during this phase as compared to 66% before and 75% after.

However, the basic resources of time and money that the project had to devote to research were in much shorter supply during the new grant. NSF was no longer providing funding earmarked for research, was not holding the project externally accountable for research products,

and there was a lot for the project to get accomplished that did not involve producing data, analyzing it, or writing it up. As Goldman explained:

> "If you're going to live in that curriculum mode, where the job is getting the curriculum out to people, then you have to be concerned with a whole different set of interests—you know how do you present yourself to the public?…How do you keep your message, but still be accessible to people?… And then on top of it, you're trying to make sure you get decent research out of it. And it's not that simple of an enterprise…. Because someone like me and Jennifer, we're like trying to…keep the vision, run the whole thing, organize everyone… between organizing and then trying to work with everyone, you're just getting the product pumped out." (Goldman/Knudsen interview 3/20/03)

Similarly, Rick Berg reported that toward the end of the project it was hard to find the time to do research on top of his curriculum development responsibilities. As he explained, "when NSF is expecting [several] units…the whole time you're just working on getting [them] done on time" (Berg interview; personal communication).

Given these challenges, it is therefore notable that more publications (16, defined as books, journal articles, book chapters, conference proceedings papers, and dissertations) were completed during this phase of MMAP's work (16 appearing in years 1996, 1997 or 1998) than during the three years beforehand (9 from '93 to '95), the three years afterwards (10 in '99-'01), or any other period of MMAP's history (see Project CV). In addition, it was during this phase that a few longtime teachers began engaging in their own action research investigations, with a couple contributing to presentations and publications for primarily practitioner audiences (Teacher EH interview; FJ interview; OO interview 5/3/04; Project CV). Some of the research that appeared during this phase was undoubtedly work that had begun during the first phase of the project, but still to have this much be completed during this phase is notable given the earlier analysis that suggested that the later stages of research compete more directly for time with reform work than earlier stages do.

However, additional resources might have helped promote increased research productivity during this phase. First, all the work that was done during the first phase to collect a large corpus of data and to work through key theoretical and empirical issues could begin to bear fruit, allowing project members to write papers more efficiently from existing data and analyses (Berg interview; Greeno personal communication). In addition, around this time more of the graduate students and post-docs on the project got to a place where they needed to get their dissertations and papers done in order to get to the next steps in their careers. Finally, additional funding sources were found to help support research activities. Goldman and her colleagues supplemented the NSF grant with $825-$850K of relatively flexible funding from industry and foundations, primarily secured through the professional and personal connections of Goldman and then IRL Director Peter Henschel (Goldman CV; Goldman/McDermott personal communications 12/29/05, 1/4/06; McDermott interview).[22] The two largest grants, which were for creating a classroom assessment tool were used to support not only the development of this tool, but also research on assessment that fed into and drew from it, resulting in several

[22] This increased level of funding contrasts with about $100-$200K of industry and foundation grants during 1992-1995 when the need was less great to have additional funding to do research (same three sources used). At least one other NSF project, Math-in-Context, pursued a similar strategy, getting an NSF ROLE grant around the same time as it was being funded for its comprehensive curriculum. MMAP did not try to get a ROLE grant as it was concerned about asking NSF for too much money and it welcomed having some money that could be used without the same level of bureaucratic demands that were characteristic of NSF. In addition, getting funding from industry and foundations would particularly please IRL director Henschel who was very interested in having IRL build connections with industry and foundations (Goldman personal communication 12/29/05).

publications (e.g., Cole, 1999; Cole, Coffey, & Goldman, 1999; Moschkovich, 1998). Similarly, Hall at Berkeley got his own NSF funding to support his own workplace-focused research that was built from MMAP's curriculum materials (Hall interview; NSF award abstract #9553648), which led to a similar number of publications (e.g., Hall, 1995, 1998; Hall & Stevens, 1995).

Negotiating with a Shifting Policy Context: The Math Wars

There was a final factor that made MMAP's work much more difficult during this phase. Soon after MMAP was committed through its NSF grant to create a comprehensive curriculum that was supposed to embody the vision of the NCTM standards, a strong backlash against this vision, now referred to as the "Math Wars," began to gather steam (Wilson, 2003). Led by mathematicians and parents, and centered in California, many of the reform-oriented curricula that MMAP viewed as being "more mainstream" were "being attacked vehemently" (Goldman/Knudsen interview 3/20/03). This directly affected many MMAP teachers as well as the funding context in which MMAP lived. In response, MMAP sought to keep its curriculum "under the radar" and "out of the line of fire" (ibid.) while individuals from the project worked behind the scenes to try to influence the policy conversation. These responses allowed the project and the curriculum it created to continue to exist during this period, but it also did not grow in prominence or in use nearly as much as it might have had the policy environment been more conducive.

However, it is striking that MMAP was successful at keeping itself from being the target of the many critics who were busy writing detailed critiques about most other reform-oriented mathematics curricula (e.g., see http://www.mathematicallycorrect.com/programs/htm).[23] This might seem surprising as at this time MMAP was being used most widely in California and had been created in Palo Alto, both of which were at the center of the emerging controversy (Wilson, 2003). However, the curriculum was less publicly known than others as it had not been published yet and was usually used by at most two self-selected teachers in any given school. In addition, MMAP's use of technology, inclusion of traditional-looking practice pages, and linkages with the mathematics used by high-status professional workers may have also helped protect it from direct attack. For example, one teacher-turned-staff-member reported doing "a couple of demonstrations [of MMAP] with parents in Palo Alto for back-to-school night" during a time of heated controversy in the district around mathematics instruction and was pleased to discover that many of "the comments from parents were 'I wish we had this.' " (Teacher TO interview). As Goldman and Knudsen (2000, p. 6) explained in a conference paper:

> "While we were in the midst of creating materials based on research in learning and our perspective on practice, the math wars began raging…. MMAP has steered clear of the math wars in large part, mostly because we are so small, and partly, we think, because of our real-world careers orientation. It's hard to call math in population models, for example, 'fuzzy math.' "

At the same time as all this was going on, MMAP staff participated in the policy conversations that were going on about mathematics instruction in Palo Alto, in California, and in the nation as detailed by Goldman, who was also a parent with a child in the Palo Alto public

[23] One might have thought that HOLD, which was the anti-reform-oriented math group in Palo Alto (http://www.dehnbase.org/hold/), would have focused its lens on MMAP at some point, but it does not seem to have done so. MMAP was, however, mentioned in passing in the famous public letter sent to then-Education Secretary Richard Riley in November 1999 protesting the Department of Education's designation of 10 mostly reform-oriented curricula as being either "exemplary" or (in the case of MMAP) "promising," although even that letter chose to illustrate its points using other curricula from that group.

schools:

> "I was on a K-12 math committee in the Palo Alto Unified School District that was in the middle of the Math Wars that started here. In 1995, I was on the California State Committee that reconsidered the 1989 standards and, for the first time, considered whether or not individual, yearly tests should be given in all California Schools. MMAP twice was invited to the National Governors Association meetings. All of the projects highlighted at those meetings were meant to influence governors and state education department officials to begin adopting technologies in schools. We also did presentations and workshops at NSF reform systemic initiatives curriculum workshops and TPD events. Jennifer [Knudsen] and Jim [Greeno] did some of this work as well. Jennifer was on the NCTM electronic standards committee. Jim gave testimony on the Newer California Standards in Sacramento hearings. [And] we all commented on the draft of the new NCTM 2000 standards document." (Goldman personal communication 7/25/05)

Still, Goldman concluded, the policy environment "pressed us harder than we pressed it," with MMAP staff having only minimal influence on the decisions that ended up mattering the most (ibid.), like the new state framework for mathematics instruction in 1997 that moved California decidedly away from any kind of reform-oriented curriculum, including MMAP's unique approach.

So although MMAP's curricula survived amidst the early phase of the math wars that occurred during this period, it did not thrive. Its choice to generally keep itself out of the limelight may have prevented it from being directly attacked, but it also likely made it more difficult for the project to promote itself and its curriculum to additional schools and teachers. And, as will be seen in the next section, as the math wars heated up, this made it increasingly difficult for MMAP's curriculum to be published, to be used by new teachers, and eventually for it even to be used by its core collaborating teachers.

Summary of This Section's Analysis

During this section, we have seen how the shift to a comprehensive curriculum and MMAP's natural evolution as a project changed the nature of MMAP's collaborative work. The problems the project worked on changed in character; authority was organized in a more asymmetric manner; and accountability to deadlines became increasingly important while other forms of accountability began to diverge between the project as a whole and its individual members. At the same time, the project improved its research productivity by identifying people and funding sources that served as resources for making that possible. Finally, it was during this time that the "math wars" began, which ended up significantly constraining what the project could do, as we will see even more dramatically in the next two sections as we discuss the history of MMAP's efforts to scale up its curriculum and how the MMAP project eventually ended.

VII. Efforts to Scale up MMAP's Curriculum (1993-2002)

In this section, I analyze the different ways MMAP worked over its history to make its curriculum available to interested teachers who were not part of the original design teams. NSF wanted to be sure that the curricula produced by its projects were used widely (NSF program officer interview) and MMAP also wished to share its materials with a much wider community of teachers. Because I am no longer focusing on the nature of the collaborations within the MMAP project, but instead how it shared its existing curricula with the world, this section will be analyzed and organized differently than the previous ones have been. The primary theoretical machinery I will use to understand the scaling up of MMAP will be Coburn's (2003) analysis of the multiple dimensions involved in scaling up educational innovations.

First, Coburn's (2003) framework will be used to analyze MMAP's scaling up priorities and strategies, eventually relating them back to the Engle & Conant (2002) principles, especially the central importance the project placed on respecting teachers' authority. Then I will illustrate these ideas by providing a historical analysis of how MMAP worked with four different partners for scaling up its curriculum: a reform-oriented network of teacher-leaders (California's Math Renaissance), individual teachers who independently became interested in its curriculum, commercial publishers, and an NSF implementation center (The Show-Me Center). Finally, I will close by summarizing the advantages and disadvantages of MMAP's curriculum from the perspective of each of its scaling-up partners, and the ways in which changing contextual conditions influenced what kinds of scaling-up strategies were most viable and when.

Analysis of MMAP's Scaling-up Priorities and Strategies

In Coburn's (2003) analysis, there are at least four key dimensions involved in the successful scaling up of a reform: spread, depth, ownership and sustainability. *Spread* is the closest to most people's intuitive ideas of what it means to scale up an intervention as it centrally includes "the spreading of a reform to greater numbers of classrooms and schools," the kinds of basic quantitative indicators that are often used to measure the broader impact of a reform (ibid., p. 7). However, in Coburn's formulation spread also includes what she refers to as *spread within*, which is the "spread of reform-related norms and pedagogical principles *within* a classroom, school or district" as when a reform's ideas influence teachers' instruction in ways not specifically targeted by the reform or somehow become institutionalized within a school's policies (ibid., p. 7). But Coburn contends that there are at least three other dimensions important for scaling up an intervention beyond just these two aspects of spread. *Depth* refers to "the nature and quality of implementation of reforms," with deep change "going beyond surface structures or procedures…to alter teachers' beliefs, norms of social interaction, and pedagogical principles as enacted in the curriculum" (ibid., p. 4). *Ownership* refers to a reform becoming "an 'internal' reform with authority for the reform held by districts, schools, and teachers who have the capacity to sustain, spread, and deepen reform principles themselves" (ibid., p. 7). Finally, *sustainability* refers to the degree to which a reform's "use is sustained in original and even subsequent schools" over time (ibid., p. 6).

MMAP's Scaling-up Priorities

It is difficult to achieve scale in all of these dimensions at once, so projects often have to make strategic choices about which aspects of scale to pursue more intensively (ibid.). In the

case of MMAP, its choices were informed by the project's core values. As seen in the top row of Table 1 (later rows will be explained later as each scaling-up partner is introduced), MMAP's number one priority in scaling up its curriculum was that teachers would have ownership over it, rather than it being something that was imposed on them from the outside (see "1, teachers" in the "Ownership" column for MMAP). MMAP had heard about examples of teacher resistance when other reform-oriented curricula were adopted by districts and imposed on teachers, and wanted to avoid having that happen with MMAP (Goldman personal communication 12/29/05). Instead, the desired target of MMAP's scale-up strategy was those individual teachers who were attracted to innovative materials with features similar to MMAP's and who thus would be interested in trying out MMAP's units in their classrooms (Berg interview). In effect, this was an extension of MMAP's core value of respecting teachers' professional authority within the project. The project wanted to have the curriculum be used only by those teachers who genuinely found something in the curriculum that they really appreciated.

As a second but still very strong priority, MMAP's desire was that new teachers would deeply implement the curriculum similarly to its collaborating teachers, by adapting it to their own local contexts. Specifically, rather than asking teachers to adhere to a single standard for how to teach a unit, or what is often referred to as fidelity, in MMAP's vision, teachers would use their professional judgment about how to best incorporate MMAP units into their instructional repertoire, adapting them as needed to better address their own contexts, purposes, and values (resulting in "2, adapted" in the "Depth" column). So depth of implementation was important to MMAP, but it was depth as created through each teacher's individual adaptations rather than by some external, MMAP-specific standard of fidelity. In effect this provided space for teachers to hold themselves accountable to what was important to them in their own contexts, another example of how the project worked to foster accountability in ways that would not undermine teachers' authority.

Finally, achieving a measure of spread was also desired by the project, but only under the

Table 1: Comparison of Priorities for Scaling up Among MMAP and Its Scaling-up Partners

	Spread	**Depth**	**Ownership**	**Sustainability**
MMAP	**3** if useful to teachers	**2** adapted	**1** teachers	–
Math Renaissance	**2** use	**1** fidelity	**1** schools	**2**
Individual Teachers	–	**2** adapted	**1** teachers	–
Publishers	**1** sales	–	–	**2** sales
NSF Implementation Center	**1 at first** sales	**1 later** fidelity with good learning outcomes	–	–

Table 2: Comparison Between MMAP and Its Partners in Preferred Strategies for Scaling up

	What to Scale Up	Through Whom
MMAP	replacement units	interested individual teachers
Math Renaissance	replacement units	interested individual teachers inside schools
Individual Teachers	replacement units	interested individual teachers, sometimes encouraged by coaches, higher education faculty, etc.
Publishers	comprehensive curriculum	superintendents
NSF Implementation Center	comprehensive curriculum	publishers as well as teachers, curriculum coordinators, and others who attend curriculum showcases

condition that teacher ownership and this form of depth of implementation had been achieved. Thus, spread was a much lower, third priority for the project. As Goldman summarized MMAP's stance on scaling up, she said she would "love MMAP to be out there, but only under the conditions in which it would work for teachers and kids." Similarly, one of MMAP's program officers talked about how MMAP's directors very much wanted their curriculum to be used with "integrity" (interview). I will return to the other rows of the table later when each of MMAP's partners and their scaling up priorities are discussed.

MMAP's Preferred Scaling-up Strategy and How It Embodied Its Priorities

MMAP's priorities around achieving scale influenced two key elements of its preferred strategy for scaling up: what to scale up and through whom.[24] As seen in Table 2 and mentioned in the previous section, MMAP generally preferred a scaling up strategy focused on distributing replacement units to individual teachers who had expressed interest in trying them out. Both choices support teacher ownership and an adaptability-based depth of implementation. Keeping individual teachers at the center of scaling-up efforts recognizes them as the primary authorities and owners of the reform. They are also the ones who might be expected to be centrally involved in adapting the curricula to the specific, local contexts of each classroom. In addition, working with replacement units makes it easier for individual teachers to have the authority for pursuing such adaptations as compared to a school adopting a comprehensive curriculum in

[24] In principle, it is not necessary that the mediator of the scaling up of an innovation be a person, although with MMAP that seems to generally have been the case.

which each teacher must coordinate any changes he or she makes with others. On the other hand, providing individual teachers with replacement units is not a particularly efficient way of encouraging spread in the most basic sense of influencing as many classrooms as possible for as much instructional time as possible. A single choice by a school to adopt a comprehensive mathematics curriculum can conceivably influence all of the mathematics instruction in every classroom in that school for a full year. In contrast a single choice by an individual teacher to use a replacement unit only has the potential for changing the mathematics instruction of just that teacher's classrooms and only for the particular weeks or months in which the unit is being used. So MMAP's preferred scaling-up strategy of encouraging individual teachers to use replacement units was perfectly suited to a project in which depth and ownership were more important than spread.

However, as also can be seen from the rest of Tables 1 and 2 and as will be detailed in the historical analysis below, MMAP's priorities and preferred strategies for scaling up often differed—sometimes subtly and sometimes dramatically—from those of the four key partners that it worked with in its scaling up efforts. In general, MMAP was most aligned with its first two partners, Math Renaissance and those individual teachers who independently bought its units, and least aligned with its last two, the NSF Implementation Center and especially commercial publishers. In the sections that follow I will historically document MMAP's work with each of these partners to scale up its curriculum, illustrating the ways in which scaling-up priorities and strategies were and were not aligned, how changing contexts contributed to this, and what impact all of this had on the relative success of the different partnerships for scaling up MMAP's curriculum in different ways.

A Historical Analysis of MMAP's Scaling-up Efforts

Using a State Reform Network to Prompt New Teachers to Try MMAP Units (1993-1996)

The first partner MMAP worked with, Math Renaissance, shared with MMAP its strategy of scaling up through replacement units, with a somewhat similar focus on teachers as a target of its efforts in hope they would come to feel ownership over the curriculum (see the first two rows of Tables 1 and 2). However, unlike MMAP, Math Renaissance was more interested than MMAP in fidelity of implementation, encouraging teachers to initially use the units exactly as written (Mumme interview). In addition, it cared more about spread, systematically working at the school level as well as at the teacher level in order to begin achieving it. Finally, my interpretation of the evidence to follow is that Math Renaissance was primarily using teacher ownership and depth of implementation as a means for eventually achieving significant spread while MMAP cared about these dimensions of scale as goals in and of themselves.

Math Renaissance was the NSF-funded state systemic initiative in California whose purpose was to encourage teachers to begin teaching in reform-oriented ways by using curriculum units as the lever of change (Mumme interview). It used a network model, beginning with teacher-leaders who could be used to support other teachers in changing their instruction through new curriculum. A year into MMAP's first main grant, Math Renaissance invited MMAP to contribute draft materials of its units to be used by California teachers participating in their project during the first phase of its work, which ended in 1996 (Mumme interview). NSF regularly let its systemic initiatives know when particular NSF curricula had been developed so it would not be surprising if that is how Math Renaissance heard about MMAP new units (NSF program officer interview). MMAP also saw Math Renaissance as the kind of group that could help it make connections with more interested teachers (Goldman/Knudsen interview 3/20/03).

Consistent with MMAP's preferred scaling up strategy, Math Renaissance used replacement units to foster change in mathematics instruction (Mumme interview). Specifically, each year, they asked participating teachers to try out a single reform-oriented replacement unit in their classroom from the three units selected for implementation in each of ten regions of the state (ibid.). By giving teachers such choices, they also promoted teacher ownership. Trained teacher-leaders who were experienced with a given unit then designed workshops in consultation with the unit's developers to support other middle-school teachers in using the new unit (ibid.). Implementation was done at the school level, with two more mathematics teachers from each school participating in the project each year (ibid.). Eventually, Math Renaissance ended up working with about 50% of the middle-school mathematics teachers in the State (ibid.).

However, rather than encouraging teachers to adapt the units to their own circumstances, Math Renaissance strongly encouraged teachers to do the units exactly as they were written, an example of encouraging a fidelity-based depth of implementation (Mumme interview). As Math Renaissance's director described the strategy, "It was behaving their way into new ideas…. A lot of people…as a result, became convinced that this kind of instruction [and] material was producing much better results than the traditional…. The long term strategy was building a market for this instructional material" (ibid.). This strategy is reminiscent of when MMAP sometimes asked teachers to try something they were skeptical of as a means of fostering teachers' accountability to new models of mathematics instruction while still not sacrificing their authority. If a teacher still liked the curriculum after trying it out in this manner, then this would increase that teacher's ownership over it.

The opportunity that was provided by Math Renaissance to share its curriculum-in-progress generally appealed to MMAP's directors Goldman and Knudsen. Although co-PI Greeno advised against sharing the early units with Math Renaissance because they were not finished, they decided to do so anyway because "we were much more about real kids and real schools getting real stuff that they could really use" (Goldman/Knudsen interview 10/27/04). Math Renaissance thought that the curriculum projects would find Math Renaissance's teachers helpful for piloting what they were developing in a wider range of classrooms (Mumme interview). MMAP concurred, feeling that it would "learn so much…[by] really see[ing] what the issues were when we were totally out of control of the situation" (Goldman/Knudsen interview 10/27/04). At the same time, it was clear to MMAP that NSF was likely to be happy to hear that they had decided to share their materials with Math Renaissance. Apparently NSF had been pleased when it was announced earlier that another project had done so (Goldman personal communication 12/29/05). At this time, NSF was very interested in spreading the reform-oriented curriculum as widely as possible (NSF program officer interview). Finally, to do so via another NSF-funded project demonstrated the kind of synergy between projects that the foundation liked (Goldman personal communication 12/22/05; NSF program officer interview).

MMAP eventually contributed three units to the Math Renaissance effort: Antarctica, Dream Home, and Codes (Mumme interview). MMAP units were selected by Math Renaissance in part because they had characteristics shared by many of the other NSF reform-oriented curricula: they were rooted in problem-solving situations, included extended activities, focused on important mathematical ideas, promoted new kinds of instruction, and were attractive enough to teachers that they would be willing to try them (Mumme interview). But there were also elements of MMAP units that made them especially attractive to Math Renaissance. Most practically, MMAP units were sufficiently separable from each other to make the replacement strategy work, something that was not true for most of the other NSF comprehensive curricula that were being

developed (ibid.). Mumme also reported that teacher-leaders "really liked the [MMAP] material a lot and…found it really engaging." (ibid.). There was also a high level of enthusiasm at Math Renaissance's workshops about MMAP, with the level of enthusiasm of these teachers being equivalent to that of core MMAP teachers in surveys that were distributed several years later (ibid.; Lichtenstein et al., 1998). In addition the creative use of technology was both a draw of the material, and a factor that limited its use because of problems with availability of computers and teacher comfort using them (Lichtenstein et al., 1998; Mumme interview).

With respect to spread, in the first year of the collaboration with Math Renaissance, the Antarctica unit was used by 75 new teachers across the State, an effort supported by MMAP teachers' development of a teacher's guide for the unit (Goldman/Knudsen interview 10/27/04; Knudsen interview 5/2/04). Compared to the dozen or so teachers who had been helping to design the units, this was a large increase in the number of classrooms using them. On the other hand, MMAP's units were used proportionately less than other units piloted by Math Renaissance primarily because of the issues around computer availability and comfort (Mumme interview; informal personal communication with former teacher-leader).

MMAP learned several things about ownership and depth of implementation of its units from working with Math Renaissance. From debriefing meetings that Math Renaissance teachers and teacher-leaders held with MMAP, the project learned that its units were "more accessible to very traditional teachers than a lot of the other curricula…because you could do them in a kind of front of the room, lecture style if you wanted to" (Goldman/Knudsen interview 10/27/04), an example of the units supporting adaptability in depth of implementation. Second, the project learned that even with new teachers using them for the first time, student engagement was high, with teachers sharing "incredibly inspiring stories" about kids who usually avoided math "coming in early in the morning and at lunch to work on things" (ibid.). Third, the project found out from Math Renaissance teachers that, in contrast to many other curricula, MMAP curricula provided support for students to not just solve mathematics problems, but pose new mathematics problems like mathematicians and math-using professionals do in their work, something that was documented systematically in Bushéy's (1997) dissertation (Goldman/Knudsen interview 10/27/04). Finally, MMAP learned that its units were providing a bridge for some teachers to change their practices to become more aware of student thinking. This occurred because MMAP activities prompted teachers to ask students authentic questions about what they were doing in order to be able to assist them on their work. When students replied, teachers would learn much more about how students were thinking. And interestingly "they didn't have to appropriate a whole new set of practices in order to carry this out. In the midst of what they were doing, they got a kind of feedback that they wouldn't get otherwise from the students" (Goldman/Knudsen interview 10/27/04). This may have engendered even more teacher ownership over the curriculum and over the value of asking authentic questions as the curriculum in effect prompted teachers to ask them so that doing so felt completely natural to them. In general, "we were happy because really when MMAP started it was about trying to keep kids engaged and give them a good safety net through middle-school mathematics that would keep them interested and ready to go on. Those kinds of stories, anecdotal though they were, made us feel like it was able to do that…[and] we didn't have to be there babysitting" (Goldman/Knudsen interview 4/10/97).

In the second phase of Math Renaissance beginning in 1996, however, MMAP was no longer involved as Math Renaissance had moved into the next phase of its larger scaling-up strategy. Now that it had created a market for reform-oriented curricula in California, its next step was to enhance depth of implementation and spread by providing professional development within

districts to allow teachers to successfully use the comprehensive reform-oriented curricula that districts had now bought for them (Mumme interview). Given that MMAP's full comprehensive curriculum had not yet been finished, let alone published, MMAP was no longer relevant (ibid.). And by the time MMAP's comprehensive curriculum was published, Math Renaissance was long over, having fallen by the wayside amidst all of the controversy in California around mathematics instruction (ibid.). So although MMAP and Math Renaissance's initial goals and strategies for scaling up were fairly well aligned for several years, as Math Renaissance moved into its next phase and the policy context around mathematics instruction began to shift, they were no longer helpful to each other.

Informally Distributing Curricula to Interested Teachers Around the Country (1997-2002)

In addition to making use of reform-oriented networks like Math Renaissance to distribute units, MMAP also sold its units to anyone who expressed interest, which was usually, but not exclusively, individual teachers. Given that this effort was led by MMAP, it is not surprising that there was a great deal of alignment in scaling-up strategies and priorities between MMAP and the individual teachers who bought its curriculum (see the first and third rows of Tables 1 and 2). However, those who bought the units did not particularly share MMAP's interest in spreading its curriculum to others. They usually bought them in order to satisfy their own interests and did not, with a few notable exceptions, make many efforts to share MMAP units with other teachers. To some extent this may reflect the fact that MMAP-interested teachers were often geographically isolated from each other in different schools. For example, some collaborating MMAP teachers talked about how they had considered sharing the MMAP curriculum with other teachers in their schools but then realized that this probably would be a wasted effort as their colleagues had such different ideas about mathematics instruction (Teacher LE interview; Teacher OO interview 5/3/04).

MMAP marketed its units by distributing an "Information Pak" that introduced MMAP's philosophy and available units, and then it took orders for the units by phone, by email, through mail, and eventually via the web. Prices were set to simply recover the anticipated costs for copying and shipping. From 1997 to 2002, prices ranged from $50 to $90 for a full Applications unit (like Antarctica or Guppies) and from $8 to $20 for each Extension or Investigations mini-unit (MMAP order sheets July 1997 to July 2002; Goldman personal communication 12/29/05).

Sales records provide evidence of moderate increases in the spread of MMAP materials across the country during this time period. Comparing the last year for which I have sales records (2001-2002) to the first (1997-1998), there was growth in orders from 40 to 137 per year. In addition, a somewhat greater percentage of orders went to addresses outside of California in 2001-2002 (84%) versus 1997-1998 (74%), with 16 states collectively represented during these two time periods. Thus, MMAP's effort at selling its units helped spread it to new locations throughout the country. In addition, although one might expect purchased units to be less likely to be used than those whose implementation was supported by a group like Math Renaissance, still the fact that several hundred orders were placed during these five years suggests that this scaling-up avenue also led to greater overall spread in terms of numbers of classrooms and teachers using the units.

In both time periods, about 60% of orders went to teachers, 24% to people in charge of supporting teachers like principals, district administrators, coaches, and professional developers, and 10% to higher education faculty, indicating that although MMAP's desired unit for scaling up was teachers, other players may have functioned as mediators between MMAP and them.

Once Extensions and Investigations were made available, most orders combined them with the relevant applications-based units, although some ordered only one type of unit or another. However, even when the full, comprehensive curriculum became available, it was rare for an order to be placed for it. Consistent with MMAP's strategy, most new teachers appear to have implemented MMAP one replacement unit at a time.

MMAP units were taken up in places outside of the Bay Area and California for a variety of reasons. Especially effective, according to Goldman and Knudsen, was:

> "Going to curriculum showcases [in] different states…those are really good places to make contacts. We went to some of those and got new teachers [on] the project. People…saw the thing and said, 'I have to do this…. I need to come to your summer institute next summer.' And often those were the best way to get teachers." (Goldman/Knudsen interview 3/20/03)

There are also some cases in which teachers found out about MMAP through university mathematics educators who knew about the project. One example of this was colleagues at the University of Alaska who brokered a collaboration with a group of Alaskan teachers who customized the Wolves and Caribou unit to make contact with local wildlife management debates. Enterprising teachers also found out about MMAP from web searches or from reading articles that mentioned the curriculum, as in the example of a whole school in Michigan whose teachers got particularly enthusiastic about MMAP. They attended the last two summer institutes, developed a whole host of additional materials to integrate the Antarctica unit with other subjects, provided footage for a CD about teaching MMAP, and then hosted MMAP's evaluator to see what they had done (e.g., Teacher TJ interview; Lichtenstein et al., 1998).

In general, MMAP had a policy of letting any interested teacher get the units and participate in project activities to the extent he or she desired. As they put it, when "teachers…found us, and for some reason this was very compelling to them, so we said, 'hey come on in' " (Goldman/Knudsen interview 3/20/03). In other words, if a teacher found MMAP units of interest and contacted them, then the project was happy to have them use the materials and become more involved in the project—no litmus test was applied for who could use the materials, exactly how they had to use them, and what their reasons might be for using them. Less effective, they felt, were "top-down" strategies, as when they made a presentation to the Board of Education of New York City (ibid.). This is not surprising given that such a strategy flies in the face of MMAP's ideals about scaling up. The units were designed by and for teachers who wanted to voluntarily transform their mathematics instruction (e.g., Berg interview) and so were not necessarily suitable for all teachers in a given school district.

Getting MMAP's Comprehensive Curriculum Published (1996-2000)

As mentioned earlier, when MMAP got its comprehensive curriculum grant, there was an expectation from NSF that every project would then get its curriculum commercially published (Knudsen interview 5/2/04). This was part of NSF's initial goal of spreading the NSF curricula to as many classrooms around the country as possible (see "Spread" column within row 5 of Table 1; NSF program officer interview). Initially, NSF and others thought that acquiring a large market for the reform-oriented curriculum would be relatively straightforward as in effect the projects had developed what the primary professional association of teachers, NCTM, had been asking for (NSF program officer interview). This initial focus by NSF on market share was aligned with most commercial publishers' primary goals regarding scaling up, which were to make a lot of money by selling curricula to as many school districts as possible and for as long as possible (see row 4 of Table 1). However, these goals for scaling up were not aligned with

MMAP, which desired to have the curriculum spread only to those classrooms with interested teachers who believed that the curriculum would be well suited for their needs (see Row 1 of the table). Once sold, publishers were not concerned with whether a given curriculum was used, let alone implemented properly, and they did not care at all whether teachers felt any degree of ownership over it. But MMAP pressed on with getting a publisher as working to become published was a requirement of their grant, and presumably there was a chance that being published might allow them to reach many more teachers who would be genuinely interested in the curriculum and ready to deeply implement it.

As it turns out, however, getting published was a very difficult process for MMAP as well as for most of the other NSF curricula. As the former NSF program officer explained, the foundation and the project came to realize that the reform curricula "were not designed the same way one designs a curriculum to get a market" (NSF program officer interview). Because of this, the projects generally needed publishers more than the publishers needed them. As a result, MMAP and many of the other projects were at the mercy of the significant consolidation and other upheaval that was going on in the publishing industry during the late 1990's. MMAP was especially vulnerable to these market forces because its curriculum was considered even more unconventional than the others, and it got started on the publication process later than most of the other curricula. On the other hand, one benefit to MMAP's late entrance to the publishing market, however, was that the project had the opportunity to learn from the other projects' negative experiences with publishers, leading them to negotiate carefully with prospective publishers for editorial control over their materials, support for teacher professional development, and copyrights on the research drafts of the materials (Goldman/Knudsen interview 10/27/04). In addition, MMAP was given significant support for finding and negotiating with a publisher by IRL, which put up its own money to hire a consultant from the publishing industry. To illustrate these points, I will first discuss factors that helped and hindered MMAP getting published and then discuss the two most significant publishing experiences the project had, first with National Geographic and then with their eventual publisher, Voyager Interactive.

The project learned through numerous inquiries and over a half dozen formal presentations made by Goldman and/or Knudsen that it was going to be difficult to convince most publishers to publish MMAP's curriculum (Knudsen interview 5/2/04; Goldman/Knudsen interview 10/27/04; Teacher TJ interview; Berg interview). First, the curriculum was highly unconventional from the perspective of publishers. It fit neither traditional expectations for mathematics curricula nor narrowing reform-oriented standards, instead including aspects of each along with some completely novel ideas. For example, MMAP thought it fit philosophically with the publisher of the high school Interactive Mathematics Project (IMP), but was rejected by them in part because mathematics was embedded within its projects rather than being the sole focus of students' activities (Knudsen interview 5/2/04). In addition, MMAP's integration of software into its curriculum was unwelcome: at this time curriculum companies did not have procedures in place to manage software development while software companies were not interested in distributing expensive paper curricula (ibid.; Goldman/Knudsen interview 3/20/03). Bottom-line issues were also central as well. MMAP's curricula were not in the format that most buyers were comfortable with, and changing that was a potentially costly and time-consuming process (Knudsen interview 5/2/04). At the same time, most major publishers only wanted to invest in curricula that could conceivably reach 50% of the market and were

much less likely to consider MMAP when Goldman and Knudsen estimated that at best it might be used by five to ten percent of teachers (Berg interview).

MMAP sought out smaller publishers, a strategy also used by many of the other NSF curricula (NSF program officer interview). However, with rapid consolidation in the industry, smaller publishers were being bought out by bigger companies, which then often shelved the reform-oriented texts from the smaller publishers in favor of the traditional ones in their portfolio that had an already large existing market (Goldman/Knudsen interviews 10/27/04, 3/20/03). In fact, on several occasions, MMAP piqued the interest of a publisher only to have that publisher bought up by another, forcing them to start over (Knudsen interview 5/2/04). As the math wars of the 90's intensified and the era of high-stakes accountability began, more recognizable mathematics curricula were increasingly favored, leaving almost no space in the market for anything like MMAP (Goldman/Knudsen interview 3/20/03).

Despite this, MMAP did have significant involvement with two publishers, each with real potential for getting their curriculum distributed more widely, first at National Geographic and then at Voyager Interactive. In both cases, particular individuals within publishing companies who believed in MMAP's vision were crucial for initiating publishing efforts, but in the end market and other larger forces ended up sabotaging them (Knudsen interview 5/2/04). At National Geographic, there was an acquisitions director in one department who "really got us," with MMAP collaboratively working with its editors to create a consistent format and terminology for all the units that Knudsen viewed as a "real improvement [that] would make them much more appealing to a broader audience" (Knudsen interview 5/2/04). However, about $20K and a year into that process, the department was informed that it had not made enough money the previous year so all new projects were dropped and the supportive acquisitions director was fired (ibid.).

Voyager was a similar story, except that MMAP materials were actually published and sold for about a year before being abruptly dropped. Voyager became interested in MMAP because its founder, who had struggled in school, believed in project-based curricula (Knudsen interview 5/2/04). In addition, MMAP was impressed that Voyager was already doing an excellent job selling similar products using an effective marketing scheme in which retired superintendents sold curricula to their former colleagues. Similar to National Geographic, for the first year MMAP and Voyager happily collaborated to make something better out of the materials (ibid.). Where relevant, standard skill exercises were added into the curriculum to make it more recognizable, something that MMAP could accept given all the changes in the policy context around mathematics instruction at that time (Knudsen interview 5/2/04; Teacher TJ interview).

But then the company decided it wanted to go for an IPO, and at that point bottom line considerations predominated: new managers were brought in to work with MMAP and the company began focusing on short-term cost cutting in order to show a profit when its stock went public (Knudsen interview 5/2/04; Goldman/Knudsen interview 3/20/03). For example, they hired "the cheapest programmer they could find" who avoided making use of then-emerging technologies like Java, handhelds, and the Web, instead creating very buggy software that implemented out-of-date programs like HyperCard (Knudsen interview 5/2/04; Goldman/Knudsen interview 3/20/03; Teacher TJ interview; TO interview). In a rush to get the materials out the door to be sold, Voyager stopped passing versions by MMAP for their feedback, and thus did many things with the materials that the project did not understand and objected to like complicating the organization of the curriculum, not providing user support for the software, and removing or not distributing key teacher support materials (Goldman/Knudsen

interview 3/20/03). All of these actions made it less and less likely that the curriculum would be implemented to the depth that MMAP desired. As Goldman described it:

> "At a certain point, they were in a real rush to get this stuff out, and they simply didn't give us the copies anymore. And I stopped pushing too. At a certain point, I just decided, they're doing their job, you know; how much can I, you know, meddle? So it just became, let's just see what happens. And I sort of held my breath and hoped that something good would happen, but it didn't." (Goldman interview 3/20/03)

The project did not have any power to object as they were under contract and there were no other clear publishing options for them on the horizon (Knudsen interview 5/2/04).

Still, the revised curriculum, renamed "Pathways to Algebra and Geometry," was published in August 1999. It was offered to districts at a price of about $2700 for materials that would support a year of instruction for a 30 student class (U.S. Department of Education Mathematics and Science Expert Panel, 1999), a price set by Voyager to be comparable with those of other reform-oriented curricula (Goldman personal communication 12/29/05).[25] Voyager apparently was very successful at selling the curriculum, with demand exceeding supply and the company garnering somewhere on the order of $2.5 million dollars of sales in its first year on the market (Goldman personal communication 12/28/05). By my rough calculations this corresponds to enough materials for a full year of instruction for about 900 classes. Voyager was especially successful at selling Pathways in Florida, Michigan, Ohio and Texas (ibid.). Thus, with respect to spread, there was more targeted distribution of Pathways to particular states, but the number of classrooms with access to MMAP curricula is much greater than in any other method attempted so far.

MMAP was conflicted about this commercial success, however (ibid.). First, their sense was that Voyager was selling the curriculum to districts with minimal attention to whether the materials were actually a good fit for their needs, thus violating MMAP's idea that spread should only be to places where the curriculum was best suited. In fact, from what MMAP directors could tell, most teachers weren't using the full set of materials as a comprehensive curriculum, instead picking out particular units to use in a supplementary or replacement unit way (Knudsen personal communication 12/29/05), thus meaning that districts had wasted a lot of money by buying the full curriculum. They also noticed, when Knudsen helped fill in on a few occasions, that what Voyager meant by providing teacher professional development paled in comparison with what they thought teachers needed, making it highly unlikely that the curriculum would be implemented to any depth or that teachers would develop any ownership over it. On the other hand, they were pleased that Voyager had done a small internal evaluation study and was planning to hire a company to do a much more extensive one in upcoming years once sufficient numbers of districts had been using the materials long enough to make that feasible.

However, a little over a year into selling MMAP's materials, Voyager abruptly decided to stop publishing Pathways and any other curricula not related to literacy. As it turned out, the Chairman of the Board of Directors of the company was a friend from Texas of incoming President George W. Bush, so when he was elected Voyager decided to ride "on his coattails" and focus on providing curricula implementing No Child Left Behind's vision for reading (Goldman/Knudsen interview 3/20/03; Knudsen interview 5/2/04). In summary, Knudsen said that working with Voyager "turned from being a very positive experience to a very confusing

[25] If my calculations are correct, this is somewhat more expensive than a year of Connected Mathematics, which at that time was being offered for about $2000 for a year of instruction (derived from figures provided in U.S. Department of Education Mathematics and Science Expert Panel, 1999). That MMAP was somewhat more expensive makes sense given that it included software.

experience to a very horrible experience to a goodbye that was actually a relief" (ibid.). Luckily, with the assistance of its publishing consultant, MMAP had negotiated its contract so it always owned the research drafts of the curriculum, which allowed it to continue to sell them informally on the side before, during, and after this process, as documented in the previous section (Knudsen interview 5/2/04).

Finally, it is important to emphasize again that the experiences that MMAP had with publishers during this time were not unique to the project. Several other NSF curriculum projects had similar experiences, with only a minority still being actively published and marketed today (Goldman personal communication 12/29/05).

MMAP's Marketing of Itself During Its NSF Implementation Center Funding (1998-2002)

The final partner that MMAP worked with in scaling itself up was the Show-Me Center, an implementation center that NSF funded in 1998 to promote public awareness of all of the NSF comprehensive mathematics curricula at the middle-school level (Goldman/Knudsen interview 3/20/03). The initial focus of scaling up within Show-Me and the other implementation centers for elementary and high schools was on promoting the sales of those comprehensive curricula that had been published (see bottom left cell of Table 1; NSF program officer interview). At the time the Show-Me Center began, MMAP had not been published yet so did not fit well with that strategy. As MMAP and other projects began to have trouble with getting published and NSF became aware of the degree to which publishers were not concerned about depth of implementation, the focus of scaling up at NSF shifted to showing that when curricula were implemented with fidelity, good learning outcomes occurred (NSF program officer interview). However, again MMAP did not fit well with NSF's scaling-up goal, as teachers used its curriculum in many different ways and thus there was no way to collect such outcomes data, had MMAP been interested in doing so and had it had the funding and expertise to collect the data (Knudsen interview 5/2/04; more about this later). However, the Show-Me Center did have a centralized website with links to information about each of the comprehensive curricula (e.g., see current version at http://showmecenter.missouri.edu), put on numerous curriculum showcases about them, and even hired a public relations person to attempt to put out fires when Math War-related controversies about curricula arose in particular districts.

The Show-Me Center also provided subcontracts of $200K per year to each project that it could use to help get the word out in hopes of further spreading the curriculum. This turned out to be an insufficient scale up strategy for MMAP. One reason was that the funds could only be used for marketing activities like developing promotional websites, creating marketing brochures, making presentations at various forums, and conducting introductory workshops (Goldman/Knudsen interview 3/20/03; Knudsen interview 5/2/04; Goldman personal communication 12/29/05; NSF program officer interview). However, funds could not be used for what MMAP needed most, namely larger-scale evaluation studies, work to further refine the curriculum, ongoing support of teachers using the curriculum, or funds to help defray publishers' greater-than-usual start-up and teacher professional development costs (Goldman personal communication 12/29/05; NSF program officer interview). For MMAP, having such support would have been especially helpful given its unconventional nature. MMAP might have been able to do further work to: evaluate the conditions under which MMAP materials were most effective; make its comprehensive curriculum materials more recognizable to a broader audience, including district leaders; support needed developments of the materials that publishers shied away from; re-program the computer applications using the latest software platforms; and

support the kinds of ongoing professional development that would allow teachers to use it well over time.

These funding restrictions contrast with NSF's earlier support for a few pioneering reform-oriented curriculum projects like the Interactive Mathematics Program, which in addition to development money was also given funds for evaluation studies, teacher professional development, several regionally based dissemination efforts, and a major curriculum revision to take into account policy changes (e.g., NSF award abstracts #9550081, 9619062, 9629168, 9633811, 9634934, 9731483, 9911936, 0101997 and 0137805).[26] However, by the time that MMAP and other projects would have benefited from such funding, NSF was no longer interested in providing it as they had moved on to other priorities and viewed the math projects as having received a lot of federal funding already (NSF program officer interview). In a few exceptional cases, NSF did provide funding for curriculum evaluations, but this was only given to projects that were already widely implemented, which was not the case for MMAP (ibid.). But without funding for the activities mentioned above, being able to become widely implemented was almost impossible for MMAP in a context in which non-traditional curricula were becoming less and less favored.

Because its implementation center funding focused on marketing, the project did not work with teachers as collaborators during this phase, instead hiring some of them to make marketing presentations about the comprehensive curriculum. As Knudsen described it:

> "We did not work with teachers in the same way. The teachers were not the co-developers of the implementation. Shelley and I directed the [MMAP] Implementation Center and we were sometimes able to include teachers and we tried really hard to include teachers. Teachers were not always really good public speakers for the materials, so a lot of it we ended up doing ourselves. I really hoped to sort of mobilize and catalyze a group of them more, but it just didn't really work out." (Knudsen interview 5/2/04)

Why might this have been the case? Some teachers were not comfortable with giving marketing presentations in which they were supposed to sell something to other adults (Teacher EH interview; Teacher OO interview 5/3/04). In addition, MMAP's teachers were being asked to promote a comprehensive curriculum that in fact none of them had used. This problem was compounded when they were asked to promote Pathways, whose applications units had been changed markedly—and not always positively—from what they had helped co-design. In addition, it did not help that the Pathways software crashed regularly, which made it difficult to convince new teachers to try out the curriculum in a context in which many were already worried about incorporating technology into their instruction (Teacher EH interview; Teacher OO interview 5/3/04; Teacher TJ interview). It was frustrating for some MMAP teachers to make presentations at these forums, to seem to have generated a lot of interest, and then to find out later that there had been only minimal follow up because of concerns about technology or the curriculum's unconventional nature (e.g., Teacher OO interview 5/3/04).

In addition, solving these marketing problems was not something these core MMAP teachers were necessarily invested in, nor particularly prepared to do. In contrast to the multi-purpose problematizing characteristic of the curriculum design phases in MMAP, learning how to better market the materials did not particularly advance any of these teachers' own goals. They did not

[26] With this comparison, I do not mean to imply any favoritism on the part of NSF toward IMP as opposed to the middle-school projects. Instead, by the time the middle-school comprehensive curricula were around, there were many more curriculum projects for NSF to support the dissemination of, with about the same amount of money available for its support (NSF program officer interview).

learn about how to be a better teacher by helping out MMAP in this way. And they no longer had the benefit of intensive interactions with a stimulating group of teachers and researchers.

Finally, making marketing presentations did not particularly depend on any of the skills most of the teachers had developed in working on the MMAP project. In fact, many of the practices that MMAP teachers and staff had learned about working with new teachers during the first two phases of MMAP's work were practices that may have been counterproductive when appropriated for use in marketing presentations at curriculum showcases. Specifically, reports about these presentations indicated that MMAP representatives at times shifted from the genre of a persuasive presentation to that of engaging with these newcomers as if they were to become new members of the collaborative design team, or at least colleagues to be supported in their development to be the best kind of math teacher they could be. Whether the new teachers actually ended up purchasing or using MMAP materials sometimes became a secondary consideration. When this happened in the context of forums in which most other curricula were doing straight sales pitches, MMAP's efforts to engage new teachers in these ways sometimes came off as implicit indictments of the curriculum by MMAP's representatives (e.g., Teacher TO interview).

For example, building on their own experience and MMAP's focus on replacement units, MMAP presenters regularly discouraged new teachers and districts from initially trying to implement the whole comprehensive curriculum, instead suggesting that they "do a unit or two in your class, see how the kids react to it, see how much math they learn" and then decide which units to do the next year (Teacher TJ interview). Although this is wise advice from a teacher professional development standpoint, the implication of this recommendation— especially as phrased above—is that students might not react to MMAP's curriculum well, or might not learn enough math with it. This unwittingly may have communicated the message that the MMAP curriculum may not be that good. As another teacher reflected, "Some people can go ahead and say 'this is just so wonderful, so exciting, it will work for all.' I do not feel comfortable in saying that you must use the unit. Instead, I am inclined to say that the unit is here and you make the choice.… And that didn't help either." (Teacher OO interview 5/3/04; personal communication 10/17/06). Although this teacher talked about this tendency as a personality characteristic, it is striking how closely it parallels how the project engaged with its collaborating teachers while the curriculum was being designed. Similarly, MMAP's sales presentations included curriculum run-throughs that provided openings for participants to provide their own evaluations of the curriculum, sometimes launching extended discussions about its perceived problems as well as benefits. In one such case, what looked like a good discussion to the presenter because people had "different ideas" made MMAP's curriculum appear "controversial" and "problematic" in the eyes of the audience (Teacher TO interview). Others have also noticed this tendency in presentations that MMAP staff and teachers have made over the years in which an open-minded, listening stance can come off as MMAP not being confident in the quality of its work or materials (Lichtenstein et al., 1998). The larger question is how can a project like MMAP maintain its moral commitment to respect the authority of teachers and guide them toward wise use of the curriculum while coming off, within a competitive marketplace, as being worthy of respect itself?

On the other hand, the written marketing materials that MMAP used like brochures, its website, and PowerPoint presentations did include the kinds of persuasive messages one might expect in documents designed to convince an audience to take a chance on a novel mathematics curriculum. For example, the slides for MMAP's marketing presentation began with a slide that

defined MMAP through its potential advantages in the classroom (MMAP presentation PowerPoint slides). Similarly, MMAP's website included a whole section devoted to compiling positive press and awards about the project (http://mmap.wested.org). Finally the standard information packet included these materials plus brochures, a curriculum guide, and some practitioner-oriented articles celebrating the merits of the project (MMAP information packet, 2000).

Still Knudsen and others believe that even had MMAP marketed its comprehensive curriculum more effectively, it was likely to have had a tough time being adopted, especially at that time, as it was not a "recognizable" curriculum that sat "on the borders of so many communities at the same time" (Knudsen personal communication 12/29/05). For example, despite MMAP's efforts to make sure all the NCTM Standards were covered in the curriculum, it did not fit the emerging consensus of what a reform-oriented curriculum should look like (Goldman/Knudsen interview 4/27/04). It rejected traditional sequencings of mathematical topics that even reform-oriented curricula followed and included consideration of design, technology, and real-world applications in ways that conflicted with the sensibilities of many mathematics educators whether reform or traditional (Knudsen interview 5/2/04; see also Hall interview). And although MMAP was more accessible to some traditionally-minded teachers, it also had many features that set it apart from conventional curricula. Finally, even though MMAP was now packaged as a comprehensive curriculum, which can be more attractive to teachers who do not have the time or inclination to make adaptations, some of MMAP's presentations made it clear that teachers would be required to do additional work to "see how you're going to be able to fit it in your curriculum to meet the goals that you want to meet," something that such teachers were not happy to hear (Teacher TJ interview). The central unsolved design dilemma was how could MMAP build something that embodied its principles while still being recognizable as worthy in the wider community (Knudsen personal communication 12/29/05).

Summary of the Analyses of MMAP's Scaling-up Efforts

In this section, we have analyzed how MMAP interacted with each of its scaling partners with respect to its scaling priorities and strategies, specific events that influenced its relationships with them, and the degree to which different dimensions of scale were achieved. To summarize, I first review the advantages and disadvantages MMAP had from the perspective of each of its scaling-up partners. Finally, I close by highlighting the ways in which changes in various contexts influenced MMAP's efforts to scale up.

Advantages and Disadvantages of MMAP to Its Scaling-up Partners

As seen in Table 3, for every advantage of MMAP to one of its scaling up partners, it eventually had just as many, if not more, disadvantages. MMAP's sophisticated use of technology impressed NSF and was a lure for some teachers who came to MMAP independently or through Math Renaissance, but it primarily functioned as an impediment for the spread and sustainable use of the curriculum; it limited the number of teachers willing to try the units, prevented even the most enthusiastic teachers from being able to use them as often as they would have liked, and made it more difficult for the project to find a good publisher who would be willing to invest extra money in writing and maintaining the software. Almost ten years later, some aspects of this problem (e.g., availability of computers in most schools) might not be so

grave, but being able to keep software like MMAP's both up-to-date, accessible, and attractive to a wide range of teachers and students would still be a daunting challenge even today.

At the same time, MMAP's goals and strategies for scaling itself up did not fully match those that most of its scaling-up partners eventually settled on. Although MMAP's approach was most consistent with that of the first phase of Math Renaissance's scaling-up strategy, as soon as Math Renaissance moved into its next phase, MMAP became irrelevant. Similarly, individual teachers who found MMAP personally appealing eventually could not sustain its use amidst trends toward externally mandated curricula. What kept MMAP being used as long as it was, was the appeal of its innovative ideas, most of the time to particular individuals who became familiar with the curriculum or saw it work in action. However, these individuals' generally separate and uncoordinated appreciations were but a thin thread for sustaining a scaling-up effort amidst the institutionalization of growing contextual forces that were working against curricula like MMAP.

The Importance of Changing Contexts in Influencing Scaling Up

In particular, this section on the scaling up of MMAP has put into high relief the central importance of contextual factors on mediating this process, especially for an unconventional innovation like MMAP (cf. Moore, 2002; Tyack & Tobin, 1994). Changes in MMAP's funding context, the publishing industry, and the technology industry were all significant during this phase. However, trumping and at times shaping several of these other contexts was the changing policy context around mathematics instruction. The policy context and its effect on these other contexts made MMAP's teacher-focused strategy for scaling up promising in MMAP's early years but increasingly less and less viable as the project proceeded. This strategy was relatively successful in the early to mid 90's when teachers had some control over their curricular choices, when states like California provided funding and professional development support for enterprising teachers to try out new curricula, and when MMAP could afford to sell its materials for cost. However, this strategy became increasingly ineffective as curriculum decision-making power moved from classroom teachers to school administrators, district curriculum coordinators, or state textbook adoption committees (Goldman personal communication 12/28/05). Since MMAP was late in becoming published, was only published briefly, and had not worked to market itself to these new players, it was usually not the curriculum they adopted. Therefore when these external players began enforcing their curricular choices through high-stakes tests, budgetary incentives, and the monitoring of the implementation of adopted curricula, this meant that MMAP was no longer a curriculum that even enterprising teachers, including its collaborating teachers, could use (e.g., Teacher FJ interview; Teacher LE interview 3/2/03; Teacher TO interview).

For scaling-up efforts to succeed therefore, there must be a close fit not just between a project's goals for scaling up and the strategies it uses for pursuing them, but also between the scaling-up strategies and the current policy and other contexts in which they are employed. And because such contexts can change rapidly over time, it is also important that projects are able to correctly predict how contexts might change in order to adjust their strategies accordingly. It is not clear how a given reform project might be able to do that reliably while also succeeding at all of its other functions.

VIII. MMAP's Ending and Continuations (1997-present)

Contextual Factors Combine to Lead to the Official End of the Project in 2002

Although MMAP kept itself going throughout the 1990's despite the many challenges it faced, several external contextual events from the late 1990's to the early 2000's combined to result in the official ending of the project in the summer of 2002.

First, the policy climate around mathematics instruction made it increasingly difficult for even core MMAP teachers to use the units in their classrooms (e.g., Teacher LE 3/2/03; Teacher TO; Goldman personal communication 12/22/05, many others). Specifically, in 1997 a new California Mathematics Framework was put into place, which moved away from the NCTM standards in several significant respects (Wilson, 2003). In 1998 and then 2001, the State adopted new middle-school mathematics curricula that did not include any of the NSF projects, including MMAP. At the same time, high-stake tests, monitoring systems, and financial incentives were used to make it more and more difficult for teachers, schools, and districts to use anything like MMAP except around the periphery like during summer school, in after school programs, during science or technology classes, in the few weeks of instruction after testing was done, and during additional math classes created for failing students (Goldman personal communication 12/29/05). Similar changes were occurring at different rates all across the country. By 2000, hardly any teachers in California were using MMAP units, with that pattern being replicated, sometimes more slowly, elsewhere in the country (Goldman/Knudsen interview 3/20/03). Only one of the MMAP teachers I interviewed is still able to use the units in a limited manner, and she is now at a parochial school (Teacher EH interview).

The second major external contextual event that helped lead to the end of MMAP was Voyager's already mentioned decision to cease publication of its MMAP-based curriculum. As already noted, this was intimately linked to the educational policy changes that were occurring nationally during this time. Because of this policy climate and changes in the publishing industry, there is little prospect of the project finding another commercial publisher in the near future. This also meant the death knell for having the curriculum undergo the large-scale test-based evaluation that Voyager had planned.

At the same time, changes in the technology industry at this time made it very difficult for the project to find other ways to distribute itself to those teachers who were still interested in using it. MMAP would have liked to have published itself electronically or teamed up with a software company, but Voyager dropped MMAP right during the lows of the dot.com bust. Industry and venture capital funding for high risk and potentially low profit ventures like distributing MMAP's software and curriculum were almost impossible to come by. In response, MMAP generated a business plan in which it would have sold the curriculum for cost, in order to make money by charging districts for professional development (Goldman personal communication 12/22/05). This strategy seemed viable as they had talked with the then chancellor of the New York City public schools and found out that large urban districts like New York's had a lot of funds for teacher professional development around curricula that could be used in extra math periods created to help students who had failed high-stakes tests (Goldman personal communication 12/28/05). However, realizing this potentially workable business plan was stymied by the fact that MMAP would have needed a significant amount of start-up funding to redevelop its software using current technologies (ibid.). The project talked with some researchers at a nearby software research center about allowing them to beta-test a software tool

that they were developing that might have streamlined this process, but in the end those researchers decided not to share their tool because of intellectual property concerns (ibid.).

To make matters worse, soon after Voyager dropped MMAP, IRL folded because of financial problems in other areas of the company. With grant funding transferable to a new institution, Goldman found MMAP and its affiliated projects a home at the educational research and development non-profit WestEd, but this ended up being a less supportive home for the project than was hoped. WestEd was well connected with policymakers, so Goldman hoped that this would give it more leverage and sources of advice for dealing with the new policy context. In addition, because WestEd was a larger organization than IRL, it would not require Goldman to spend as much time on organizational management tasks, allowing her to focus on finding new footholds for MMAP's curriculum (Cole interview).

However, the project quickly discovered that West Ed was a less collaborative place than IRL (Cole interview; Goldman/Knudsen interview 3/20/03). There was less interaction between projects at West Ed, which felt to some MMAP participants like a "lonely, quiet, [and] sterile hotel" that one rent rooms in as compared to IRL, which felt more like a large, boisterous extended family (Cole interview; Goldman/Knudsen interview 3/20/03). In addition, because of space constraints, West Ed had to put many of MMAP's staff in offices far from each other, sometimes on different floors or in the case of Knudsen and Goldman, in different cities (ibid.; Berg interview; Cole interview; Goldman/Knudsen interview 3/20/03). This made coordination between different members of the MMAP team difficult (Cole interview). As a result of these and other factors, several key staff members, including Berg, Cole, and eventually Goldman, either did not choose to move to West Ed or left before MMAP's implementation grant was over (Berg interview; Cole interview; Goldman/Knudsen interview 3/20/03).

The final death knell for the project occurred when the director of the Show-Me Center decided in late 2001, when the Center was applying for its second round of NSF funding, that she would not be willing to direct the project if MMAP continued being a part of it. Given the backlash against reform-oriented curriculum, apparently she did not want to deal with a project that was much harder to explain and, in her eyes, defend. In addition, the fact that MMAP was no longer being published made it an even less attractive member of that group (Knudsen interview 5/2/04). Therefore, when funding for the MMAP part of the Implementation Center ran out in the summer of 2002, the MMAP project officially ended. Soon thereafter any references to MMAP or Pathways were removed from the Show-Me Center's website. Knudsen then left West Ed to work on projects at SRI.

Together these external contextual changes provided not just a 1-2 punch, but a 1-2-3-4-5 punch to the MMAP project. There might have been ways for the project to have responded more effectively to some of them, but probably not well enough to have prevented the eventual ending of the project around this time.

Research During the Implementation Center Phase and Beyond (1998-Present)

While MMAP was being supported through implementation center funding, it had no funding for research, and many fewer paid staff. People who had begun research papers and strongly cared about them seeing the light of day found ways to get them done mostly on their own time. Not surprisingly, researchers who were or became university-affiliated were more likely to publish their work while those who were not university-affiliated were less likely to do so.

There was some additional empirical research to inform the design of the scaling-up strategy that the project wanted to do but was not able to do during this phase (Knudsen personal communication 12/22/05). First, the project staff wanted to do a field-test of the whole comprehensive curriculum once it was completed, but they "didn't have the expertise or connections to make this happen" (ibid.). This would have required getting a whole district to adopt the curriculum, which was especially challenging at that time, particularly in California (Wilson, 2003). In addition, the project's directors did not have any relevant connections in districts to build on. Failing that, the project wanted to do studies or at least get information on how districts evaluate curricula for adoption as a basis for revising their comprehensive curriculum to be more attractive and usable from the perspective of district decision-makers, but again it did not have the district contacts to make that possible (Knudsen personal communication 12/29/05). The project had hoped that their publisher would have helped them with that, but that was clearly not in the cards once Voyager dropped them.

However, the data that MMAP collected is continuing to be analyzed by former MMAP researchers, with several papers currently in press. There even have been some plans to publish a book of MMAP research. Given that, MMAP's impact on the research community is ongoing.

Continuing Interest in MMAP Curriculum Materials (2002 -Present)

Similarly, although the MMAP project has been completed and no commercial publisher is in sight, it is not clear that the curriculum is dead either. For example, several years ago the Codes unit was refashioned for handhelds and used within a summer school (Goldman/Knudsen interview 3/20/03). As I directly observed during several of my interviews (e.g., Goldman/Knudsen interviews 3/20/03, 10/27/04), teachers continue to contact Knudsen and Goldman about getting copies of the curriculum, which is frustrating as in most cases schools no longer have computers that can run the software. The curriculum also continues to be cited by other researchers as representing an exemplary use of technology for supporting inquiry-based mathematics learning (e.g., Linn et al., 2000). In addition, during the several years in which I have been involved in researching this case, the project has received several promising inquiries to re-create the curriculum at a fairly large scale in other countries, for non-school contexts and the like that Goldman and her colleagues have pursued. In fact, just as this case was being finalized, I found out from Goldman that the Global Education and Learning Community (https://edu-gelc.dev.java.net), founded by former Sun Microsystems CEO Scott McNeely, will be making MMAP curricula freely available on the web, beginning in 2007 with the Antarctica Project and its Extensions and Investigations (personal communication 10/26/06). The GELC's mission is "empowering teachers, students and parents with self-paced, web based, free and open content (curriculum resources, assessments)," which seems, on the face of it, to be a pretty good fit with MMAP's scaling-up priorities. So MMAP's journey to scale up its curriculum is definitely not over yet.

IX. Various Impacts of the Project

In this section, I summarize the various impacts that the MMAP project had by combining my own data and analyses with results of an evaluation of the project that was conducted in 1998 at the conclusion of MMAP's comprehensive curriculum grant. This evaluation study was conducted by a private evaluation firm, Quality Evaluation Designs (Lichtenstein, Weissglass, & Ercikan-Alper, 1998), and drew upon the following data sources: surveys of 11 core MMAP teachers, 20 teachers exposed to MMAP units as part of Math Renaissance, and 72 students participating in MMAP units; interviews with students, teachers, and MMAP staff; and observations of classroom sessions and teacher workdays. The evaluation investigated the degree to which the project had achieved the goals it set out to achieve in its comprehensive curriculum grant and also reported quantitative and qualitative results on various teachers' and students' views of the units and the project. In the following sub-sections I consider the degree to which the project achieved its stated goals, its reform-related impacts, its research-related impacts, and the role of the project in influencing the professional trajectories of the people who participated in it.

Achievement of the Project's Stated Goals

The project had a variety of stated goals that evolved from grant to grant. Goals for the second MMAP comprehensive curriculum grant were drawn from those quoted in the Lichtenstein et al. (1998) evaluation while others were inferred from interviews, the MMAP website, and MMAP publications. I will outline the goals for each grant and then summarize the degree to which each was achieved.

At the outset of the first full NSF grant, the MMAP project had at least five primary goals:

1) To "test the feasibility of an applications approach to math learning and to figure out what would that be like and how" (Goldman interview 3/20/03; also on the MMAP website; Goldman & Greeno, 1998; and elsewhere).
2) Through this approach, to promote equity in mathematics education by "design[ing] MMAP curriculum units specifically for students who have been traditionally underserved by school mathematics—girls, minorities, inner city, and rural students." (Goldman & Greeno, 1998, p. 13).
3) "To bring to the curriculum design process a collaborative community that included education researchers, teachers and teacher educators, curriculum developers, math-using professionals, and students" (http://mmap.wested.org/pathways/history.html).
4) "To conduct research to improve on our materials design and to generate new understandings of mathematics teaching and learning" (http://mmap.wested.org/pathways/history.html)
5) "To work with teachers to learn about the issues they face as they make changes in their perspectives and practices" (http://mmap.wested.org/pathways/history.html)

In 1992-1995, the project made significant progress on all five goals, successfully co-designing with teachers applications-based units that appeared to engage underserved students, something that was partly validated in the 1998 evaluation report (see "Impact of the Project on Students" below). In the process, the project became clearer about what it meant for a unit to be applications-based, became aware of the issues teachers faced in using MMAP units, and found new ways of engaging with math-using professionals to draw upon a wide range of professionals while not allowing them to unduly influence the collaborative process. With respect to goal 4, extensive research was conducted to improve on the materials, which provided an initial foundation that eventually resulted in at least 45 publications to be detailed later. In addition to making progress on these goals, the project got a head start on disseminating its curriculum when

it shared three units with Math Renaissance, exposing MMAP units to as many as 160 teachers who did not collaborate with the project (inferred from Lichtenstein et al., 1998, p. 30).

When MMAP became officially tied to the other NSF middle-school mathematics curriculum projects during its comprehensive curriculum grant, it had the following four additional goals, all of which were considered to have been addressed by the project by Lichtenstein et al. (1998):

6) "To develop a comprehensive set of materials that stimulate sixth-eighth graders to learn and apply mathematics to real-world problems.… All materials will link to the NCTM and the California Mathematics Framework" (1994 MMAP II Proposal, pp. 7, 8, as cited in Lichtenstein et al., 1998, p. 1)
7) "Provide materials and means to empower math teachers to develop customized curriculum aligned with NCTM and state standards" (1994 MMAP II Proposal, p. 11, as cited in Lichtenstein et al., 1998, p. 7)
8) "Spearheading Assessment Materials and Practices…to develop ways to see mathematics in all middle-schoolers' work, and to create professional development materials that enhance teachers' understanding of students' math activity; and to develop innovative assessments to accompany MMAP II curriculum materials…." (1994 MMAP II Proposal, p. 13, as cited in Lichtenstein et al., 1998, p. 8)
9) "Galvanizing a community of math professionals." (1994 MMAP II Proposal, p. 15, as cited in Lichtenstein et al., 1998, p. 12)

Specifically, the project was cited for: finishing its comprehensive curriculum package, making "materials and guidelines available to educators via an on-line and hard copy catalog and order form," producing a "Curriculum Guidebook," "offer[ing] innovative approaches to assessments…[with] assessment opportunities embedded in the curriculum materials," "pull[ing] together a cohesive group of teachers, providing them a variety of rich opportunities for professional development," and making "numerous substantive contributions to non-MMAP teachers and educators nationwide" (Lichtenstein et al., 1998, p. iii).

Finally, during MMAP's subcontract from the NSF implementation grant, NSF gave MMAP the goal of scaling up by:

10) Commercially publishing its comprehensive curriculum
11) Through publication, its own efforts, and those of the NSF implementation center, being successfully disseminated and implemented by numerous teachers around the country.

MMAP was at best partially successful at achieving these goals. After many failed attempts, MMAP's comprehensive curriculum was finally published by a small publisher but was withdrawn from the market a year later for reasons relatively independent of MMAP. Both in-house and commercial versions of MMAP materials were disseminated around the country to a moderate degree given the non-traditional nature of the materials. MMAP estimates that approximately 60,000 kids used MMAP units before Voyager published them (McDermott interview).[27] Voyager sold their version of the comprehensive curriculum to at least 30 districts (Knudsen interview 5/2/04), making at least 2.5 million dollars in sales during the first year (Goldman personal communication 12/28/05).

However, given the current policy climate around mathematics instruction and the fact that most MMAP software is now obsolete, with a few notable exceptions MMAP materials basically have disappeared from current teachers' classrooms, including those of all but one of the core MMAP teachers I interviewed. When MMAP units are used these days, they appear at the fringes—after school, during the summer, as part of technology instruction, or in a dramatically shortened form (Goldman/Knudsen interview 3/20/03; Knudsen interview 5/2/04). They are

[27] Just before 1998, Goldman & Greeno (1998) estimated that MMAP units had been used by over 200 teachers and 40,000 students.

now used at best as supplementary curricula in the context of other core mathematics programs, rather than being used either as a comprehensive curriculum or even as full replacement units.

Reform-Related Impacts

External Evaluations of the MMAP Comprehensive Curriculum

Two Department of Education expert panels, one on mathematics curricula and the other on technology curricula evaluated MMAP's comprehensive curriculum. It was selected as a "promising" curriculum in both reviews. For example, with respect to "Program Quality," the mathematics curriculum expert panel found that:

> "Strengths of MMAP include integrated use of technology in the core curriculum, adaptability to local and state curriculum expectations, an emphasis on real-world contexts and work-place simulations, and the ability to meet students at their skill level and move them forward…. Reviewers found that MMAP's goals are aligned with NCTM standards…. The instructional design of the program is creative, flexible, appealing, and motivating both to students and teachers…. Scenarios in which students learn in context provide purpose for learning rigorous mathematics and are one of the program's strengths…. Reviewers noted that the activities provide multiple avenues for connections within mathematics itself and with other subject areas (for example writing, geography, and science), and therefore attend to a broad range of student interests." (U.S. Department of Education Mathematics and Science Expert Panel, 1999, p. 53)

All five NSF middle-school mathematics curricula applied for recognition from the mathematics curriculum panel, and of them, only MMAP and Connected Mathematics were selected for any recognition (Goldman personal communication 12/29/04). In addition, MMAP was the only curriculum honored in both the technology and mathematics curriculum categories. Like Connected Mathematics, MMAP could have been cited as an "exemplary" as opposed to simply a "promising" mathematics curriculum, but was not as it did not have any large-scale studies of effectiveness that involved standardized tests. Connected Mathematics did have these data, having been out in the market for a few years, giving its publisher sufficient time to have conducted such a study.

Impact of MMAP Units on Students

In the 1998 evaluation, all students who had recently participated in the Antarctica unit in several different classrooms across schools filled out a 3-page survey asking them to rate the Antarctica unit and their participation in it as well as provide their reasons for preferring Antarctica or their regular textbook, and any suggestions they had for improving the unit. The first author of the evaluation interviewed at least sixteen students and observed several classes around the country. As summarized in the report, MMAP generally achieved its goal of engaging many different kinds of students in its mathematics units:

> "All data suggest that a broad range of students (regardless of gender, ethnicity, or ability level) were motivated by MMAP materials. Most students enjoyed the novelty of computer-based curriculum and the opportunity to work in groups. Furthermore, MMAP seems to motivate students who don't like math generally. Students' responses to surveys further indicated that MMAP activities effectively targeted students' ability levels. No significant differences were found between boys and girls along most variables studied, suggesting that MMAP activities engage each gender equally. In addition, no significant differences emerged between special education students and mainstream students along most variables studied. Yet, special education students tended to prefer groupwork and MMAP more often than mainstream students." (Lichtenstein, Weissglass, & Ercikan-Alper, 1998, p. iv)

At the same time, the interviews revealed that students did have difficulty recognizing that they were studying real math during MMAP units as they viewed them as too fun and as not looking or acting like the kinds of math activities they had done in the past, something that MMAP

struggled with throughout its history (ibid., p. 69; also Teacher OO interview 5/3/04). However, twice as many students preferred MMAP to textbooks (ibid.).

However, no measures of student learning were used in this evaluation of MMAP or any others because the evaluator did not feel it was appropriate to do so with MMAP as each teacher implemented the units differently (ibid.). Specifically, the problem was that because their curriculum was so "modifiable…we couldn't figure out what we thought was an intellectually honest way" to collect evidence of student learning "where we could say you can attribute this positive outcome to our curriculum" (Knudsen interview 5/2/04). As an illustration of this problem, inspired by another design-based research study, MMAP designed a sophisticated transfer task in one teacher's classroom in order to be able to conduct a pre/post study that would assess transfer across all of its classrooms using a given unit (Hall interview). However, when MMAP tried to use the same task in another teacher's classroom, it did not make sense to the students given how the teacher was implementing the unit and thus did not seem to be measuring the same concepts (Goldman personal communication 12/28/05). There were, however, reports of one study in a San Francisco classroom that showed that MMAP students made gains on standardized tests that were slightly better than those of similar students using traditional curricula (U.S. Department of Education Mathematics and Science Expert Panel, 1999, p. 54; Teacher OO interview 5/3/04).

Several former members of the project now wish that MMAP had spent more effort documenting and compiling evidence of student learning (Knudsen interview 5/2/04; Teacher OO interview 5/3/04). As one teacher speculated, "it didn't have that carry-over that I thought it would even though new teachers trying out the units were so excited about them.… I think I was amiss in not keeping records to show 'wait a minute, it does work.' " (Teacher OO interview 5/3/04; personal communication 10/17/06). For example, that teacher reported that when she began using MMAP so many of her students began passing her school's test for early entrance into algebra that her principal ended up raising the cut-off score because there was not enough funding for that many classes. Unfortunately, however, she had long since thrown this assessment data away, which she regrets now requires people to trust her that what happened did indeed happen. In the context in which MMAP originally began, however, there was little push for such evidence, and MMAP's commitment to teacher adaptation makes it very difficult to produce.

Impact of the Project on Teachers' Classroom Practices

Almost every MMAP teacher I interviewed talked about how using MMAP units and engaging with MMAP staff influenced the character of their teaching practices. For example, one teacher talked about how much she learned about keeping students productively engaged in groupwork by learning to ask off-task groups substantive questions about their work rather than directly telling them to get back to work (Teacher LE interview 3/2/03). As Goldman (2001) specified:

> "Teachers planned differently, set up new physical spaces in the classroom, and innovated ways to move students through activities, space, and time. They thought about new approaches to mathematics content and new ways to assess mathematics learning. Restructuring their classrooms became a never-ending job."

In addition, this pattern of results was consistent with teacher interviews and surveys in the evaluation report, which especially highlighted the impact of MMAP on increasing teachers' technological sophistication and use of progressive pedagogy (Lichtenstein et al., 1998, p. 57). Interestingly, a very similar pattern of results was found for Math Renaissance teachers who had

used MMAP units as for core MMAP teachers, which indicates that it is not necessary for teachers to have collaboratively designed MMAP units to have had MMAP make an impact on their teaching practices.

Research-Related Impacts

Nature of MMAP's Impact on the Research Literature

So far at least 50 publications have been produced from the MMAP project, including 1 book, 19 journal articles, 13 book chapters, 15 proceedings papers, and 2 dissertations. The project also produced numerous working papers and made presentations at various forums, including AERA (e.g., at least 62 of those, primarily drawn from 1992 to 1995).[28]

There have been four primary types of publications that the project has produced so far: theoretical articles, empirical studies, vision pieces, and how-to articles. Twenty-nine percent of MMAP's publications have been theoretical pieces in which new perspectives on learning or mathematics education were illustrated with examples from MMAP classrooms or curricula (e.g., Greeno & Hall, 1997; Greeno & MMAP, 1997, 1998; Moschkovich, 1998). Another 24% have consisted of empirical studies from MMAP classrooms, usually conducted using ethnographic and case study methods (e.g., Bushéy, 1997; Cole, 1995a, 1995b; Jurow, 2004; Moschkovich, 2002). A similar proportion of publications (22%) were what I call "vision pieces" in which one or more aspects of MMAP's vision for mathematics curriculum or instruction (e.g., its use of technology, methods of assessment, or embedding of mathematics within applications) were briefly presented in an engaging fashion, often in articles for practitioners or in conference proceedings for fellow researcher-designers (e.g., Goldman & Milanese, 1995; Goldman, Moschkovich, & MMAP, 1995; Goldman & Greeno, 1998). Finally, the last major class of publications (11%) completed by MMAP provided practical tips for practitioners about how to address various dilemmas around mathematics instruction (Cole, Coffey, & Goldman, 1999; Goldman, 2001; Goldman & Knudsen, 2004).

In order to characterize the topics that MMAP researchers wrote about, I coded the primary topic or topics address by each of the forty-five publications in my list. From this I found that more of MMAP's publications (33%) have focused on the implications of embedding mathematics within real-world applications than any other topic. A more moderate proportion of publications (16-20%) included extensive discussion of each of the following issues: rethinking assessment practices, addressing equity issues, and mediating mathematics learning through representations and other communicative practices. Finally, there was a smaller but still significant degree of treatment of the following topics, which each appeared in about 9-11% of MMAP publications: situativity theory, how the project organized its collaborations, students' learning of specific mathematics concepts, and students' learning of new problem solving and other mathematical practices. Another two or three publications (4-6%) included significant treatment of teaching practices or methodology.

To assess the impact of MMAP's publications on the research community, I looked at how frequently MMAP publications were cited in other publications and presentations. To determine that, I excluded the 15 conference proceedings papers, and compiled Google Scholar's citation counts for the 26 out of the 35 dissertations, books, journal articles, and book chapters that were listed in its database. The median MMAP publication has been cited three times, which I judge

[28] However, because MMAP and its participants did not consistently record these presentations, especially after 1995, I will not report any more specifics about them.

to be a respectable but not astounding figure. The mean citation rate, however, is much higher, a very healthy 17 citations per publication. However, the mean is skewed upward because one MMAP article (Greeno & MMAP, 1998) has been cited over two hundred times while five others have been cited more than twenty times (Goldman & Greeno, 1998; Greeno & Hall, 1997; Greeno & MMAP, 1997; Hall, 1996; Hall & Stevens, 1995). Most but not all of these six highly cited MMAP publications were focused on theoretical issues and addressed to research audiences.

In addition to affecting the literature, MMAP's ideas are having an impact on other researchers through direct contact that they have with former MMAP researchers. Basically, former MMAP researchers are taking their favorite ideas from MMAP and sharing them with their new collaborators, who often still find them novel and innovative (Goldman personal communication 12/29/05; Goldman/Knudsen interview 3/20/03; Hall interview; Knudsen interview 5/2/04; Moschkovich interview). In addition, MMAP's core researchers are regularly asked to consult with others who want to achieve similar goals with technology, with curriculum, or in collaborating with teachers, and the MMAP units and what was done during the project provide helpful resources for grounding such conversations (Goldman personal communication 12/29/05).

Overall, given all this, I think it is safe to conclude that MMAP has had a more than respectable impact on the research community, one that has been distributed across a fairly wide range of research topics and types of publications. And this does not take into account the fact that MMAP researchers are continuing to publish papers about the work they did on the project.

Impact of MMAP's Research on Other Practitioners

It is much more difficult to assess the impact of MMAP's research on practitioners besides the teachers with whom MMAP directly interacted around its research. The ways in which such an impact would likely be seen would not be through citation rates, but instead in changes in the choices that practitioners might make about what to do in schools and classrooms. What we can say is that MMAP did regularly write for practitioner audiences, making it possible that its research might have had a more general impact on practitioners. Overall, 24% of MMAP's publications were written for such outlets. Interestingly, despite the fact that MMAP's preferred mediator for scaling up was the classroom teacher, the project did not just target its publications for teachers, but instead wrote for venues like *Principal Magazine, New Horizons for Learning, Phi Delta Kappan,* and *Educational Leadership* that are targeted at principals and other educational leaders and policymakers.

The Role of MMAP in Transforming Biographies

Most importantly, MMAP lives on in very deep ways with the people who were part of its core community. With few exceptions, participation in MMAP was a life-changing, or at least career-changing experience for its core participants. In fact most of the people I talked with have made active efforts to apply MMAP's insights to the work that they have done since then in many different types of careers beyond just research and teaching. For example, after MMAP, one teacher began taking a leadership role in writing curriculum for his district, later seeking affiliations with another curriculum project to be able to do that (Teacher FJ interview). Another teacher who was inspired by MMAP's video-analysis sessions ended up adapting its methodology in his work with new teachers (Teacher TO interview). Similarly, inspired by MMAP's ideas, former researcher and curriculum developer Cole has used them to start a

business in which she helps parents learn to better support their school-aged children's learning in ways that keep them engaged more deeply (Cole interview). As part of that, she has created a website filled with helpful resources (see http://www.biglearning.com), sends out a regular online newsletter to a growing set of subscribers, and conducts workshops and other supporting enterprises (Cole personal communication). Berg has applied MMAP principles to interactive web-based activities at the San Jose Children's Discovery Museum (Goldman/Knudsen interview 3/20/03). The examples of former MMAP participants applying what they learned in the project to their current work could go on and on. It is this function of MMAP in having (in the words of Ray McDermott) "transformed the biographies" of its participants that I believe will ultimately be one of the deepest legacies of the project (McDermott interview).

X. Conclusions: Lessons Learned from MMAP about Reconfiguring the Usual Relationships Between Research and Practice

There are at least four important lessons that this case of design-based research provides for others interested in reconfiguring the usual relationships between research and practice. First and foremost, the MMAP case provides many ideas about methods that other projects might wish to use to support productive collaborations between researchers and practitioners. Second, the case of MMAP makes it clear how substantially a design-based research project's trajectory can be shaped by its funding, policy, and other external contexts. Third, the latter phases of the project provide important lessons about the costs and benefits of different strategies for attempting to bring an innovative design to scale. Finally, the project provides several useful methods for addressing the challenge of making progress on both research and reform within a design-based research effort. In the remaining subsections, I elaborate a bit on each of these lessons.

Methods for Supporting Productive Collaborations Between Researchers and Practitioners

Perhaps the most powerful set of lessons that MMAP provides is wisdom about how to organize teams that make use of the contributions of a diverse group of stakeholders. This is considered by many to be an ongoing challenge in educational research and development (e.g., Burkhardt & Schoenfeld, 2003; Kaestle, 1993; Lagemann, 1997; Pelligrino & Goldman, 2002). As was seen throughout the report, MMAP can be understood to have done this by embodying the principles of problematizing, authority, accountability, and resources in its own unique ways. In Table 1, I summarize how the project did that over its history.[29] For example, other projects might be able to make use of these and other strategies that MMAP found particularly helpful:

- Identifying activities relevant to joint work, but that will level the playing field of assumed expertise between participants;
- Mediating the contributions of especially dominant participants using other participants (as MMAP did with math-using professionals); and
- Encouraging participants to change their own practices in sought after directions while still not undercutting their authority by selectively but regularly asking them to account for why they are doing what they are doing (as MMAP did with teachers whose practices were not sufficiently consistent with key MMAP values).

In later projects, Goldman and Knudsen were able to work much more quickly and effectively to establish productive collaborative teams by directly applying what they had learned during MMAP. Specifically, they used these ideas in a subsequent project called PRIMES, which among other things involved fostering effective collaborations between parents and teachers around mathematics instruction (Goldman/Knudsen interview 3/20/03). In fact, they put out a booklet in which they specified several different models for effective workshops for parents and teachers to explore mathematics instruction together (ibid.). They have also informally consulted with a few other design-based research projects about what they had learned in this area (Goldman personal communication 12/28/05; Knudsen personal communication 12/22/05).

[29] Many thanks to Jennifer Knudsen (personal communication 12/29/05) for getting this table started for the comprehensive curriculum phase and to Brigid Barron for first encouraging me to create such tables for Engle & Conant (2002)

Table 1.

Summary of How MMAP Embodied the Engle & Conant (2002) Principles to Support Its Collaboration with Teachers

Problematizing	Authority	Accountability	Resources
Assembling Team and Conducting an Orientation			
• found people with diverse ideas and experiences • found people interested in MMAP goals and issues • introduced equity problem • shorthand as case of problem with math instruction	• found people inclined to respect authority of teachers *exception:* scientists • encouraged contributing by all during introduction and shorthand discussion • shorthand equalizes expertise	• found people with some shared values, providing basis for developing accountability • establishing norm of innovating to help students learn • encouraged participants to be responsive to others	• new staff and teachers aligned with principles • calculator, email, etc. • IRL as a place to meet, with supportive norms & interesting people
Designing Replacement Units			
• curriculum design relevant, open-ended problem for all • designing teachers' guide relevant, open-ended problem for all	• teachers given choices or input to agenda (agency) • teachers' ideas show up in next versions (contributors) • teachers treated as authorities about their work • valuing of distributed expertise • addressing domination of math-using professionals by mediating their participation thru teachers	• teachers account for how their unit ideas relate to what math-using professionals do and what "applications-based" units are • teachers asked to account for non-MMAP-like practices *but* • accountability selective to allow contextual specification	• time to negotiate ideas • money to buy time by paying substitutes • money to signal respect • informal tone and contexts to interact in • providing computers to support use of units • data from field-tests

Creating Comprehensive Curriculum

- coordinating curricula is a fairly constrained problem
- assessment design relevant, open-ended problem for all

units,

- long time staff lead subgroups in pieces of curriculum design
- authority around expertise developed through project or brought in for that purpose
- distribution of expertise across teams as well as individuals

- external accountability to NSF internalized

but

- project accountable for comprehensive curriculum

but teachers are not, and some staff have reservations

- lessons learned from earlier designs
- data from field-tests

but:

- fewer iterations for new

and no field-testing of full comprehensive curriculum

Scaling up Its Curriculum

- sometimes allowed sales demonstrations to turn into discussions of pro's and con's of the curriculum
- encouraged teachers to have authority over whether and how to use the curriculum
- completed curriculum

but

- software often crashed

The Powerful Influence of Funding, Policy, and Other External Contexts

Although to some extent MMAP was able to create a "third space" for participants that was relatively sheltered from various external contexts (Wenger, 1998), it is also clear from the project's history that sustaining such a sheltered environment is very difficult to do. Design-based research projects like MMAP and all projects that seek to reconfigure research and practice are always inextricably connected to a whole host of other contexts that cannot be ignored. Figure 1 provides one representation of many of the contexts that the MMAP project interacted with during its trajectory, with arrows used to indicate which contexts influenced which others.

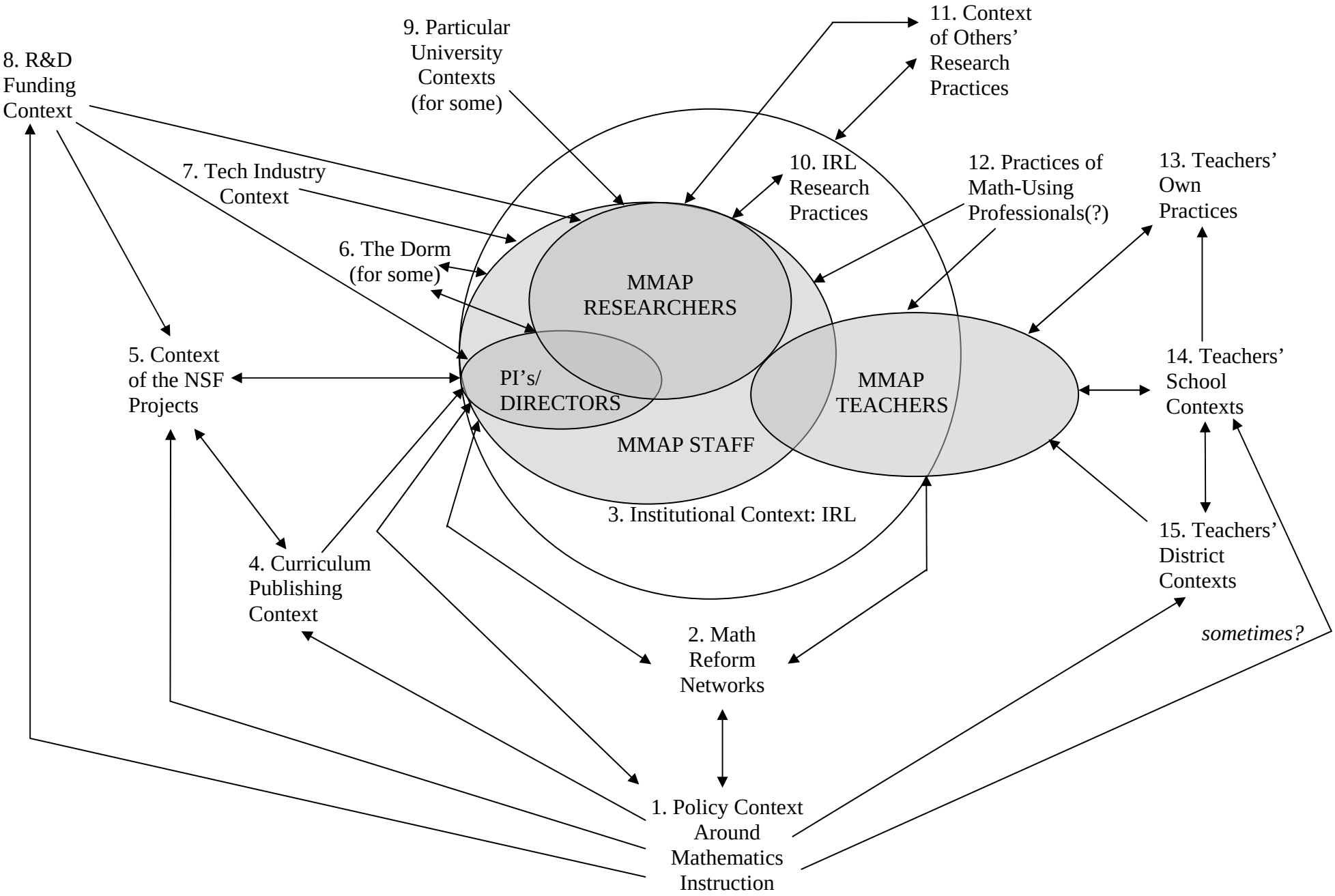
As I think my analysis makes clear, MMAP's funding and policy contexts were especially central to shaping its trajectory. MMAP's complex relationship with the National Science Foundation and the other curriculum projects that it became associated with through the NSF, shaped the project in many ways. It changed the kinds of products the project was responsible for providing, helped it learn things it might not have learned otherwise, and encouraged it to scale up its efforts. The effect of NSF on MMAP was also amplified by the fact that Goldman and other members of the project were well attuned to the fact that they were using valuable taxpayers' money during the project (Goldman personal communication 12/29/05). Therefore, Goldman often felt the project needed to do everything it had promised to NSF plus more (ibid.). So it is not just that NSF encouraged (or even pushed) the project in certain directions, but also that MMAP felt it had a duty to be especially responsive to NSF's encouragement.

Costs and Benefits of Different Strategies for Bringing an Innovative Design to Scale

One way external contexts had their impact on MMAP was in shaping the success of various strategies that the project or one of its partners developed for attempting to bring MMAP's curriculum to scale. MMAP's priorities for dimensions of scale to emphasize (Coburn, 2003) often did not correspond very closely with the priorities of its scaling-up partners or with the values increasingly institutionalized in the larger policy context. For example, MMAP's primary desire for teacher ownership of its curriculum became moot once teachers no longer had decision-making authority for which mathematics curricula to use. Similarly, MMAP's desire that teachers deeply implement its curriculum by adapting it to their local contexts became less and less viable as teachers were required to use mandated curricula in increasingly standardized ways. Even MMAP's desire to have its curriculum be spread widely among those classrooms in which teachers viewed it as being best suited was no longer relevant. What united all of these aspects of MMAP's scaling-up priorities was a belief that teachers should have the authority about whether and how MMAP's curriculum should be used. This worked fine in the early 1990's and within the core design team as then the project had the power to make sure that teachers were treated as authorities. However, in the larger schooling environment MMAP could not control trends that moved the authority for making curricular decisions further and further away from teachers. That MMAP did everything it could to continue treating teachers as authorities no longer was sufficient for making them function that way in the environments in which they worked, and in which MMAP's curriculum was competing.

Another aspect of bringing an innovative design to scale that this case raises is what kind of organization is best suited for navigating this process. As I argued, MMAP's generally successful ways of interacting with new teachers during the collaborative design process might have been counterproductive when these same ways of interacting were used in forums in which

Figure 1. *MMAP participants and how they interacted with various contexts while the project was at IRL*



projects were expected to sell themselves. This raises the possibility that, just like high tech start up companies which need different organizational structures to manage the challenges of developing their products and getting them to early adopters versus selling them more broadly (Moore, 2002), I will suggest that the demands of scaling up an ambitious educational intervention like MMAP are likely to require a different kind of organization than is required for designing and testing it. The key open question is what kind of organization and strategy are needed for this later phase in which the goal is to, in the words of Moore (2002), "cross the chasm" from just having a few early adopters using an innovation to having it be something in much more mainstream use. This case suggests several challenges involved in this process.[30]

Methods for Supporting Progress on Both Research and Reform

Finally, the MMAP case provides some strategies projects can use to help keep research going while being intensively involved in reform activities. First, the project made good use of tasks that helped to simultaneously advance both research and reform goals like curriculum design and the design of teachers' guides that did not just serve as practitioner documents but also helped researchers to better understand what was involved in embedded mathematics in applications and how teachers learned how to teach in this manner. In addition, the project found that it was helpful to have dedicated personnel, funding, and time to support research activities so that progress would keep being made despite the lure and day-by-day accountability of working responsibly with practitioners. Still, making progress on research and reform simultaneously is something that MMAP needed to continually work on, a characteristic I claim is common to design-based research projects in which both research and reform are pursued simultaneously.

[30] Interestingly, being in Silicon Valley, MMAP's directors were aware of, and were trying to learn from Moore's work when it first came out in the early 90's, but did not have anyone on their team from a start-up marketing background who might have been able to help them to implement any of these ideas (Knudsen personal communication 12/29/05).

XI. References

Brown, A. L., Ash, D., Rutherford, M., Nakagawa, K., Gordon, A., & Campione, J. C. (1993). Distributed expertise in the classroom. In G. Salomon (Ed.), *Distributed cognitions: Psychological and educational considerations* (pp. 188–228). New York: Cambridge University Press.

Brown, J. S., Collins, A., & Duguid, P. (1989). Situated cognition and the culture of learning. *Educational Researcher, 18*(1), 32-41.

Burkhardt, H. & Schoenfeld, A. J. (2003). Improving educational research: Toward a more useful, more influential, and better-funded enterprise. *Educational Researcher, 32*(9), 3-14.

Bushéy, B. J. (1997). *Student reflection in emergent mathematical activity.* Dissertation, Stanford University School of Education.

California Department of Education (1985). *Mathematics framework for California public schools, K-12.* Sacramento, CA: Author.

Clancey, W. J. (1991). Situated cognition: Stepping out of representational flatland. *AI Communications—The European Journal on Artificial Intelligence 4*(2/3), 109-112.

Clancey, W. J. (1992). Overview of the Institute for Research on Learning. In *Proceedings of CHI'92* (pp. 571-572), May 3-7, 1992.

Coburn, C. E. (2003). Rethinking scale: Moving beyond numbers to deep and lasting change. *Educational Researcher, 32*(6), 3-12.

Cole, K. (1994, April). The mouse trap: Alternatives for judging and encouraging equity in computer based group work. Paper presented at the annual meeting of the American Educational Research Association, New Orleans, LA.

Cole, K. (1995a). Equity issues in computer-based collaboration: Looking beyond surface indicators. In J. L. Schnase & E. L. Cunnius (Eds.), *Proceedings of CSCL '95: The First International Conference on Computer Support for Collaborative Learning.* Mahwah, NJ: Erlbaum.

Cole, K. A. (1995b). *Structuring academic engagement in classrooms.* Dissertation, Stanford University School of Education.

Cole, K. (1999). Walking around: Getting more from informal assessment. *Mathematics Teaching in the Middle School, 4*(4), 224-227.

Cole, K. A., Coffey, J., & Goldman, S. (1999). Using assessments to improve equity in mathematics. *Educational Leadership, 56*(6), 56-58.

Collins, A., Bielaczyc, K., & Joseph, D. (2001, January). Design experiments: Theoretical and methodological issues. Presentation at the conference, Commemorating the Contributions of Ann Brown to Children's Learning. Berkeley, CA.

Collins, A., Joseph, D., & Bielaczyc, K. (2004). Design research: Theoretical and methodological issues. *Journal of the Learning Sciences, 13* (1), 15-42.

D'Amico, L. (2005, December). *The Center for Learning Technologies in Urban Schools: A case of design-based research in education.* Technical report for the MacArthur and Spencer Foundation Meta-study on projects that sought to reconfigure the usual relationships between research and practice.

Engle, R. A. (2004, April). Revisiting previous discussions to deepen teachers' engagement with mathematics and pedagogy. In M. S. Smith (Chair), *Developing a knowledge base for*

teaching: Learning content and pedagogy in a course on patterns and functions. Poster symposium at the annual meeting of the American Educational Research Association, San Diego, CA.

Engle, R. A. & Conant, F. R. (2002). Guiding principles for fostering productive disciplinary engagement: Explaining an emergent argument in a community of learners classroom. *Cognition and Instruction, 20*(40), 399-483.

Engle, R. A. & Faux, R. B. (2006). Towards productive disciplinary engagement of prospective teachers in educational psychology: Comparing two methods of case-based instruction. *Teaching Educational Psychology, 1*(2).

Goldman, S. R. (2001). Technology in the classroom: Guidelines from the field. *ERIC/IT Update, 22*(2). http://ericit.org/newsletter/Volume22-2/goldman.shtml.

Goldman, S. & Greeno, J. G. (1998). Thinking practices: Images of thinking and learning in education. In J. G. Greeno & S. Goldman (Eds.), *Thinking Practices in Mathematics and Science Education* (pp. 1-16). Hillsdale, NJ: Erlbaum.

Goldman, S. & Knudsen, J. (2000). Learning sciences research and wide-spread school change: Issues from the field. *Proceedings of ICLS 2000*.

Goldman, S. & Knudsen, J. (2004). Principles for making middle school math more equitable. *Classroom Leadership, 7*(6).

Goldman, S., & McDermott, R. (in press). Staying the course with video analysis, In Goldman, R., Pea, R., Barron, B., & Derry, S. (Eds.), *Video Research in the Learning Sciences*. Hillsdale, NJ: Lawrence Erlbaum.

Goldman, S. & Milanese, M. (1995, May). Middle school math: A new approach. *Principal Magazine*, 39-40.

Goldman, S., Moschkovich, J. N., and the MMAP Project Team (1995). Environments for collaborating mathematically. In J. L. Schnase & E. L. Cunnius (Eds.), *Proceedings of CSCL '95: The First International Conference on Computer Support for Collaborative Learning.* Mahwah, NJ: Erlbaum.

Greeno, J. G. (1989). A perspective on thinking. *American Psychologist, 44,* 134–141.

Greeno, J. G. & Goldman, S. R. (Eds.), (1998). *Thinking practices in mathematics and science learning.* Mahwah, NJ: Erlbaum.

Greeno, J. G. & Hall, R. P. (1997). Practicing representation: Learning with and about representational forms. *Phi Delta Kappan, 78*, 361-367.

Greeno, J. G., McDermott, R., Cole, K., Engle, R. A., Goldman, S., Knudsen, J., Lauman, B., & Linde, C. (1999). Research, reform, and aims in education: Modes of action in search of each other. In E. Lagemann & L. Shulman (Eds.), *Issues in education research: Problems and possibilities* (pp. 299-335). San Francisco: Jossey-Bass.

Greeno, J. G. & the Middle-school Mathematics through Applications Project (1997). Theories and practices of thinking and learning to think. *American Journal of Education, 106*, 85-126.

Greeno, J. G. & the Middle-school Mathematics through Applications Project (1998). The situativity of knowing, learning, and research. *American Psychologist, 53,* 5-26.

Hall, R. (1995). Exploring design-oriented mathematical practices in school and work settings. *Communications of the ACM, 62.*

Hall, R. (1996). Representation as shared activity: Situated cognition and Dewey's cartography of experience. *Journal of the Learning Sciences, 5*(3), 209–238.

Hall, R. (1998). Following mathematical practices in design-oriented work. In C. Hoyles, C. Morgan, & G. Woodhouse (Eds.), *Rethinking the Mathematics Curriculum, Volume 10, Studies in Mathematics Education Series* (pp. 29-47). London: Falmer Press.

Hall, R. P., Knudsen, J., & Greeno, J. G. (1995/1996). A case study of systemic aspects of assessment technologies. *Educational Assessment, 3,* 315-361.

Hall, R., and Stevens, R. (1995). Making space: A comparison of mathematical work in school and professional design practices. In S. L. Star (Ed.), *The Cultures of Computing,* (pp. 118-145). London: Basil Blackwell.

Huberman, M. (1999). The mind is its own place: The influence of sustained interactivity with practitioners on educational researchers. *Harvard Educational Review, 69*(3), 289-319.

Jurow, A. S. (2004). Generalizing in interaction: Middle school mathematics students making mathematical generalizations in a population-modeling project. *Mind, Culture, and Activity, 11*(4), 279-300.

Kaestle, C. (1993). The awful reputation of education research. *Educational Researcher, 22*(1), 23-31.

Labov, W. (1972). The transformation of experience in narrative syntax. In W. Labov, *Language in the inner city* (pp. 354-396). Philadelphia: University of Pennsylvania Press.

Lagemann, E. C. (1997). Contested terrain: A history of educational research in the United States, 1890-1990. *Educational Researcher, 26*(9), 5-17.

Lave, J. & Wenger, E. (1991). *Communities of practice: Legitimate peripheral participation.*

Lichtenstein, G., Weissglass, J., & Ercikan-Alper, K. (1998). *Final evaluation report: Middle-school Mathematics through Applications Project, MMAP II (1994-1998).* Denver, CO: Quality Evaluation Designs.

Linde, C. (1993). *Life stories: The creation of coherence.* New York: Oxford University Press.

Linn, M. C., Kessel, C., Lee, K., Levenson, J., Spitulnik, M., & Slotta, J. D. (2000). *Teaching and learning K-8 mathematics and science through inquiry: Program reviews and recommendations.* Santa Monica, CA: Metiri Group.

McKnight, C., Crosswhite, J., Dossey, J., Kifer, E., Swafford, J., Travers, K., et al. (1987). *The underachieving curriculum: Assessing U. S. mathematics from an international perspective.* Champaign, IL: Stipes.

Moore, G. A. (2002). *Crossing the chasm: Marketing and selling disruptive products to mainstream customers.* Revised Edition. New York: Harper Collins.

Moschkovich, J. N. (1996). Learning mathematics in two languages. *Proceedings of the Twentieth Annual Meeting of the International Group for the Psychology of Mathematics Education*, Spain.

Moschkovich, J. N. (1998). Rethinking authentic assessments of students' mathematical activity. *Focus on Learning Problems in Mathematics, 20*(4), 1-18.

Moschkovich, J. N. (2000). Learning mathematics in two languages: Moving from obstacles to resources. In W. Secada (Ed.), *Changing the Faces of Mathematics (Vol. 1): Perspectives on multiculturalism and gender equity* (pp. 85-93). Reston, VA: NCTM.

Moschkovich, J. N. (2002). Bringing together workplace and school mathematical practices in the classroom. In M. Brenner & J. Moschkovich (Eds.), *Everyday and academic mathematics: Implications for the classroom. Journal for Research in Mathematics Education,* Monograph Number 11, 93-110.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.

Pellegrino, J. W. & Goldman, S. R. (2002). Be careful what you wish for—you may get it: Educational research in the spotlight. *Educational Researcher, 31*(8), 15-17.

Schein, E. H. (1996). Organizational learning: What is new? Invited address to the Third Biennial International Conference on Advances in Management, Framington, MA, June 28, 1996. Also appears as working paper # 3912, July 1996, MIT Sloan School of Management.

Schoenfeld, A. H. (2004). The math wars. *Educational Policy, 18*(1), 253-286.

Schwartz, D. L. & Bransford, J. D. (1998). A time for telling. *Cognition and Instruction, 16*(4), 475-522.

Stein, M. K. & Coburn, C. E. (2003, August). Toward producing usable knowledge for the improvement of educational practice: A conceptual framework. Paper presented at the biannual meeting of the European Association for Research on Learning and Instruction, Padova, Italy, August 26, 2003.

Tyack, D. & Tobin, W. (1994). The "grammar" of schooling: Why has it been so hard to change? *American Educational Research Journal,* 31(3), 453-479.

U.S. Department of Education Mathematics and Science Expert Panel (1999). *Exemplary and promising mathematics programs.* Washington, CD: Author.

Wenger, E. (1998). *Communities of practice: Learning, meaning, and identity*. Cambridge, UK: Cambridge University Press.

Wilson, S. (2003). *California dreaming: Reforming mathematics education.* New Haven, CT: Yale University Press.

XII. Appendix 1
Specifics About Interviews

1. Teacher Interviews

Who	Interview Date	When Joined MMAP	Teaching Experience	School Type(s)	Other Paid Roles on MMAP
FJ*	05/05/04	1st cohort '92	Small	Urban/Suburban	Writer, Presenter
TO	05/18/04	1st cohort '92	Moderate	Urban	Prof. developer, Writer
OO*	05/03/04	1st cohort '92	Extensive	Urban	Presenter
EH	05/04/04	2nd cohort '93	Small	Urban/Suburban	Writer, Presenter
TJ	10/26/04	2nd cohort '93	Moderate	Suburban	Materials Organizer
LE*	03/02/03	2nd cohort '93	Extensive	Urban	
CA*	05/03/04	2nd cohort '93	Extensive	Suburban	

All interviewed teachers were with the project until the end of its teacher component in 1998, except LE who stopped participating a year or two earlier. EH, OO, TO, and TJ continued participating past 1998 as staff or paid consultants. OO and TO also read a draft and commented on it October 2006.

* For these teachers, I also have rough transcripts of interviews the project conducted with them in 1994 for MMAP's paper about how it coordinated research with reform (Greeno et al., 1999) for which I served as a research assistant.

2. Interviews with Other Staff (not including project leads: see #4 on next page for them)

Who	Interview Date	When in MMAP	Background	Roles on Project In Addition to Curriculum Design
Rick Berg	03/20/03	1992-2000	Cognitive Development Research	Research, Writing, Software Development
Karen Cole	05/05/05	1991-2001	Technology Curriculum Design & Educational Psychology Research	Research, Writing, Assessment Coordinator
Larry Gallagher	10/26/04	1992-1995	Software Engineering	Software Development
Jim Greeno*	03/24/03	1990-1998	Psychological & Educational Research	Co-PI, Research
Rogers Hall	06/23/04	1990-1996	Artificial Intelligence & Educational Research	Research, Writing
Ray McDermott	10/27/04	1990-1998	Anthropological & Educational Research	Co-PI, Research
Marisa Milanese	08/12/04	1993-1994	Writing	Writing
Judit Moschkovich	10/28/04	1993-1998	Mathematics Education Research	Research, Assessment Coordinator

I checked quotes with all of these people in 2005-2006. In addition, Gallagher, Greeno, McDermott, and Moschkovich commented on a draft in October 2006.

* I also had two brief follow-up conversations with Greeno on 5/19/05 and 12/29/05 that are listed as personal communications in the text.

3. Other Interviews

Who	Date	Relationship to Project
George Pake	10/14/96 by MMAP	Co-PI, First Director of IRL, Content Consultant on MMAP
Judith Mumme	04/07/04	Director of Math Renaissance While MMAP Was Being Used
Anonymous	05/09/06	Former NSF Program Officer Working with MMAP

In addition, Mumme checked quotes and commented on the section about the history of MMAP's involvement with Math Renaissance in May 2006.

4. Interviews and Personal Communications with Project Leads
Shelley Goldman (Principal Investigator) and Jennifer Knudsen (Project Manager)

Who	Date	Format	Key Topic(s)
Goldman & Knudsen	03/20/03	Interview	Project history and general orientation
Knudsen	05/02/04	Interview	Sharing curriculum via publishing, Math Renaissance, and NSF Implementation Center; Hiring of writers and programmers; Refinement of "applications-based"
Knudsen	05/05/04	Interview	Changes in Antarctica curriculum over time; Description of documents being shared
Goldman, Knudsen, McDermott	10/27/04	Interview	Wide ranging interview to fill in various gaps (McDermott soon leaves for appointment); Description of videos being shared (only Goldman)
Goldman, Knudsen	07/25/05, 07/30/05	Email	Reactions to draft MMAP context diagram
Goldman	08/12/04	Email	Answering question about MMAP orientation
Knudsen	08/30/04	Email	Jennifer's background and joining of MMAP
Goldman	08/31/04	Email	Work on planning grant
Knudsen	11/13/05	Email	Reactions to draft of 1990-1995
Knudsen	11/14/05	Phone	Answering questions on draft reactions & 1995-1998
Goldman	12/10/05	Email	Reactions to draft of 1990-1995
Goldman	12/22/05	Email	Reactions to draft of 1995-2002
Goldman	12/28/05	Email	Responses to my questions about draft reactions
Knudsen	12/28/05	Phone	Answering remaining questions about 1995-2002
Knudsen	12/29/05	Email	Additional comments on draft of 1995-2002
Goldman	12/29/05	Phone	Answering remaining questions
Goldman	01/04/06	Email	Reactions to draft about research on project
Goldman	5/12/06	In Person	Recorded comments about Greeno's role in the project while preparing Festschrift presentation

XIII. Appendix 2

MMAP Project Curriculum Vita

Sources:

- Researchers' CV's: Cole, Goldman, Gallagher, Greeno, Hall, Knudsen, McDermott, and Moschkovich
- http://mmap.wested.org/research/ResearchArticles.html, which appears to have been written in 1996 or 1997
- References to research products during interviews
- Google Scholar and Web of Science searches

This CV systematically under-represents presentations in later years as they were often not included on participants' CV's and the website that did list them was current only until about 1996 or 1997. I am undoubtedly also missing several papers that have been submitted in the last few years.

 # = at least one author was university-affiliated
% = at least one author is a MMAP teacher or former MMAP teacher

1 Book

Greeno, J. G. & Goldman, S. R. (Eds.), (1998). *Thinking practices in mathematics and science learning.* Mahwah, NJ: Erlbaum.

19 Articles

Cole, K. (1999). Walking around: Getting more from informal assessment. *Mathematics Teaching in the Middle School 4*(4): 224-227.

Cole, K. A., Coffey, J., & Goldman, S. (1999). Using assessments to improve equity in mathematics. *Educational Leadership 56*(6): 56-58.

Eckert, P., Goldman, S., & Wenger, E. (1997, Summer). The school as a community of engaged learners. *Wingspread Journal*. Special issue on learning productivity.

Goldman, S. (2001). Technology in the mathematics classroom: Guidelines from the field. *ERIC Update*, *22* (2). Available at http://ericit.org/newsletter/Volume22-2/goldman.shtml.

Goldman, S. (2004). Project-based learning and teaching—Engaging students, energizing teachers, involving parents. *TechScape Voices. 1*(1).

Goldman, S. & Knudsen, J. (2004). Principles for making middle school math more equitable. *Classroom Leadership, 7*(6). Available at: http://www.ascd.org/publications/class_lead/200403/goldman.html.

Goldman, S. & Milanese, M. (1995, May). Middle school math: A new approach. *Principal*, 39-40.

Greeno, J. G. & Hall, R. P. (1997). Practicing representation: Learning with and about representational forms. *Phi Delta Kappan, 78,* 361-367.

Greeno, J. G. & the MMAP Group (1997). Theories and practices of thinking and learning to think. *American Journal of Education*, *106*: 85-126.

Greeno, J. G. & the Middle-school Mathematics through Applications Project Group (1998). The situativity of knowing, learning, and research. *American Psychologist, 53,* 5-26.

Hall, R. (1996). Representation as shared activity: Situated cognition and Dewey's cartography of experience. *Journal of the Learning Sciences, 5*(3), 209-238.

Hall, R. P., Knudsen, J., & Greeno, J. G. (1995/1996). A case study of systemic aspects of assessment technologies. *Educational Assessment, 3,* 315-361.

Hall, R. (1995, September). Exploring design-oriented mathematical practices in school and work settings. *Communications of the ACM*, 62.

Jurow, A. S. (2005). Shifting engagements in figured worlds: Middle school mathematics students' participation in an architectural design project. *The Journal of the Learning Sciences, 14*(1), 35-67.

Jurow, A. S. (2004). Generalizing in interaction: Middle school mathematics students making mathematical generalizations in a population-modeling project. *Mind, Culture, and Activity, 11*(4), 279-300.

Kahn, T. & Taber Ullah, L. K. (1997). Learning by design: Integrating technology into the curriculum through student multimedia design projects. *New Horizons for Learning.*

Moschkovich, J. N. (1998). Rethinking authentic assessments of students' mathematical activity. *Focus on Learning Problems in Mathematics 20*(4), 1-18.

Moschkovich, J. (2002a). An introduction to examining everyday and academic mathematical practices. In M. Brenner & J. Moschkovich (Eds.), *Everyday and academic mathematics: Implications for the classroom. Journal for Research in Mathematics Education*, Monograph Number 11, 1-11.

Moschkovich, J. N. (2002b). Bringing together workplace and school mathematical practices in the classroom. In M. Brenner & J. Moschkovich (Eds.), *Everyday and academic mathematics: Implications for the classroom. Journal for Research in Mathematics Education*, Monograph Number 11, 93-110.

13 Book Chapters

#Goldman, S. (2002). Instructional design: Learning through design. In J. Guthrie (Ed.), *Encyclopedia of Education* (pp. 1163-1169). Second Edition. New York: Macmillan.

Goldman, S. & Mathison, A. (in press). Bringing the Internet to your classroom? Read me first. In M. Furlong (Ed.), *We teach in cyberspace*. Wilsonville, OR: Franklin, Beedle Associates.

Goldman, S., Knudsen J., & Latvala, M. (1998). Engaging middle schoolers in and through real-world mathematics. In L. Leutzinger (Ed.), *Mathematics in the middle* (pp. 129-140). Reston, VA: National Council of Teachers of Mathematics.

Goldman, S. & Greeno, J. G. (1998). Thinking practices: Images of thinking and learning in education. In J. G. Greeno and S. Goldman (Eds.), *Thinking practices in mathematics and science education* (pp. 1-16). Hillsdale, NJ: Lawrence Erlbaum Associates.

Goldman, S. & McDermott, R. (in press). Staying the course with video analysis. In Goldman, R., Pea, R., Barron, B., & Derry, S. (Eds.), *Video research in the learning sciences.* Hillsdale, NJ: Erlbaum.

Greeno, J. G. (in press). A situated perspective on cognition and learning in interaction. In T. Koschmann (Ed.), *Theorizing learning practice.* Hillsdale, NJ: Erlbaum.

Greeno, J. G., McDermott, R., Cole, K., Engle, R. A., et. al. (1999). Research, reform, and aims in education: Modes of action in search of each other. In E. C. Lagemann & L. S. Shulman (Eds.), *Issues in education research: Problems and possibilities* (pp. 299-335). San Francisco: Jossey-Bass.

Hall, R. (1998). Following mathematical practices in design-oriented work. In C. Hoyles, C. Morgan, & G. Woodhouse (Eds.), *Rethinking the Mathematics Curriculum, Volume 10, Studies in Mathematics Education Series* (pp. 29-47). London: Falmer Press.

Hall, R. (2000). Video recording as theory. In A. Kelley & R. Lesh (Eds.), *Handbook of Research Design in Mathematics and Science Education* (pp. 647-664). Mahwah, NJ: Lawrence Erlbaum.

Hall, R. & Stevens, R. (1995). Making space: A comparison of mathematical work in school and professional design practices. In S. L. Star (Ed.), *The cultures of computing*, (pp. 118-145). London: Basil Blackwell.

McDermott, R. P. (in press). Does "learning" exist? In T. Koschmann (Ed.), *Theorizing learning practice.* Hillsdale, NJ: Erlbaum.

Moschkovich, J. N. (2000) Learning mathematics in two languages: Moving from obstacles to resources. In W. Secada (Ed.), *Changing the Faces of Mathematics (Vol. 1): Perspectives on multiculturalism and gender equity* (pp. 85-93). Reston, VA: NCTM.

Stenning, K., Greeno, J. G., Hall, R., Sommerfeld, M., & Wiebe, M. (2002). Coordinating mathematical with biological multiplication: Conceptual learning as the development of heterogeneous reasoning systems. In M. Baker, P. Brna, K. Stenning, & A. Tiberghien, (Eds.), *The role of communication in learning to model* (pp. 3-48). Mahwah, NJ: Lawrence Erlbaum Associates.

2 Dissertations

Bushéy, B. J. (1997). *Student reflection in emergent mathematical activity*. Doctoral dissertation, Stanford University.

Cole, K. (1995b). *Structuring academic engagement in classrooms*. Doctoral dissertation, Stanford University.

8 White Papers, Working Papers, or Technical Reports

Berg, R. (date unknown). *The creative side took over there: The social construction of creativity in math classrooms.* MMAP working paper.

Cole, K. (1996). *How do I know they are learning? Designing assessment systems for project-based curriculum* . Menlo Park, CA: Institute for Research on Learning.

Crawford, W. S. (date unknown). *Transforming the third "R": Teaching middle school mathematics through applications*. MMAP Working Paper.

% Habecker, D. (~1996). *Using journals in math class*. MMAP working paper.

Knudsen, J. (1995). *What MMAP teachers do: Dimensions of reform-oriented mathematics teaching*. MMAP working paper.

Moschkovich, J. N. (1996). *Rethinking authentic assessments of students' mathematical activity.* Menlo Park, CA: Institute for Research on Learning.

% Strong, S. L. (date unknown). *Bridging the MMAP: A needs assessment of the outreach component for the Middle-school Mathematics Through Applications Project*. MMAP working paper.

Syer, T. A. (date unknown). *Classroom with 18 Power Macs: Blessing or Chaos?* Working paper.

At Least 16 Conference Proceedings Papers (many may be missing after 1996)

#Cole, K. (1995). Equity issues in computer-based collaboration: Looking beyond surface indicators. In J. L. Schnase & E.L. Cunnius (Eds.), *Proceedings of CSCL '95: The First International Conference on Computer Support for Collaborative Learning*. Mahwah, NJ: Erlbaum.

Cole, K. (1999). Greasing the wheels: Powerful assessment in a technology-rich curriculum. In D. A. Thomas (Ed.), *The International Conference on Mathematics/Science Education and Technology: Proceedings of M/Set 99*. Charlottesville, VA, Association for the Advancement of Computing in Education.

Cole, K. A. (2000). Technology and beyond: Project-based teacher partnerships. *Proceedings of SITE 2000*. San Diego.

Cole, K. & Syer, C. (1999). From research to teacher professional development to technology and back again: The development of a video exploration of classroom assessment. In B. Collis & R. Oliver (Eds.), *ED-Media: The World Conference on Educational Multimedia, Hypermedia & Educational Telecommunications. Volume 2* (pp. 1105-1110). Seattle, WA: Association for the Advancement of Computing in Education.

Goldman, S., Cole, K., & Syer, C. (1999). The technology/content dilemma. In *Online proceedings of the Secretary's Conference on Educational Technology*. Washington, D.C.: Department of Education.

Goldman, S. & Knudsen, J. (2000). Learning sciences research and wide-spread school change: Issues from the field. *Proceedings of ICLS 2000*.

% Goldman, S., Knudsen, J., & Muniz, M. (1995). *When promise outweighs problems: Technology integration in math classrooms*. Available at: http://www.ndex.sesp.nwu.edu/ndec/ProjectPages/mmap/mmap.html.

Goldman, S. & Moschkovich, J. (1998). Technology environments for middle school: Embedding mathematical activity in design projects. In J. Bruckman et. al, (Eds.), *Proceedings of the ICLS 98*: *International Conference of the Learning Sciences* (pp. 112-117). Atlanta: Georgia Tech University.

Goldman, S., Moschkovich, J. N., & the MMAP Project Team (1995). Environments for collaborating mathematically. In J. L. Schnase & E. L. Cunnius (Eds.), *Proceedings of CSCL '95: The First International Conference on Computer Support for Collaborative Learning.* Mahwah, NJ: Lawrence Erlbaum Associates, Inc.

Greeno, J. G. & the Middle-school Mathematics through Applications Group (1997). Participation as fundamental in mathematics education. In J. A. Dossey, J. O. Swafford, M. Parmantie, & A. E. Dossey (Eds.), *Proceedings of the Nineteenth Annual Meeting, North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 1-15). Columbus, OH: ERIC Clearinghouse for Science, Mathematics, and Environmental Education.

Hall, R. (1994). Visual reasoning in instruction. within N. H. Narayanan (Chair), Symposium on visual reasoning

in discovery, instruction and problem solving. In A. Ram & K. Eiselt (Eds.), *Proceedings of the Sixteenth Annual Conference of the Cognitive Science Society* (pp. 980-984). Hillsdale, NJ: Erlbaum.

Hall, R. (1995). Realism(s) for learning algebra. In C. B. LaCampagne, W. Blair, & J. Kaput (Eds.), *The Algebra Initiative Colloquium* (Vol. 2, pp. 33-51). Washington, D.C.: U.S. Department of Education, Office of Educational Research and Improvement.

Hall, R. & Stevens, R. (1996). Teaching/learning events in the workplace: A comparative analysis of their organizational and interactional structure. In *Proceedings of the Eighteenth Annual Conference of the Cognitive Science Society*. Hillsdale, NJ: Lawrence Erlbaum Associates.

Moschkovich, J. N. (1994). Assessing students' mathematical activity in the context of designing projects: Defining "authentic" assessment practices. In D. Kirshner (Ed.), *Proceedings of the Sixteenth Conference of the Cognitive Science Society*. Baton Rouge, LA: University of Louisiana.

Moschkovich, J. N. (1996). Learning mathematics in two languages. *Proceedings of the Twentieth Annual Meeting of the International Group for the Psychology of Mathematics Education*, Spain.

Moschkovich, J. N. (1997). Mathematical practices in insurance agents' work. *Proceedings of the 19th Annual Meeting of Psychology for Mathematics Education, North American chapter* (pp. 143–149). Columbus, OH: ERIC Clearinghouse for Mathematics, Science, and Environmental Education.

At Least 33 Other Conference Presentations (missing most after 1996 as not in CVs)

Berg, R., Chiu, M., & Hall, R. (1994, April). Interactive construction of models in middle school mathematics design projects. Paper presented at the American Educational Research Association annual meeting, New Orleans, LA.

Berg, R. & Goldman, S. (1996, April). Why design activities involve middle schoolers in learning mathematics. In *Learning through design: Contextualizing inquiry for science and mathematics*. Symposium presented at the annual meeting of the American Educational Research Association, New York, NY.

Bushéy, B. J. & Greeno, J. G. (1994, April). Versions of mathematics: Multiple epistemologies in the classroom. Paper presented at the American Educational Research Association annual meeting, New Orleans, LA.

Cole, K. (1994, April). The mouse trap: Alternatives for judging and encouraging equity in computer based group work. Paper presented at the American Educational Research Association annual meeting, New Orleans, LA.

Cole, K. (1996). Structuring engagement in middle-school mathematics classrooms. Paper presented at the annual meeting of the American Educational Research Association, New York, NY.

Cole, K. (1998). Socialization and engagement: Social relationships with academic content. Paper presented at the annual meeting of the Pacific Sociological Association, San Francisco, CA.

Cole, K. A. (2000). Issues in developing classroom assessments as part of a design experiment. Paper presented at the annual meeting of the American Educational Research Association, New Orleans, LA.

Goldman, S., Knudsen, J., & Gallagher, L. (1993, April). An applications approach to math learning: The middle-school mathematics through applications project. Paper presented at the National Council of Teachers of Mathematics Conference, Seattle, WA.

Goldman, S. & McDermott, R. (1994, April). Is it math yet? Kids' communication and math in framework-inspired classrooms. Paper presented at the American Educational Research Association annual meeting, New Orleans, LA.

Greeno, J., Hall, R., & Knudsen, J. (1992, April). Participatory design of assessment systems. Paper presented at the American Educational Research Association annual meeting.

Greeno, J. (1992, April). Generality in the specificities of situations: Understanding of concepts viewed as attunement to families of constraints. Paper presented at the American Educational Research Association.

Greeno, J. (1992, June). The situation in cognitive theory. Paper presented at the American Psychological Society.

Greeno, J. (1993, June). Understanding concepts in activity. Paper presented at the Conference on Discourse Comprehension (in honor of Walter Kintsch), Boulder, CO.

Greeno, J. (1993, May). A theory of concepts and understanding. Paper presented at the Society of Experimental Psychologists, Seattle, WA.

Greeno, J. (1994, August). The situativity of learning: Prospects for syntheses in theory and practice. Paper presented at the American Psychological Association, Los Angeles, CA.

Greeno, J. (1994, July). Reasoning and understanding as social practices. Paper presented at the American Psychological Society symposium, Washington, D.C.

Goldman, S. (1996, April). Using video to support case-based professional development. In *Designing innovative video for teachers' professional development*. Symposium presented at the American Educational Research Association Meeting, New York, NY.

% Goldman, S. & Manak, R. (1996, April). Participatory research as a model for designing classroom assessments. In *Researching reform in math classrooms: The Middle-school Mathematics through Applications Project.* Symposium presented at the annual meeting of the American Educational Research Association, New York, NY.

Greeno, J. & Hall, R. (1992, February). Participatory design of assessment systems. Paper presented at the Inaugural Conference of the Center for the Study of Education and Child Development in a Diverse Society, University of California at Los Angeles.

Hall, R. (1995). Getting into trouble with videotape. Paper presented at the American Educational Research Association annual meeting, San Francisco, CA.

Hall, R. (1994, June). Exploring "late algebra": Algebraic reasoning in adult design work. Paper presented at the National Center for Research in Math and Science Education: Early Algebra Conference, University of Wisconsin, Madison.

Hall, R. (1994, May). Mathematical practices in and out of school. Paper presented at the National Science Foundation "Sugarloaf" Conference on New Research Directions, Chestnut Hill, PA.

Hall, R. (1995, April). Exploring mathematical practices in design-oriented work. Paper presented at the American Educational Research Association, San Francisco, CA.

Hall, R. (1995, October). CSCL/CSCW? Invited panel discussion at the Computer Support for Collaborative Learning conference. Indiana University.

Hall, R. (1996, August). Mathematical practices in school and work settings. Talk at an invited conference on Workplace Simulations. Learning Research and Development Center, University of Pittsburgh, Pittsburgh, PA.

Hall, R. & Stevens, R. (1995, September). Sorting termites and tracking wolves: Constructing quantity in and out of school. Invited paper for a symposium on "Symbolizing, Communicating, and Mathematizing." Vanderbilt University, TN.

Lauman, B. & Moschkovich, J. N. (1996, April). Students' understanding of functions concepts embedded in design units. Symposium paper presented at the annual meeting of the American Educational Research Association, New York: NY.

Moschkovich, J. N. (1993, December). A multicultural perspective of teaching mathematics. Paper presented at the Thirty-sixth Annual Conference of the California Mathematics Council, Asilomar, CA.

Moschkovich, J. N. (1994, July). Access to mathematics in middle school: The MMAP curriculum and assessment system. Paper presented at the Quality Education for Minorities Conference, San Francisco, CA.

Moschkovich, J. N. (1994, April). Assessing students' mathematical activity in the context of designing projects: Defining "authentic" assessment practices. Paper presented at the American Educational Research Association annual meeting, New Orleans, LA.

Moschkovich, J. N. (1995, April). Making math or making change? Contrasting views of mathematics classrooms. In *Communities of practice in mathematics classrooms: Reconciling everyday and academic mathematics?* Symposium presented at the annual meeting of the American Educational Research Association, San Francisco, CA.

Moschkovich, J. N. (1996, April). "Making change" and "Making mathematics" as proposals for mathematics classroom practices. Symposium paper presented at the annual research pre-session of the National Council of Teachers of Mathematics. San Diego, CA.

Moschkovich, J. N. (1996, July). Design environments for collaborating mathematically. Paper presented at the Meeting of the International Council of Mathematics Education; Sevilla, Spain.

17 Funder, University, or Other Non-peer Reviewed Presentations through 1996

Goldman, S. & Knudsen, J. (1993, May). Learning math by designing solutions to real-world problems. Paper presented at the National Science Foundation Principal Investigators Meeting for State Systemic Initiatives,

Washington, D.C.

Goldman, S. & Knudsen, J. (1994, May). MMAP technologies for math learning. Paper presented at the Computerworld Smithsonian Awards Education Summit, Washington, D.C.

Goldman, S. (1993, November). Seas, sees, and seize: MMAP and the challenges of classroom ethnography. Paper presented at the Education in Math, Science, and Technology Colloquium, University of California at Berkeley.

Greeno, J. (1993, May). Reform efforts in mathematics and science education. Paper presented at the Stanford Alumni Club, Pittsburgh, PA.

Greeno, J. (1992, January). Concepts in situated cognition. Paper presented at the Cognitive Science Group, Georgia Institute of Technology.

Greeno, J. (1992, January). Concepts in situated cognition. Paper presented at the Cognitive Science Center, McGill University.

Greeno, J. (1994, February). A situativity theory of concepts and understanding. Paper presented at the Institute for Research in Cognitive Science, University of Pennsylvania, Philadelphia.

Greeno, J. (1994, January). A situativity theory of understanding concepts. Paper presented at the School of Psychology Colloquium, Georgia Institute of Technology.

Greeno, J. (1994, February). A situativity theory of concepts and understanding. Paper presented at the Institute for Research in Cognitive Science, University of Pennsylvania, Philadelphia.

Greeno, J. (1994, September). Situativity theory of cognition and learning. Paper presented at the Institute for Educational Research, University of Oslo.

Greeno, J. (1995, November). A situative analysis of reasoning and understanding. Paper presented at the SESAME Program, University of California at Berkeley.

Hall, R. (1992). Locating models in activity. Paper presented at the Educational Testing Service Working Groups, Princeton, NJ.

Hall, R. (1993, October). Using videotape for research and design in the Middle-school Mathematics through Applications Project. Paper presented at the National Design Experiments Consortium Meeting, Learning and Technology Center of Vanderbilt University.

Hall, R. (1994). Representational practices in mathematics. Paper presented at the Special Education Graduate Program, University of California at Berkeley.

Hall, R. (1994, November). Designing for mathematical understanding. Paper presented at the Seminar on Human-Computer Interaction: Project on People, Computers, and Design, Stanford University.

Hall, R. (1996). Managing realism(s) in elementary and middle school mathematics classrooms. Talk at a Postdoctoral Fellows Forum for the NAE/Spencer Foundation. New York, NY. Also presented at the 1996 NCTM Research Pre-session Meetings, San Diego, CA.

Shared, A. (1995). *A Call to Action.* Report of the California State Mathematics Task Force.