Week 3: Fixed Effects

- R wrap-up
  - Types & Type Conversion
  - NA values
  - Getting help
  - Installing packages

- Fixed Effects
  - Introduction to Fixed Effects
  - Running the model in R
  - Model Formulae & Interactions
  - Model Fitting
  - Hypothesis Testing
  - Fitted Values, Residuals, & Outliers

- New dataset: http://www.scottfraundorf.com/R/Stroop.csv
Types

- R treats continuous & categorical variables differently:
  - **Numeric**
  - **Factor**: Variable w/ fixed set of categories
  - **Character**: Freely entered text (e.g., open response question)
Types

- Get our Week 2 data again:
  - `experiment <- read.csv('week2.csv')`
- R's heuristic when reading in data:
  - Letters anywhere in the column → factor
  - No letters, purely numbers → numeric
Type Conversion: Numeric → Factor

- Sometimes we need to correct this
  - Room 4 is not “twice as much” Room 2

- Create a new column that's the factor (categorical) version of TestingRoom:
  - `experiment$Room.Factor <- as.factor(experiment$TestingRoom)`

- Or, just overwrite the old column:
  - `experiment$TestingRoom <- as.factor(experiment$TestingRoom)`
Conversion: Character → Factor

- When `ifelse()` results in words, R creates a **character** variable rather than a **factor**
  - Need to convert it

Wrong:
- `experiment$FaveRoom <- ifelse(experiment$TestingRoom==3, 'My favorite room', 'Not favorite')`

Right:
- `experiment$FaveRoom <- as.factor(ifelse(experiment$TestingRoom==3, 'My favorite room', 'Not favorite'))`
**Type Conversion: Factor → Numeric**

- To change a factor to a number, need to turn it into a character first:
  - `experiment$Age.Numeric <- as.numeric(as.character(experiment$Age))`
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We might have run into some problems trying to change **Age** into a numerical variable...

<table>
<thead>
<tr>
<th>Age.Numeric</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min. : 18.00</td>
</tr>
<tr>
<td>1st Qu.: 29.25</td>
</tr>
<tr>
<td>Median : 42.50</td>
</tr>
<tr>
<td>Mean  : 41.46</td>
</tr>
<tr>
<td>3rd Qu.: 59.00</td>
</tr>
<tr>
<td>Max.  : 63.00</td>
</tr>
<tr>
<td>NA's  : 240</td>
</tr>
</tbody>
</table>

NA means “not available”...
- Characters that don't convert to numbers
- Missing data in a spreadsheet
- Invalid computations
If we try to do computations on a set of numbers where *any* of them is `NA`, we get `NA` as a result...

- `sd(experiment$Age.Numeric)`

R wants you to think about how you want to treat these missing values

- We’ll talk more in the Missing Data week about what the good solutions are
NA – Solutions

- To ignore the NAs when doing a specific computation, use `na.rm=TRUE`:
  - `mean(experiment$Age.Numeric, na.rm=TRUE)`
- To get a copy of the dataframe that excludes all rows with an NA (in any column):
  - `experiment.NoNAs <- na.omit(experiment)`
- Change NAs to something else with logical indexing:
  - `experiment[is.na(experiment$Age.Numeric)==TRUE,]$Age.Numeric <- 23`
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Getting Help

• Get help on a specific known function:
  - `sqrt`
  - `write.csv`

• Try to find a function on a particular topic:
  - `logarithm`

- External resources:
    - Mailing list for using R in language research
  - Web tutorials like http://www.statmethods.net
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R Packages

• R has lots of add-ons for many kinds of statistical analysis (e.g., structural equation modeling)

• lme4: Package for mixed effects models
**Downloading the Package: RStudio**

- **Tools** menu -> **Install Packages**
- Type in `lme4`
- Leave **Install Dependencies** checked
  - Grabs the other packages that `lme4` makes use of
  - Only need to do this once per computer!
Downloading the Package: R

- Packages & Data menu -> Package Installer -> Get List
- Find lme4
- Make sure to check Install Dependencies
  - Grabs the other packages that lme4 makes use of
- Only need to do this once per computer!
library() command

• Need to do this in each script where you’ll use the package:
  • `library(lme4)`

• Tells R to load up the lme4 package you downloaded
  • If you had a lot of add-on packages, loading them all automatically would make R really slow to start
  • So, we only load the packages needed for this analysis
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Mixed Effects Models!

- Next two weeks: Basics of a mixed effects analysis with continuous/numerical variables
  - This week: **Fixed** effects (effects of interest)
  - Next week: **Random** effects (e.g., subjects, items)

- After that: categorical variables
  - As outcome
  - As predictors
http://www.scottfraundorf.com/R/Stroop.csv

• Stroop task dataset
  • `Stroop <- read.csv( ... )`

Task: Say the INK COLOR of the printed word, ignoring what the words spells out
Introduction to Fixed Effects

• Predicting one variable as a function of others

Latency to name color (RT) = Baseline + # of previous trials + Font Size + Subject + Item + Error
Introduction to Fixed Effects

- Predicting one variable as a function of others

Latency to name color = Baseline + # of previous trials + Font Size

NEXT WEEK!
Introduction to Fixed Effects

- Predicting one variable as a function of others

\[ \text{Latency to name color} = \text{Baseline} + \text{# of previous trials} + \text{Font Size} \]

**Fixed effects** that we’re trying to model
Introduction to Fixed Effects

- Predicting one variable as a function of others

\[ y_{i(jk)} = \text{Baseline} + \# \text{ of previous trials} + \text{Font Size} + \text{Error} \]

(Bryk & Raudenbush, 1992; Quene & van den Bergh, 2004, 2008)
Introduction to Fixed Effects

- Predicting one variable as a function of others

\[ Y_{i(jk)} = Y_{000} + \text{# of previous trials} + \text{Font Size} + \text{Error} \]

Latency to name color
Baseline

(Bryk & Raudenbush, 1992; Quene & van den Bergh, 2004, 2008)
Introduction to Fixed Effects

- Predicting one variable as a function of others

\[ y_{i(jk)} = Y_{000} + Y_{100} + Y_{200} + \text{Error} \]

- Latency to name color = Baseline + # of previous trials + Font Size + Error

(Bryk & Raudenbush, 1992; Quene & van den Bergh, 2004, 2008)
**Introduction to Fixed Effects**

- Predicting one variable as a function of others

\[
Y_{i(jk)} = Y_{000} + Y_{100}X_1 + Y_{200}X_2 + \text{Error}
\]

- Latency to name color
- Baseline
- # of previous trials
- Font Size
- Error

(Bryk & Raudenbush, 1992; Quene & van den Bergh, 2004, 2008)
Introduction to Fixed Effects

• Predicting one variable as a function of others

\[ y_{i(jk)} = \gamma_{000} + \gamma_{100} x_1 + \gamma_{200} x_2 + e_{i(jk)} \]

Latency to name color = Baseline + # of previous trials + Font Size + Error

(Bryk & Raudenbush, 1992; Quene & van den Bergh, 2004, 2008)
Introduction to Fixed Effects

• What if we aren’t interested in predicting specific values?
  • e.g., We want to know whether a variable matters or the size of its effect

• But: Can ask if knowing about this variable (reliably) improves the model predictions
  • If so, it’s related to the DV
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Running the Model in R

Linear Mixed Effects Regression
Running the Model in R

- **Time to fit our first model!**
  - `model1 <- lmer(RT ~ 1 + PrevTrials + FontSize + (1|Subject) + (1|Item), data=Stroop)`
    - Name of our model, like naming a dataframe
    - Linear mixed effects regression (function name)
    - Dependent measure comes before the ~
    - Intercept (we’ll discuss this more very soon)
    - Variables of interest (**fixed effects**)
    - Random effect variables
    - Name of the dataframe where your data is

- **Here it is as a single line:**
  - `model1 <- lmer(RT ~ 1 + PrevTrials + FontSize + (1|Subject) + (1|Item), data=Stroop)"
Running the Model in R

• Time to fit our first model!
• \texttt{model1 <- lmer(} \\
  \texttt{RT ~ 1 +} \\
  \texttt{PrevTrials + FontSize + (1|Subject) + (1|Item), data=Stroop)}

  Name of our model, like naming a dataframe
  Linear mixed effects regression (function name)
  Dependent measure comes before the \texttt{~}
  Intercept (we’ll discuss this more very soon)
  Variables of interest (\texttt{fixed effects})
  Random effect variables
  Name of the dataframe where your data is

• Quick note: This version of the model makes assumptions about the random effects that probably aren’t true. We’ll deal with this next week when we discuss random effects.
Running the Model in R

• Where are my results?
  • Just like with a dataframe, we’ve saved them in something we can view later

• To view the model results:
  • `summary(model1)`
    • Or whatever your model name is is

• We save the model so that we can:
  • Compare models later
  • View our results again
  • Get stuff from the model (like residuals)
Sample Model Results

**Formula:** Variables you included

**Data:** Dataframe you ran this model on

Check that these two matched what you wanted!

Relevant to model fitting. Will discuss soon.

Random effects = next week!

Number of observations, # of subjects, # of items

Results for fixed effects of interest (next slide!)

Correlations between effects

• Probably don’t need to worry about correlations between effects unless they’re very high (Friedman & Wall, 2005; Wurm & Fisicaro, 2014)
Parameter Estimates

- Estimates are the $y$ values from the model notation
- Each additional trial of experience $\approx 18$ ms decrease in RT
- 1-point increase in font size $\approx 13$ ms increase in RT
- Intercept: Baseline RT if # of previous trials, font size are 0
Parameter Estimates

- Don’t worry; we’ll get to hypothesis testing soon!

WHERE THE @$^@$ ARE MY P-VALUES!??
Confidence Intervals

• 95% confidence intervals are:
  • Estimate ± (1.96 * std. error)

• Try calculating the confidence interval for the **font size** effect

  • Function that does this for all fixed effects
  • Need to load it first with: `source('Path to File on Your Computer/summary.ci.R')`
  • Then: `summary.ci(model1)`
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Model Formulae: Interactions

- Hang on, what if I think that the **font size** and **serial position** will interact?
  - Font size effect might get weaker as you get practice with the task
- Add an interaction to the model:
  - `model2 <- lmer(RT ~ 1 + PrevTrials + FontSize + PrevTrials:FontSize + (1|Subject) + (1|Item), data=Stroop)`
  - : means **interaction**
Model Formulae: Interactions

• A shortcut!: *

• * means the interaction plus all of the individual effects
  • `model3 <- lmer(RT ~ 1 + PrevTrials*FontSize + (1|Subject) + (1|Item), data=Stroop)`
  • For factorial experiments, usually what you want

• Scales up to even more variables: YearsOfStudy*WordFrequency*NounOrVerb
  • 3-way interaction, all 2-way interactions, all main effects
Model Formulae Practice

• What do each of these formulae represent?
  • CollegeGPA ~ 1 + SATScore + HighSchoolGPA
  • PerceivedCovariation ~ 1 + StrengthOfRelation + PriorBelief + PriorBelief:StrengthOfRelation
  • DetectionRT ~ 1 + Brightness*Contrast + PreviousTrialRT
Model Formulae Practice

• What do each of these formulae represent?
  • CollegeGPA \sim 1 + SATScore + HighSchoolGPA
    • College GPA predicted by SAT score & high school GPA, no interaction
  • PerceivedCovariation \sim 1 + StrengthOfRelation + PriorBelief + PriorBelief:StrengthOfRelation
    • Perceived covariation predicted by strength of relation, prior belief, and their interaction
  • DetectionRT \sim 1 + Brightness*Contrast + PreviousTrialRT
    • Detection RT predicted by brightness, contrast, & their interaction, plus previous trial RT
Model Formulae Practice

• Write the formula for each model:
  • 1) We’re interested in the effects of family SES, prior night’s sleep, and nutrition on math test performance, but we don’t expect them to interact
  • 2) We factorially manipulated sentence type (active or passive) and plausibility in a test of text comprehension accuracy
Model Formulae Practice

• Write the formula for each model:
  • 1) We’re interested in the effects of **family SES**, **prior night’s sleep**, and **nutrition** on **math test performance**, but we don’t expect them to interact
    • \( \text{MathPerformance} \sim 1 + \text{SES} + \text{Sleep} + \text{Nutrition} \)
  • 2) We factorially manipulated **sentence type** (active or passive) and **plausibility** in a test of **text comprehension accuracy**
    • \( \text{ComprehensionAccuracy} \sim 1 + \text{VerbType} + \text{Plausibility} + \text{VerbType:Plausibility} \) or
    • \( \text{ComprehensionAccuracy} \sim 1 + \text{VerbType}*\text{Plausibility} \)
Interpreting Interactions

- Doesn’t look like much of an interaction

```
Fixed effects:                           Estimate  Std. Error t value
(Intercept)                       954.47234   26.21396    36.41
PrevTrials                       -16.71998    1.90486    -8.78
FontSize                         13.03436   0.44372    29.38
PrevTrials:FontSize             -0.02414   0.03311   -0.73
```

- What the interaction mean if it existed?

\[ y = 954 + -17 \times \text{PrevTrials} + 13 \times \text{FontSize} + (-0.02 \times \text{PrevTrials} \times \text{FontSize}) \]

Amplifies the \text{PrevTrials} effect
(larger number = smaller RT)
if large font size

Reduces the \text{FontSize} effect
(larger number = longer RT)
if more previous trials

When would this decrease RT the most? (most negative number)
- When \text{prev trials} is large
- When \text{font size} is large
Interpreting Interactions

• Doesn’t look like much of an interaction

<table>
<thead>
<tr>
<th>Fixed effects:</th>
<th>Estimate</th>
<th>Std. Error</th>
<th>t value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Intercept)</td>
<td>954.47234</td>
<td>26.21396</td>
<td>36.41</td>
</tr>
<tr>
<td>PrevTrials</td>
<td>-16.71998</td>
<td>1.90486</td>
<td>-8.78</td>
</tr>
<tr>
<td>FontSize</td>
<td>13.03436</td>
<td>0.44372</td>
<td>29.38</td>
</tr>
<tr>
<td>PrevTrials:FontSize</td>
<td>-0.02414</td>
<td>0.03311</td>
<td>-0.73</td>
</tr>
</tbody>
</table>

• What the interaction mean if it existed?
  • Negatively-signed interactions (like this one) amplify negatively signed effects and reduce positively signed effects
  • Positively-signed interactions amplify positively signed effects and reduce negatively signed effects
Interpreting Interactions Practice

• Ambiguity: + effect on reading time (slower reading)
• Animate subject: No effect
• Ambiguity x animacy interaction is +
  • Interpretation: Animate subject amplifies the syntactic ambiguity effect

• L2 proficiency: + effect on translation accuracy
• Word frequency: + effect on accuracy
• Frequency x proficiency interaction is -
  • Interpretation: Word frequency effect gets smaller with high proficiency
  • (Or: Proficiency matters less when translating high frequency words—different way of stating the same thing)
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Model Fitting

• We specified the formula

• How does R know what the right parameter estimates are for this model?
Model Fitting

- Solve for $x$:
- $2(x + 7) = 18$
$2(x + 7) = 18$

- Two ways you might solve this:
  - Use algebra
    - $2(x+7) = 18$
    - $x+7 = 9$
    - $x = 2$
    - Guaranteed to give you the right answer
  
- Guess and check:
  - $x = 10$? -> $34 = 18$  *Way off!*
  - $x = 1$? -> $9 = 18$  *Closer!*
  - $x = 2$? -> $18 = 18$  *Got it!*
  - Might have to check a few numbers
Model Fitting

- Two ways you might solve this:
  - **t-test**: Simple formula you can solve with algebra
    \[ t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{s_{X_1 X_2}}{n}}} \]
  - **Mixed effects models**: Need to *search* for the best parameters

- **Analytic Solution**

- **Non-Analytic Solution**
Model Fitting

- In particular, looking for the *model parameters* (results) that have the greatest (log) *likelihood* given the data
  - Maximum likelihood estimation

- Likelihood is like the reverse of probability. Probability is about a *result* given a *model*. Likelihood is about a *model* given the *results*.
  - “Given a fair coin, what’s the *probability* of heads?”
  - vs.
  - “I got heads 83 out of 100 times. How *likely* is this to be a fair con?”
Model Fitting

• In particular, looking for the *model parameters* (results) that have the greatest (log) *likelihood* given the data
  • Maximum likelihood estimation

• Not guessing randomly. Looks for better & better parameters until it *converges* on the solution
  • Like playing “warmer”/“colder”
Model Fitting—Implications

• More complex models take more time to fit
  • `lmer(DV ~ formula, data=Stroop, verbose=2)`
    • `verbose=2` shows R’s steps in the search
    • Probably don’t need this; just shows you how it works

• Possible for model to **fail to converge** on a set of parameters
  • Issue comes up more when you have more complex models (namely, lots of **random effects**)
  • We’ll see more next week about when this might happen & what to do about it
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**Hypothesis Testing—t test**

- We don’t have p-values, but do we have t values:

  - A t test comparing this γ estimate to 0
    - 0 is the γ expected under the null hypothesis that this variable has no effect

![Fixed effects table](image)
Great! A $t$ value. This will be really helpful for my inferential statistics.

But you also need the degrees of freedom! And degrees of freedom are unclear for mixed effects models. GOT YOU!

Au contraire! In this context (when I have lots of observations), the degrees of freedom might not even matter so much.

Curses! Foiled again!
**Hypothesis Testing—t test**

- For a $t$ test, critical value for significance depends on the degrees of freedom
Hypothesis Testing—t test

- For a t test, critical value for significance depends on the degrees of freedom

Critical values for 2-tailed t-test with alpha=.05

- Critical value for z distribution (~1.96)
- Degrees of freedom vs. Critical t value graph
Hypothesis Testing—$t$ test

- For a $t$ test, critical value for significance depends on the degrees of freedom.
- But, by about 150-200 d.f., has converged to 1.96.
Hypothesis Testing—$t$ test

- In many mixed effects models, enough degrees of freedom that the $t$-distribution has converged to the normal distribution (i.e., the distribution of $z$ scores)

- Baayen (2008) suggests critical value of $t = 2.00$
  - Accounts for the fact that we never exactly converge to 1.96
  - Conservative
Hypothesis Testing—Likelihood Ratio Test

• Another way to test effects…

• What if we made a model where we included NO practice effect?
  • Then, could see whether this model makes worse predictions

• Fit two models:
  • `model1 <- lmer(RT ~ 1 + PrevTrials + FontSize + (1|Subject) + (1|Item), data=ESL)`
  • `model1.NoPT <- lmer(RT ~ 1 + FontSize + (1|Subject) + (1|Item), data=ESL)`
Hypothesis Testing—Likelihood Ratio Test

• Now, compare them with `anova()`:  
  • `anova(model1, model1.NoPT)`

• Note: Order of the two models in the `anova()` command doesn’t matter
Hypothesis Testing—Likelihood Ratio Test

• Now, compare them with `anova()`:
  • `anova(model1, model1.NoPT)`

Differences in log likelihoods are distributed as a chi-square
• d.f. = number of variables we added or removed
• Here, $\chi^2(1) = 380.91, p < .001$

We’ll discuss what this means next week (don’t worry; it’s what we want)
Hypothesis Testing—Likelihood Ratio Test

- Now, compare them with `anova()`: `anova(model1, model1.NoPT)`

- Which model fits better?
  - In general: Higher log likelihood (less negative)
  - When you’re adding or removing 1 effect: The model with the effect always fits better. Question we’re testing is whether it fits **significantly** better.
Hypothesis Testing—Likelihood Ratio Test

• Now, compare them with `anova()`:  
  
  `anova(model1, model1.NoPT)`

• Is there a particular level of likelihood that’s “good”?  
  
  No.  
  
  Likelihood is also influenced by sample size and how much variability there is in the dataset overall.  
  
  Only makes sense to compare likelihoods of different models of the same data
Hypothesis Testing—Likelihood Ratio Test

• Another way to understand the likelihood ratio test is that there are two theoretical models of the Stroop task being considered here:
  - Theoretical model #1: There is a practice effect
  - Theoretical model #2: There is no practice effect

• With the likelihood ratio test, we’re testing which of these models is a better account of the data
  • In this way, the likelihood ratio test directly relates to comparing different theoretical models!
Hypothesis Testing—Conclusions

• The tests are less different than you think!

• Forcing an effect to be 0 just means removing it from the model

\[ Y_{i(jk)} = \gamma_{000} + \gamma_{100} \text{FontSize} + \gamma_{200} \text{PrevTrials} \]
Hypothesis Testing—Conclusions

• The tests are less different than you think!

• Forcing an effect to be 0 just means *removing it from the model*

\[ Y_{i(jk)} = \gamma_{000} + \gamma_{100} \text{FontSize} + 0 \text{PrevTrials} \]

Any number of previous trials multiplied by 0 is 0, so # of previous trials just drops out of the equation
Hypothesis Testing—Conclusions

- The tests are less different than you think!
  - **t-test**: Tests whether an effect differs from zero, based on this model
  - **Likelihood ratio**: Compare to a model where the effect actually is zero

- **Likelihood ratio** test is less likely to detect spurious differences, so better test
- But, large differences uncommon
Hypothesis Testing—Other Options

• Also a new add-on package, \texttt{lmerTest}, that estimates the d.f. for the t-test
  • Similar to correction for unequal variance
  • But, likelihood ratio test is still best

• Previously, a function called \texttt{pvals.fnc()} that did nonparametric statistics
  • Didn’t work well with “weak” random effects (low variance)
  • Not currently available
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Predicted Values

- A model implies a predicted value for each observation:
  \[ y = 954 + -17 \times \text{PrevTrials} + 13 \times \text{FontSize} + 0 \]
- For a trial with 10 previous trials and a font size of 36, what do we predict as the RT?

- See all of the predicted/fitted values:
  - \texttt{fitted(model1)}
  - Make them a column in your dataframe:
    - \texttt{Stroop$PredictedRT <- fitted(model1)}
Predicted Values

- How well do these match up with what we know actually happened?
- $R^2$: $\text{cor}(\text{fitted(model1)}, \text{Stroop$RT$})^2$
  - But, this includes what’s predicted on basis of subjects/items
  - Compare to the $R^2$ of a model with just the subjects & items
Residuals

- How far off are our individual predictions?
- **Residuals**: Difference between predicted & actual for a specific observation

- “2% or 3% [market share] is what Apple might get.”
  - former Microsoft CEO Steve Ballmer on the iPhone

- Actual iPhone market share (2014): **42%**
- Residual: **39 to 40** percentage points
Residuals

- `resid(model1)`
- Residuals are on the same scale as the original DV (e.g., milliseconds or Likert ratings)
  - `abs(scale(resid(model1)))`
  - z-scores them so they're in *number of standard deviations*
- Can use this to identify & remove outliers
  - `Stroop.OutliersRemoved <- Stroop[abs(scale(resid(model1))) <= 3, ]`
  - Outliers after accounting for all of the variables of interest, subjects, and items
- How many data points did we lose?
  - `nrow(Stroop) - nrow(Stroop.OutliersRemoved)`
How Should Outliers Change Interpretation?

• Effect reliable with and without outliers?
  • Hooray!

• Effect only seen if outliers removed?
  • Effect characterizes most of the data, but a few exceptions

• Effect only seen with outliers included?
  • Suggests it’s driven by a few observations

• No effect either way?
  • Weep softly at your desk
**Conclusion**

- Fixed effects are the variables of interest
  - Estimated with maximum likelihood estimation
  - Characterize relation of predictors to outcome
  - Defined by model formula
  - Can test their contribution to the model
    - t-test and likelihood ratio test
    - Residuals can help detect outliers

- Next week: Random effects
  - **Babyak**: Good general introduction to overfitting (not specific to mixed effects models)
  - **Barr et al.**: Model comparison in context of random effects