Using Theory and Measurement to Sharpen Conceptualizations of Mathematics Teaching in the Common Core Era[1]

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Abstract

We argue that large-scale, standards-based improvements in the teaching and learning of mathematics necessitate advances in our theories regarding how teaching effects student learning and progress in how we measure instruction. Our theory—an embodiment of the interaction of high and low levels of two constructs that past research has shown to influence students' development of conceptual understanding (explicit attention to concepts and students' opportunity to struggle)—guided the development of survey, video-, and artifact-based measures of teaching. Here, we develop a validity argument for the inferences that can be drawn about teaching from these measures by identifying claims and empirical evidence about the extent to which those claims are born out in practice. Results suggest our theory is capturing four patterns of teaching and that it can successfully predict different types of student learning – skills efficiency measured on the state standardized test and conceptual understanding as measured through open-ended task sets.

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Once again, we find ourselves at the threshold of a new policy era aimed at improving student learning through the introduction of challenging academic standards and accompanying student examinations. The Common Core State Standards (CCSS) in mathematics and English Language Arts, as well as the Next Generation Science Standards, have shifted states' and school districts' agendas toward ensuring that all students leave high school "college- and career-ready." Even for those states that have not adopted the CCSS and aligned examinations, many are reassessing and realigning their standards and their testing programs to "live up to" to these new, more ambitious standards. In mathematics, this means that students must develop not only skill efficiency, but also more rigorous ways of thinking and reasoning and deeper levels of conceptual understanding.

The changes that must happen in order for students to reach these goals are immense and, once again, teachers find themselves as both the targets and the agents of reform (Cohen, 1990). They are targets because poor instruction is implicitly identified as the cause of unacceptably low levels of student performance; they are agents because teachers are widely seen as representing our nation's best opportunity to foster enhanced student learning. There are, however, few guidelines and little support for teachers to learn how to become agents of reform. Standards do not prescribe how to teach. At the same time, there is a lack of consensus in the research and practice communities regarding the specific features of teaching that foster deeper levels of student learning.

We argue that making headway on this challenge will require advances in our theories of how teaching affects student learning and progress in how we measure instruction. Theory is

required to draw our attention to particular features of teaching that matter for students' development of conceptual understanding and to guide the development of measures; measurement is required to devise new ways of ascertaining the status of instructional practice at scale.

This article draws from a larger investigation of instructional practice and student learning that we are conducting in the state of Tennessee. After spending Race-to-the-Top resources on statewide teacher professional development in mathematics, state education leaders wanted to be able to monitor the range and variability of mathematics instruction across the state in ways that could inform decisions about future allocation of resources in the new standards-based era. Tennessee was one of the first states to adopt the CCSS and, although the state has transitioned to Tennessee standards, these new state-based standards are aligned with the Common Core. In this context, we are conducting a study of natural variation in grades 4-8 mathematics teaching and student learning across the state.

The overall purpose of this article—the first in a series grounded in the Tennessee work—is to describe the theoretical framework that anchors our work, and to generate and test claims grounded in that framework. In so doing, we take the first steps toward constructing a validity argument regarding the inferences that can be made about teaching based on teachers' responses to survey items aimed at measuring key features of mathematics instruction that matter for student learning in the Common Core era. The article unfolds in four sections. We begin by describing our theoretical framework and its relevance for studying and improving teaching in the Common Core era. In this section we also lay out a set of claims, patterns and relationships that we predict to hold based on the theoretical framework. In the next section, we describe our methods for measuring and analyzing instruction and student learning aligned with the

theoretical framework. In the third section, we present findings from these analyses that provide mostly convergent evidence to support the viability of our measures and the validity of the inferences that we draw from them, although we highlight some puzzling findings. We conlcude with a discussion of the implications of this work for measurement at scale and for improving teaching in the Common Core era.

How Mathematics Teaching Influences Student Learning

There are few well-developed theories of how teaching influences student learning (Hiebert & Grouws, 2007; Floden, 2001). There are, however, patterns that emerge across empirical studies describing how ***different*** *kinds of teaching support* ***different*** *kinds of student learning*. We know the most about features of teaching that lead to improvement in skill efficiency defined as the accurate, smooth and rapid execution of procedures (Gagne, 1985; Hiebert & Grouws, 2007) because the most readily available student learning outcomes have been from state-standardized tests that feature multiple-choice items at relatively low levels of complexity (Lane, 2003; Webb, 1999).

In the policy context created by the Common Core, however, the field needs evidence regarding the features of teaching that influence students' development of conceptual understanding, defined here as connections among mathematical facts, procedures and ideas (Brownell, 1935; Hiebert & Carpenter, 1992; Hiebert & Grouws 2007). The development of the Common Core was heavily influenced by the need to avoid "the mile-wide, inch-deep problem" (Schmidt, McKnight, & Raizen, 2002) by stressing the need for students to develop **deeper** conceptual understanding of **fewer** key ideas (CoreStandards.org). A call for deeper conceptual understanding demands a review of what is known and not known regarding teaching practices that foster the development of students' conceptual understanding.

In a review of studies that examined the relationship between teaching and student learning ***of concepts,*** Hiebert & Grouws (2007) identified Explicit Attention to Concepts (EAC) and Students Opportunity to Struggle (SOS) as key teaching features that foster conceptual understanding (page 383). **Explicit attention to concepts** is defined as the public noting of connections among mathematical facts, procedures and ideas (Heibert & Grouws, 2007) This can be done through discussions about the mathematical meaning underlying procedures; by noting how different solution strategies are similar or different, by reminding students of the main point of the lesson and how that point fits into the bigger picture. **Students' opportunity to struggle** is defined as students expending effort to make sense of mathematics, to figure something out that is not immediately apparent; it connotes solving problems that are within reach and wrestling with key mathematical ideas that are comprehensible, yet not well-formed (Hiebert & Grouws, 2007).

Although other features (e.g., use of concrete materials, asking higher order questions) were sometimes associated with conceptual understanding, they were "too specific and too closely tied to particular classroom conditions to support claims that they apply across classrooms" (page 391). EAC and SOS, on the other hand, were observed to operate effectively across a range of contexts and teaching systems. EAC, in particular, was found to be quite robust as it appeared across a variety of studies that used different research designs, that are situated in different approaches to teaching (e.g., teacher- versus student-centered), and that vary in terms of how concepts are developed (e.g., through discourse versus through specially designed materials).

Like EAC, students' opportunity to struggle is especially relevant for today's Common Core era. According to the CCSSM, conceptual understanding develops when students engage in one or more of the 8 mathematical practices (CoreStandards.org). Several of these practices

relate to students' grappling or struggling with important mathematics (e.g., make sense of problems and persevere in solving them, construct viable arguments and critique the reasoning of others). SOS also enjoys support in the research literature. In addition to empirical studies in mathematics education that demonstrate the effects of cognitive demand on student learning outcomes (Stigler & Hiebert, 2009; Stein & Lane, 1996), SOS has garnered theoretical and empirical attention in the learning sciences more broadly, most recently under the label of productive failure (Kapur, 2008; Kapur & Bielaczyc, 2012). Research has begun to recognize that not all struggle is productive and to identify the degree and kind of structure needed to facilitate (or undermine) the productive effects of students' struggle (Puntambekar & Hubscher, 2005; Tobias & Duffy, 2009). The importance of EAC and SOS was also confirmed in the 1999 TIMSS video study in which they identified *the engagement of students in active struggle with core mathematics concepts and procedures* as shared by all high achieving countries (Stigler & Hiebert, 1999, page 34).

A Focus on Interaction and Grain Size

Our theoretical framework starts with these two constructs, but takes them a step further by examining their interaction. Past measures of classroom teaching have typically concentrated on isolated features (e.g., process-product measures such as wait time) ***or*** have relied on very broad labels of teaching approaches (e.g., reform vs. conventional instruction) which, although comprised of a multitude of interacting features, do not specify or measure those interactions. Our examination of the interaction of EAC and SOS takes an approach that is different from either of these. It is anchored in our belief that specific instructional features achieve their impact through *interaction with one another* rather than independently and/or additively (Stigler & Heibert, 2009).

In selecting EAC and SOS, we also strove to adopt features that exist at a medium grain size (suggesting that they may be the most noticeable/observable among a constellation of features that typically interact and travel together but would be too small to measure reliably). The TRU Math Framework represents a similar effort to identify features of effective teaching that operate at a medium grain size (Schoenfeld, Floden, & the Algebra Teaching Study and Mathematics Assessment Project, 2014). In this scheme, 5 dimensions of effective mathematics teaching are identified (1) the mathematics; (2) cognitive demand; (3) access to mathematical content; (4) agency, authority, and identity; and (5) uses of assessment. Schoenfeld and colleagues claim that, at this level of analytic grain size, "the five dimensions, broadly construed, encompass the essentials of productive mathematics classrooms" (p. 3). EAC and SOS are most strongly aligned to the first two dimensions, although issues of agency, authority, and identity are implicated in some of our measures as well. Additionally, TRUMath focuses on " . . . *minimally overlapping* dimensions of mathematics classroom activity" (page 2); our framework, on the other hand, emphasizes the interaction among features.

Placing EAC and SOS in interaction with one another, we argue, offers a way to deepen—and challenge—conventional ways of measuring instruction in mathematics education research. Studies typically conflate either high or low levels of EAC and SOS. For example, under the label of "reform instruction," researchers often blur a variety of features such as students grappling with challenging problems (high SOS) and attention to conceptual understanding (high EAC), the assumption being that they co-occur (at either high or low levels) and, together, lead to effects on student learning. Similarly, under the label of "direct instruction," researchers often fuse students practicing what they already know (low SOS) with instruction that is devoid of mathematical concepts or ideas (low EAC).[1]

Instead of avoiding the potential for interaction between high and low levels of EAC and SOS, we feature interactions as a key part of our theoretical framework. Doing so enables us to test for the existence of the "off diagonals" (e.g., High EAC/Low SOS) and yields testable hypotheses that demonstrate the consequences of interactions among them for student learning.

Building an Initial Instantiation of the Theoretical Framework

Our first step toward understanding the influence of EAC and SOS on student learning is to explore how their interaction produces different profiles of teaching. As shown in Figure 1, a simple 2x2 matrix of high and low levels of SOS and EAC produces four quadrants.

[Insert Figure 1 Here]

Quadrants 1 and 4 describe typically drawn profiles of reform versus traditional teaching. In the first quadrant (high SOS and high EAC) students are provided with open-ended tasks for which there is not a predictable, well-rehearsed approach or pathway to solve the task. Students have to exert considerable cognitive effort as they invent and test different strategies for solving the task. Student work on the task provides the primary fodder for class discussions, and attention is also paid to connecting student work with important mathematical concepts and ideas.

The fourth Quadrant represents instruction in which there is both low tolerance for student struggle and limited attention to concepts. This often occurs as worksheet-driven instruction in which the teacher demonstrates the procedure she wants students to use and then they do a set of similar problems using that same procedure with no reference to why the procedure works or when it is appropriate to use it. This profile of teaching accounts for the majority of instruction in the U.S. (Schmidt, McKnight, & Raizen, 2002) and is often referred to as "traditional" or "direct" instruction.[2]

Quadrants 2 and 3 invite us to break away from the conventional labels associated with "reform" and "traditional" instruction. Quadrant 3 does not align with any espoused view of teaching and learning, but it does describe a profile of teaching that can materialize when teachers unsuccessfully enact Quadrant 1 teaching (Stein, Grover, & Henningsen, 1996).[3] When students are provided with open-ended problems for which they do not have the prior knowledge or a strategy for solving, they can struggle, but not in productive ways. Without teacher scaffolding of students' thinking toward the important mathematical ideas embedded in the task, students do not engage with mathematical concepts.

Quadrant 2 teaching involves explicit attention to concepts, but, unlike Quadrant 1 instruction, it provides less opportunity for sustained student struggle. Although we suspect that teaching in this quadrant can take a variety of forms, it often involves teacher demonstration of a general procedure for solving a problem with time taken to explain concepts as they relate to procedures and to encourage and entertain student questions. Multiple representations are enlisted to explain concepts along with drawing connections between those representations and the procedure that is being taught. This kind of instruction can be viewed as high on EAC but as curtailing student struggle by suggesting a pathway that students follow to a solution (rather than having them invent their own approaches). That doesn't mean, however, that students can mindlessly follow the pathway, but rather have to think about what they are doing and why.

There are other ways in which student struggle could be constrained in a high EAC Quadrant 2 profile of teaching. To our knowledge, however, the mathematics education literature does not contain many other examples of Quadrant 2 teaching. In the learning sciences, however, there is a long history of research on the relationship between procedural and conceptual learning (e.g., Rittle-Johnson, Siegler, & Alibali, 2001) and design-based studies have begun to explore

when and how structure can be introduced to mitigate or build on student struggle (see for example, Schwartz & Bransford, 2009). Hiebert and Grouws (2007) refer to the possibility of conceptual lessons that contain no elongated episodes of struggle but do contain more *bounded* forms of struggle (e.g., smaller explorations of targeted concepts embedded in more highly structured lessons).

In summary, recent descriptions of teaching and learning have tended to classify instruction into one of two systems: "reform" and "traditional" teaching. Although representing an advance in some ways,[4] these labels have also proven to be problematic, mostly because they "group together features of instruction in ill-defined ways and connote different kinds of teaching to different people" (Hiebert & Grouws, p. 380). Moreover, the labels have become flashpoints in the "math wars," a series of unproductive back-and-forths between reformers and mathematicians that have polarized the field (Munter, Stein, & Smith, in press; Schoenfeld, 2004).

Here, we purposefully move beyond these labels using a theoretical framework to guide testable claims and begin to acquire evidence for a validity argument. We believe that the theoretical framework helps advance our understanding in a number of ways. For example, although one is likely to see the same kinds of curricula and instructional tasks in Q1 and Q3 profiles, the separation of the two quadrants makes clear the key contribution of EAC by illustrating the lack of it in Q3 teaching. The distinction between Q1 and Q2 teaching opens the door for exploring the role that SOS plays in both teaching and student learning. In research guided by our theoretical framework—as reported herein—we demonstrate how we are able to describe concrete differences in teaching between the quadrants (when we use our theoretical

framework to identify teachers into quadrants using patterns of responses to survey data) and test whether there are associated differences in student learning.

Building a Validity Argument

Along with our partners in the Tennessee Department of Education, our ultimate goal is instructional improvement *at scale*. This led us to the use of surveys as a vehicle for measuring mathematics teaching practice across the state. Our validty question thus becomes: what kind of inferences can we make about teaching based on survey items aimed at measuring two key features of instruction?

Historically, a common approach that has been used to demonstrate construct validity of survey responses is to triangulate them with other criterion measures collected via different methods (e.g., McDonald, 2008). Indeed, researchers in education have primarily used criterion measures to demonstrate how surveys can provide valid inferences about a teachers' mathematics instruction and, therefore, be of use at scale (e.g., Mayer, 1999). For example, Stetcher (2006) made a validity argument by comparing vignette-based items to other (criterion) measures of mathematics teaching practice. Similarly, Mayer (1999), using more traditional items organized around latent constructs consistent with the NCTM standards, made arguments for the validity of inferences that could be made about teaching from his results.

We attempt to build and elaborate on this research. To ensure as much alignment between the survey and our criterion measures as possible we anchor not only our survey measures, but also our criterion measures of teaching practice (video, and teacher assignments and student work) in our theoretical framework. In so doing, we are testing whether teachers' responses to survey items and their resulting classification into quadrants are observable in practice and whether they correlate with other measures in ways that are expected.

Current investigations of validity typically involve the collection of a wide range of evidence that provides a scientific basis for a specific score interpretation (AERA, APA, & NCME, 2014; Kane, 2006). Most measures (typically assessments, but here teaching practice) are designed to serve specific purposes, and each purpose involves an interpretation of scores that should be subjected to a validity investigation. In our case, we view our theoretical framework as a useful heuristic for generating claims and undergoing hypothesis testing of those claims in order to acquire evidence for a validity argument. Our primary purpose in developing survey measures aligned to our theoretical framework was to test whether such measures capture meaningful differences in teaching practice and, therefore, can be expected to be useful for large-scale research studies.

Most current work on validty is based on Kane's (1992) argument-based approach. Following this method, we have elucidated the assumptions[5] that undergird the claims.[6] As we frame our validity argument, we explicitly state claims regarding the meaning of our survey-based quadrant placements, then test these claims using empirical evidence. Specifically, our claims – and subsequent inferences for empirical testing – are the following:

1. Amidst calls for "reform", teachers have responded in different ways, resulting in teaching practices similar to profiles defined by our theoretical framework. Thus, self-reported teaching practices on the survey should reflect a tendency for teaching within one of the four quadrants.
2. When teachers respond in patterned ways to surveys, suggesting a teaching tendency aligned with a given quadrant, their quadrant placement ought to reveal instructional practices that are theoretically consistent with the core features of mathematics teaching (EAC and SOS) illustrated by our two-by-two matrix (see Figure 1). Specifically, while Quadrant 1 and

Quadrant 4 teaching will be characterized by instructional practices aligned with high- and low- EAC and SOS, respectively, we expect that teachers whose patterns of survey responses place their tendencies in Quadrant 2 would have practices aligned with high EAC and low SOS and Quadrant 3 teachers to have practices aligned with the opposite.

3. A teacher's self-identification of their tendency for teaching within a particular quadrant ought to be related to more objective, independent, measures developed from video observations and teacher-provided assignments with accompanying student-work artifacts. For example, a tendency for Quadrant 1 teaching should be characterized by: high conceptual and high struggle video scores; a higher percentage of assignments asking for extended writing; a higher percentage of high cognitive demand tasks; fewer problems per task; and problems requiring students to generate an additional representation or solution path. Similar hypotheses hold for teaching tendencies within the remaining quadrants.
4. Differences in teaching tendencies within the quadrants ought to be related to student learning in theoretically coherent ways. Specifically, a tendency for teaching within the quadrants should be meaningfully related to student performance on assessments of students' skills-efficiency and conceptual understanding. For example, students who experience Quadrant 1 teaching should out-perform students from other quadrants on non-routine, open-ended assessments as well as on assessments of skills and procedural fluency. These students should outperform students from Quadrants 3 and 4 classrooms because of the lack of explicit exposure to concepts in those quadrants.. Students from Quadrant 1 classrooms are also predicted to outperform students who have primarily experienced Quadrant 2 teaching because in addition to being exposed to high levels of EAC and SOS, they have had repeated opportunity to work on similar tasks throughout the year. Students who experience Quadrant

2 teaching, however, should outperform students from Quadrants 3 and 4 classrooms on assessments that measure higher-level concepts.

Methodology

Sample

Our investigation draws on data from the first (2013-14) and second (2014-15) years of a study of natural variation in mathematics teaching in the state of Tennessee. We recruited a volunteer sample of teachers of mathematics in grades 4 through 8 throughout the state to participate in the study, with teachers opting to be part of our larger "survey only" sample or a more intensive sample including additional forms of data collection. 256 teachers took the surveys at time 1 and time 2 and have complete data. These teachers form our primary analytic sample.

[Insert Table 1 Here]

A subset of these teachers have video- (n=27) and artifact-based (n=54) data about their teaching practice. This sample is largely representative of 4th through 8th grade Tennessee mathematics teachers. As shown in Table 1, teachers are slightly more likely to have taught in 4th or 5th grade, but each grade level is well represented. Although most teachers in our sample are female (90%) and white (90%) these proportions are not unusual considering the population of teachers across the state. The teachers reported a wide range of teaching experience and, on average, they taught about 300 minutes a week of mathematics instruction. Teachers in the sample are also geographically diverse coming from all regions of the state. On average we have almost as many schools represented as we have teachers in our study. Finally, we have achievement data for all students for 193 of our 256 teachers on two achievement measures (described later) given statewide in the year prior to our survey administration.

Data and Measures

We begin with a description of the survey, including how our design process helps to mitigate common problems associated with survey-based research. We then proceed to describe two additional measures (assignments and student work and videos of classroom lessons) and the measures of student performance that we used.

Survey. The survey consisted of a variety of item types. In addition to general questions (years teaching, number of math classes taught per week), there were vignettes, items about teachers' beliefs and practices, and items assessing teachers' mathematics knowledge for teaching (Hill et. al., 2008). The vignettes were short descriptions of a mathematics lesson; they were designed to vary according to theoretically based profiles of teaching in each quadrant. After reading the vignette, teachers were asked to answer a bank of questions about it and to use a slider bar to estimate the amount of time their lessons resembled the vignette (0% to 100%).

A host of problems have been documented with regards to self-report research including surveys (e.g., Stone et al., 1999). Difficulty with memory retrieval for frequency counts (e.g., Schwartz and Sudman, 2002; Menon and Yorkston, 2000) and social desirability (e.g., Paulhus and Vazire, 2007) are commonly known problems in social-behavioral surveys. These are among the reasons that Camburn et al. (2015) (through a comparison of self-reports of frequency counts on an annual survey to self-reports of daily log data on comparable items) found that 75% of teachers over-reported on the annual survey. However, we agree with researchers who suggest the answer is not to throw the baby out with the bathwater (Paulhus & Vazire, 2007; Donaldson & Grant-Vallone, 2002). Instead, we have attended to these known problems in both our survey design and analyses.

First, one way in which scholars seek to enhance the accuracy of self-reports in medical or social science surveys is to find better ways of asking questions of respondents. For example,

asking neutral questions can reduce social desirability (Paulhus & Vazire, 2007), contextualizing items can aid memory recall (Tourangeau, 1999) and improve the accuracy of self-reports (Paulhus & Vazire, 2007), and asking indirect as well as direct questions can provide more 'honest' appraisals of sensitive topics (Dalal, 2012). We adhered to these principles in designing multiple items across different item formats, all anchored to our theoretical framework.

The design of our questions also benefited from cognitive interviews conducted before our first administration of the survey. Teachers were asked to complete the survey and, at pre-determined intervals, to talk about how they were interpreting items. We were particularly interested in their response to the vignette-based items (see Appendix A). We found that different individuals found different vignettes to be socially desirable; specifically they found most desirable those that they thought best described their teaching. Thus, our contextualization of quadrant teaching through the vignettes may be considered an instantiation of neutral items, reducing concerns about social desirability. Another contextualization strategy that we employed involved creating an item about perceived constraints in teacher's schools/districts that prevented them from teaching in the way they would like to. This item was inserted right before a set of items about their teaching practices. By providing them with the opportunity to acknowledge that their teaching is shaped by external factors, our intention was to put them in a mindset in which they would be more likely to report their practices honestly. All of these design considerations, combined with the low-stakes nature of our survey, we think mitigates any serious concerns about social desirability (Chan, 2009).

1. Survey Items for Analyzing Patterns of Self-Report Aligned with our Theoretical Framework. Our work began with the development of survey items, utilizing different item formats and response scales, aligned with our theoretical framework. To understand and test our

first claim we used several measures across the survey and analyzed patterns of responses using a latent profile analysis (described below). First, using the same mathematics topic, we created one vignette per quadrant, but varied the tasks and teaching approaches for implementing the tasks to align with our theory of instruction in each quadrant. Following each vignette, we asked teachers the proportion of time they spent teaching like the vignette and we asked teachers to agree or disagree with after-vignette items providing rationale for why certain teaching decisions were made (see Figures A1a through A4c in Appendix A for the four vignettes and items). A fifth vignette was created to gauge teachers' tendencies to continue to facilitate students' struggle with the mathematics in the task or to limit the struggle within the context of a specific teaching event (see Figures A5a and A5b in Appendix A). The proportion of time they self-report persisting in maintaining high struggle (Mr. Clayton) was a fifth item for subsequent use in a latent profile analysis.

In addition, we generated an additional four quadrant scale scores based on our theoretical framework. For each quadrant (e.g., Quadrant 3 exemplified by Mrs. Jones' vignette) we identified particular items throughout the survey that a teacher with a high tendency for teaching like Mrs. Jones would agree to[7]. We then used a partial credit system for each item (e.g., strongly agree=1; agree=.5; otherwise=0) and totaled across these items providing signal for a tendency toward Quadrant 3 teaching. Items contributing to each quadrant scale score (i.e., Q3 scale score in this case) were chosen theoretically and tended to be inclusive of several after-vignette and other likert scale items (see Tables A1 through A4 in Appendix A for the full set of items for each scale for each quadrant). The four quadrant scale scores (i.e., Q1 scale score, Q2 scale score, etc.) were constructed to be used in the subsequent latent profile analysis.

Finally, we administered the proportional reasoning items from the Mathematics Knowledge for Teaching (MKT) middle-grades assessment (see, e.g., Hill et. al., 2008). We used this as a proxy for teachers' pedagogical content knowledge. We assumed that including a proxy for teachers' knowledge might help generate profiles of teaching at the intersection of knowledge and teachers' decision-making about practice – such that our profiles might be indicative of "usable knowledge" (Kersting et al., 2015). This variable was a summed score of each teacher's correct items out of the 30 on this scale of the MKT. This item was our tenth, and final, variable included in our LPA.

2. Survey Items for Confirmatory Factor Analysis for EAC and SOS. More generally, the survey consisted of instructional Likert-scale items contributing to measurement of our two main constructs (EAC and SOS). Indirect items were designed to elicit teachers' reasoning about teaching decisions and how this influences students' learning about mathematics (e.g "More important than extended math discussions, students need a lot of practice with math problems"). Practice items were more traditional and direct. Here, we asked teachers to report the frequency with which they engaged in specific instructional activities associated with EAC and SOS (e.g., "Students listen to and critique others' reasoning and solution strategies"). All of these items appear in Table A5 in Appendix A organized around theoretical sub-constructs within EAC and SOS.

Video-based instructional measures. The subset of 27 teachers on whom we have video data were similar to our overall sample except they included a greater proportion of elementary teachers. Teachers were videotaped in the spring of 2014 for at least 120 minutes of instruction, typically spread over two consecutive lessons. Teachers were told to do "what they normally do" and videographers were trained to capture both whole-class and small-group interactions.

Specifically they were trained to follow the teacher as she moved from group to group and to record (as much as possible) of the conversation and student work.

Lessons were first divided into tasks based on the artifacts used to conduct the lesson,[8] a process that was done by two independent coders with 91% agreement.[9] Segmenting the lesson into identifieable tasks served to assure that coders were using the exact same portion of the lesson when ascribing codes.

Each task was coded individually for explicit attention to concepts and opportunities for student struggle using a protocol developed specifically for this project (see Appendix B). Videos were assigned randomly to three coders with expertise in mathematics education and/or coding of observations. Coders were trained to criterion with an agreement of 80% with an expert coder before beginning coding assignments.[10] 20% of the videos were double coded with an inter-reliability of 65% exact agreement across all codes and 82% agreement on scoring items as "high" or "low" across each code.[11]

Selected scores from the dimensions described on the coding protocol were used to create two composite scores; EAC and SOS. Specifically, the EAC score was a combination of the explicit attention to concepts code along with the degree of consolidation and tie to canonical representations codes. The SOS score was a combination of the opportunities for student struggle code with codes for the type of discourse present in the classroom and the cognitive demand of the task as it was enacted. Finally, because of variability in the number and length of tasks, as well as differences in amount of time for the math lesson among teachers, codes for each dimension for each task were weighted by the proportion of time that respective task lasted relative to the amount of time devoted to the lesson.

Artifact-based instructional measures. Teachers were asked to submit their assignments and samples of student work for five days that overlapped with the classroom videotaping. Teachers were asked to submit every assignment and instructional task used in their class along with two samples of high-quality and two samples of medium-quality student work. Each assignment had an accompanying cover sheet on which the teacher described the nature of the assignment/task and how she would judge student performance on that assignment. All three of these items (cover sheet, assignment, and student work) were coded by 3 individuals using a protocol (see Appendix C). The participating teachers were randomly assigned to one of three coders that scored all their submitted packets (including cover sheets, assignments and student work). Coders were trained to a criterion of 80% agreement with an expert coder before beginning their coding assignments.[12] All packets for 20% of the teachers were double coded with an exact agreement of 86%.[13]

Student Learning Measures. We employ two available measures of student learning. While acknowledging that the boundaries between items measuring procedures and concepts are not always clear cut, we make the case below that one assessment was primarily a skills-based measure and one was more conceptually based. The Tennessee Comprehensive Assessment Program (TCAP) was administered to all students in grades 4-8 in Tennessee in 2012-13. Items on the TCAP were multiple-choice covering the following range of topics: mathematical processes, numbers and operations, algebra, geometry and measurement, and data analysis, statistics and probability. The distribution of topics varied by grade level. For example, number and operations items represented 50% of the 4th grade TCAP, but only 24% of the 8th grade TCAP. Most important for our analytic purposes, the vast majority of TCAP items require a single correct answer and students receive credit based only on whether or not they've achieved

that answer. We infer that most TCAP items are skills and/or procedures based on (a) a review of their released items (see examples in Appendix D); (b) the fact that TCAP was operational during the NCLB-era in which questions about the cognitive complexity of state tests were being raised (Lane 2003); and (c) Tennesse's standards for proficiency were found to be far below those of the National Assessment of Educational Practice (NAEP) which has a reputation as including multiple-choice items that are more conceptual in nature (Resnick, Stein, & Coon, 2008).

Our second learning measure is from a Constructed Response Assessment (CRA) administered in 2012-13. Students at each grade level were provided 4 different tasks (an example item from grade 7 and its associated scoring rubric is included in Appendix D). The rubrics were designed to assess both mathematical content and mathematical practices consistent with the CCSS. These items are non-routine problems, most of which require novel solution strategies, strategies. Many can be solved in more than one way; students' achieve "credit" for being correct and for demonstrating conceptual understanding through appropriate representations or explanations, not only for getting the correct answer. The CRAs were scored by *Measurement Inc.* under contract with the state of Tennessee. A technical report delivered to the state demonstrated adequate reliability characteristics for the CRA scoring (*Measurement Inc.*, 2013).

We examined differences in student growth by teacher on these two different outcomes. We used statewide data for grades 4 through 8 in the following manner: we standardized our outcome within-grade so each student had a standardized score relative to their same grade peers; we combined students across grades so that we could utilize all of the classrooms in our sample. We then generated covariate-adjusted value-added scores for teachers using all 4^{th} through 8^{th}

grade classrooms across the state on each of our two outcome measures[14] (See Appendix E for a write-up of the model). While this is a simpler model than many value-added models in use today (see, e.g., limits of covariate-adjusted versus cross-classified models in Rowan et al. (2002) and for a description of different value-added approaches for the same outcome McCaffrey et al., (2004)), we expect there is more to be gained from comparing and contrasting across our two outcomes. For example, Papay (2011) observed large variation in value-added scores by outcome, more variation than typically experienced by using different model specifications.

Analytic Methods

Our data analyses were constructed to seek evidence that enables us to evaluate our four central claims. Consequently we organize discussion of analytic methods around these claims.

Confirmatory Factory Analysis (Claim 1). In order to evaluate our first claim (that teaching practices reflect a tendency for teaching in one of the four quadrants), the underlying structure of the survey-based Likert indirect and direct "practice" items was theorized to measure two distinct constructs, namely EAC and SOS. Additionally, several sub-constructs were hypothesized to be present in the survey (see Table A5, e.g., the subconstruct of making *connections* among solution strategies or mathematical ideas was hypothesized to be highly related to EAC). We used MPlus Version 3.01 (Muthen & Muthen, 2010) to conduct a Confirmatory Factor Analysis.

We assessed the adequacy of our measures vis-à-vis our theoretical framework in two different ways. First, we examined the fit of our data to our hypothesized higher order structure. More specifically, we sought evidence for confirmation of the presence of the overarching EAC and SOS factors as well as the sub-constructs within them. This hypothesized structure dovetails

with the typical second-order factor structure in which the lower level sub-factors are substantially correlated while the presence of the higher order factor explains the relationship between those sub-factors (Chen, West, & Sousa, 2006). In other words, the sub-constructs are correlated because they share a common source – namely, the second-order factor of EAC or SOS (Reise, Moore, & Haviland, 2010). Examination of absolute and comparative fit indexes provide evidence for how well the data fit the proposed model.

Second, we also conducted a model comparison to understand whether our hypothesized second-order two-factor model fit the data better than a second-order one-factor model (Hoyle, 2000). Such a model comparison provides evidence about the dimensionality of the measured construct(s), EAC and SOS. If, for example, "reform" versus "traditional" instruction were the dominant explanation then the one-factor model would demonstrate a better fit to the data and would be preferred for its model parsimony. Because the one-factor model is fully nested in the two-factor model a comparison of χ^2 statistics, adjusting for differences in degrees of freedom, provides a statistical test for model fit to the data (Hoyle, 2000).

Latent Profile Analysis (Claim 2). To understand the extent to which teachers had a tendency for teaching practices consistent with our theoretical framework, we examined patterns of item responses utilizing a Latent Profile Analysis. Latent Profile Analysis (LPA) was chosen for this purpose as it is helpful in illuminating the relationship of a single categorical latent variable (teaching quadrant) with a set of continuous indicators (survey responses) (Vermunt & Magidson, 2002). We then interpreted the resultant profiles to be indicative of a tendency for teaching within one particular quadrant.

Ten variables were used to estimate the LPA model. Five of these variables were the percent of time (0%-100%) teachers reported their instruction resembled the practices portrayed

in each of five instructional vignettes previously described. The next four variables were the quadrant scale scores generated from theoretical notions of the pattern of responses teachers would record for particular items if they had a tendency towards one quadrant profile. The final variable was scores on the Mathematical Knowledge for Teaching (MKT).

MPlus Version 3.01 (Muthen and Muthen, 2010) was used to conduct our latent profile analyses. Evaluation of the output from extracting 4, 5, 6, 7 and 8 classes resulted in the selection of a 6-class solution. Indicating a classification utility similar to prior research (Pastor et al., 2007), the entropy values for this model were .86. We used the output from this model to decide which of the four quadrants each of the 6 latent classes belonged in.

To uncover whether teaching tendencies carried meaning related to our theoretical framework, we explored patterns of responses on our sub-constructs. We took the factor scores generated on our sub-constructs from the confirmatory factor analysis in order to understand differences in group means based on quadrant placements from the LPA. The sufficiency of our quadrant placements was determined by examining the means for factors and sub-factor scores associated with EAC and SOS and the consistency of the response patterns on those factors and sub-factors with the expected responses based on the operational definitions of the quadrants in our theoretical framework (i.e., quadrant 2 teaching is defined by high levels of EAC and low levels of SOS, while quadrant 3 teaching is the opposite).

Descriptive analyses of between-group differences (Claim 3). In order to evaluate our third claim (a teacher's tendency for teaching within a particular quadrant ought to be related to more objective measures developed from video- and artifact-based measures of teaching), we checked for convergence between our quadrant placements and our other objective measures of teaching; video-based scores of teaching on concepts and struggle (n=27 teachers) and a subset

of features of assignment-based scores (n=54 teachers). Given the small number of teachers overall, and in particular the fact that only two quadrant 2 and three quadrant 4 teachers were included in the video sample, we limit our analysis to a description of group means.

ANOVA for between-group differences for student learning (Claim 4). In order to evaluate our fourth claim (differences in teaching tendencies aligned with the quadrants ought to be related to student learning in theoretically meaningful ways), we explored whether there were between group (quadrant) differences in the mean scores for our measures of student learning using ANOVA. The measures captured teachers' valued added scores for the TCAP, a predominately skills-based assessment, and the CRAs, an assessment with considerably more attention to students' conceptual understanding.

Results

Claim 1: Teaching practices reflect a tendency for teaching in one of the four quadrants

Confirmatory factor analyses were used to investigate the factor structure of eight hypothesized sub-constructs. Three models were used for comparative purposes. Model 1 tested a single general factor using all 38 items (i.e., items nested in one "reform" factor). Model 2 tested whether a second-order single-factor model was a better fit to the data (i.e., all eight sub-constructs were nested in one "reform" factor). Finally, Model 3 tested whether the hypothesized second-order two-factor model (i.e., three sub-constructs in EAC and five sub-constructs in SOS) was a significant improvement beyond the second-order single-factor model. Of primary interest was the comparison between model 2 and model 3 because it tests whether participants responded differently to items thought to be theoretically contributing to EAC and SOS. The fit

indexes for the three models shown in Table 1 confirm that model 3 is the best fit to the data, suggesting EAC and SOS were distinct constructs captured through our survey.

[Insert Table 2 Here]

Model 3 in Table 1 has the best demonstrated fit to our data. This is true in terms of absolute and comparative fit, where model 3 (RMSEA=.06; CFI=.794; TLI=.779 is better than model 1 (RMSEA=.09; CFI=.524; TLI=.497) or model 2 (RMSEA=.07; CFI=.718; TLI=.698).Additionally, the χ^2 difference tests reveal similar improvement in model fit from model 1 to model 2 [$\Delta \chi^2$ (Δ df=8, N=256) = 571.95 (p<.001)] and from model 2 to model 3 [$\Delta \chi^2$ (Δ df=1, N=256) = 220.74 (p<.001)] The fact that the two-factor model is a better fit suggests that our survey captured data from teachers about their practice where our hypothetical constructs EAC and SOS are correlated (r=.40) but they also remain distinct (i.e., respondents were sometimes high on one construct but low on the other and vice-versa).

Claim 2: Teachers had a tendency to endorse teaching practices consistent with the theoretical framework

LPA findings with respect to latent profile covariates. In order to separate teachers according to their tendencies for teaching within a given quadrant, we examined patterns of their responses across 10 variables, five of which allowed them to indicate their preference for one type of teaching versus another (i.e., percentage of their time teaching in a manner similar to the teacher featured in each vignette). Our LPA identified six distinct groupings of teacher responses. We placed each of the 6 empirically identified latent classes into our theoretical framework (i.e., the four quadrants) based on the group means reported in the output for each latent cluster. Certain latent clusters had very distinct profiles. For example, Quadrant 4

teachers (See Figure 2, purple line; n=42 teachers) formed a distinct group because their observed means demonstrated a stark contrast from the other groups in terms of strong tendencies to report teaching similar to the Ms. Smith vignette and strong tendencies to reject teaching like the other vignettes. Quadrant 1 teachers (see Figure 2, blue line; n=77 teachers) separated into two different latent classes with reported tendencies for a) *not* teaching like Ms. Smith and b) teaching in ways similar to all other quadrants, especially Quadrant 1. When combined, Quadrant 1 teachers had observed means that were distinct from Quadrant 4 teachers because their tendencies were highest for Quadrant 1 and lowest for Quadrant 4, essentially mirroring the purple line.

[Insert Figure 2 Here]

These two diametrically opposed profiles represent just about half of all teachers surveyed. Thus, using patterns of responses to survey items we identified many teachers whose tendencies in their mathematics teaching do not neatly fall into "reform" or "traditional" teaching designations. For example, Quadrant 3 teachers (see Figure 2, green line; n=90 teachers) are most distinct in that they reject Quadrant 4 teaching (i.e., Ms. Smith), but they fail to self-report a strong tendency for teaching like teachers in any of the other vignettes. Finally, Quadrant 2 teachers (see Figure 2, red line; n=47 teachers) also separated into two different latent classes that embraced elements of low-struggle teaching like instructional practices in Quadrant 4 (i.e., Ms. Smith) but differed from Quadrant 4 teachers on their average response to the fencing task vignette about allowing students to struggle in context (Mr. Clayton). Furthermore, they also express the highest tendency for Quadrant 2 (i.e., Ms. Evans), and the second-highest for Quadrant 1 (Ms. Park). What is distinct about Quadrant 2 and Quadrant 3 teachers would be lost if we tried to measure "reform" teaching along just a single dimension, as teachers in both

quadrants would likely be lumped in the same group together somewhere between the blue and purple lines.

LPA findings with respect to EAC and SOS factor scores from the CFA.

Corresponding to our second claim, as an empirical test of whether our interpretation of group means within our latent classes identified in the LPA resulted in theoretically meaningful quadrant designations, we examined the scores of teachers within each quadrant on the sub-constructs identified in the CFA. We think of this empirical test as one of cross-validation within the survey itself, while also testing whether teachers have responses to survey items fitting our theoretically defined quadrants (i.e., are there high EAC teachers who also provide limited SOS?). Our theory suggests Quadrant 1 teachers (see Figure 3, blue line) ought to be high on both EAC and SOS, and vice-versa for Quadrant 4 teachers (purple line). Our theory further suggests that there would be teachers with differential scores on EAC and SOS; specifically Quadrant 2 teachers ought to be high on EAC and low on SOS (and vice-versa for Quadrant 3).

[Insert Figure 3 Here]

In general, these propositions hold (note the green and red lines crossing for EAC and SOS), helping to substantiate the existence of our theoretical profiles in actual practice. Furthermore, we find Quadrant 2 teachers provided additional convergent evidence because of their pattern of responses on the sub-constructs. Take, for example, how the blue and red lines cross within EAC, which is theoretically consistent because Quadrant 2 teachers are likely to reject elements of students being asked to struggle on their own connoted here by "minimal teacher input." Likewise, the crossing of the red and purple lines within SOS is seen as further convergent evidence because Quadrant 2 teachers seem to embrace some elements of "productive struggle"

(i.e., because they believe in providing meaning beyond procedures, we would expect they would score higher on the latter three sub-constructs than the previous 2 sub-constructs, which they do). Importantly, the pattern of findings represented in Figure 3 suggests that our quadrant placements correspond to the theory of teaching specified initially to which we anchored our measurement work.

Claim 3: There is some general correspondence between survey-based quadrant placements and other measures

In particular, video scores among our intensive video sample (see Figure 4) followed expectations. For example, consistent with theoretically based expectations, struggle scores were highest in Quadrants 1 (μ=9.15) and 3 (μ=6.91) and lowest in Quadrants 2 (μ=4.88) and 4 (μ=5.60). In addition, conceptual instruction was highest in Quadrant 1 (μ=10.74). However, while Quadrant 2 teaching should theoretically be high on EAC, this was not the case for our 2 teachers with video evidence (μ=6.64). There are simply too few teachers, and too few days of sampled instruction to know if this constitutes divergent evidence.

[Insert Figure 4 Here]

We also examined mean differences for some of our assignment scores. Here, too, we find some differences consistent with our quadrant placements (see Figure 5). For example, Quadrant 4 teachers provided tasks with more problems (μ=14.9). Combined with the finding that they only provided high cognitive demand tasks 4% of the time, this seems to suggest they provided students with more problems for repeated practice applying procedures. Conversely, Quadrant 1 and Quadrant 2 teachers tended to provide fewer problems per task (μ=8.2 and μ=7.8, respectively), which is consistent with the finding that approximately a quarter of their tasks were high cognitive demand (μ=27% and μ=22%, respectively). All in all, more objective

measures of teaching practice both through video- and artifact- based scoring suggest convergence with our survey-based quadrant placements.

[Insert Figure 5 Here]

Claim 4: Differences in teaching tendencies aligned with the quadrants were related to student learning in theoretically meaningful ways

Finally, One-Way ANOVAs suggest no significant mean differences across quadrant designations when covariate-adjusted value-added scores from the TCAP were analyzed ($F=1.87$; $df=189$, $p=.136$). However, there were significant mean differences across quadrant designations when covariate-adjusted value-added scores from the CRA were analyzed ($F=4.80$; $df=189$, $p=.003$). Post-hoc tests using bonferoni adjustments revealed significant mean differences between value-added estimates for students exposed to Quadrant 1 teaching relative to Quadrant 3 (ES=.64, $p=.004$) and relative to Quadrant 4 teaching (ES=.68, $p=.022$). While the trend for value-added scores on the TCAP were in the same direction as the CRA (see Figure 6), only the more conceptual test was sensitive to instructional differences implied by our survey quadrant placements. This pattern of findings is consistent with prior studies showing greater instructional sensitivity for assessments with greater attention to students' conceptual understanding.

[Insert Figure 6 Here]

Discussion

This work is being undertaken as a first step toward the building of a larger body of validity evidence that supports the development of self-report research measures aligned with our theory of teaching and learning. We examined whether teachers' instructional tendencies

matched our quadrant theory; whether teachers' self reports of their teaching practices aligned with the level of EAC and SOS theorized to comprise the quadrant in which they were placed, and whether there was alignment between teaching tendencies within the quadrants and video- and artifact- based measures and patterns of student learning as described by the theoretical framework that the measures were designed to operationalize.

Our findings are encouraging along a number of fronts. The results of the Latent Profile Analysis give credence to our claim that teachers are responding in a variety of ways to the demands placed upon them by higher state standards and accompanying examinations. The strength with which the LPA captured reform (Quadrant 1) and traditional (Quadrant 4) teaching was not unexpected. The fact that only half of the teachers were "covered" by those two profiles, however, provides us with impetus to continue to build and refine our multi-faceted profile theory.

Although not included as part of the claims and evidence reported here, our video-based scoring of classroom lessons has unveiled a variety of lesson formats that appear to fit Quadrant 2 teaching (i.e., high EAC, low SOS) that do not conform to the quintessential version of Quadrant 2 instruction (tightly structured, conceptually based instruction with minimal tolerance for student struggle). For example, a lesson might incorporate bounded moments of student struggle into a larger, teacher-controlled classroom discussion about deriving a procedure. Understanding the different varieties of teaching within our larger quadrants is important not only from a measurement perspective, but, even more important , from a development perspective (more on this below). As we continue this work, we will seek to better understand, and perhaps label these different patterns of teaching, even within the four quadrants espoused here.

It is one thing to have Latent Profile Analysis create four groups of teachers with teaching tendencies that appear fairly well aligned with our four quadrants. It is another thing to demonstrate that teachers who "fall into" a particular quadrant also report engaging in instructional practices consistent with the 2X2 matrix in Figure 1. With the evidence we've presented for Claim 2 we were able to show the expected finding that Quadrant 1 and Quadrant 4 teachers were universally high and low, respectively, on EAC and SOS. However, we also showed that Quadrant 2 and Quadrant 3 teachers were alternately high on one dimension, but low on the other. We think this is important variation to attend to, not only for researchers, but for mathematics educators.

Quadrant placements also aligned well with our expected patterns of student learning. That we found significant differences in value-added scores on the CRA that were consistent with our theoretical predictions, is a particularly important signal that we are on the right track in our measurement and analytic endeavors.

Limitations

The theoretical framework was useful for testing research-based claims, but it is not without limitations. First, the techniques employed in this manuscript relied on quadrant placements as a heuristic, which we have demonstrated to be useful as an initial instantiation of multi-dimensional measurement of mathematics teaching. We do not mean to propose that quadrant placements are an end in themselves, as we ascribe to the view that measurement of teaching is our goal as opposed to the placement of teachers.[15] Furthermore, the quadrants are reductionist by nature. On our surveys we observed more than one latent class per quadrant, indicating more than four patterned responses on the surveys. When coding our videos we have observed similar variations in teaching within quadrants. Thus, we think there are more "patterns

of teaching" than quadrants. By examining mean differences we gloss over some of this variability. We think this variability is worthy of further exploration and do not mean to imply that four quadrants are sufficient to describe mathematics teaching in all its complexity.

Second, we have acknowledged well-known problems relying on self-reported behavior on surveys and how we have tried to address these by examining patterns of survey responses to many items. After ascribing teachers to quadrants using this method, we describe these quadrant profiles using factor scores from the same surveys. The factor scores and some of the items contributing to quadrant placements are not independent of one another.

Finally, we think it is important to reflect on the replicability of this work. It will be important to examine test-retest reliability of our survey as well as understand if there are any maturation effects. Additionally, the method of latent profile analysis for placing teachers in quadrants is highly reliant on the variables entered into the analysis. Going forward we will seek to refine the variables in order to try to better distinguish elements of teaching that are unique to each quadrant. Latent profile analysis is also sensitive to the sample, as patterns of responses for a teacher are being compared to other teachers' patterns. This led to some quite distinct patterns among teachers as described for quadrants 1 and 4 in the manuscript. However, quadrant 3 appears less distinct as the means for this group are closest to average for any of the groups. This could represent a legitimate pattern, but the concern could also be that we have pulled out distinctive patterns, resulting in one group that is less coherent than the others.

Contribution of a Theoretical Framework

What role did our theoretical framework play in our endeavors? First it drew our attention to empirically supported features of teaching that matter for students' development of conceptual understanding. We took a bet on the constructs of EAC and SOS—not only because

of their empirical backing (Hiebert & Grouws, 2007)—but also because they appeared to operate at an optimal grain size meaning that they probably contain within them a coherent and somewhat stable constellation of sub-constructs (such as those we identified and tested in claim 2). Yet they are not so large that they mix together dozens of discrete features whose interactions remain unexamined. Our approach contrasts with the use of larger categories of teaching—such as reform versus traditional instruction—that are used to refer to a wide variety of features (e.g., students' working in small groups, the use of manipulatives, etc.) that—in different and often ill-defined configurations—are taken to define the category.

Second, the theoretical framework guided the development of all of our measures which ensures common constructs as a focus of measurement and provides the optimal environment for empirical tests for our claims contributing to a validity argument. Thus, we are testing whether teachers' responses to survey items and their resultant quadrant placement are observable in practice and whether they correlate with other measures in ways that are expected and explainable by the theory.

Third, the theoretical framework provides *meaning* to instructional features and their interrelationships, as well as their relationship to what and how students learn. As researchers, theory helps us to *understand* what we are studying and to make predictions. Theory is useful to teachers as well, in that it provides a conceptual framework for making sense of research findings and their own teaching. Without theory, teachers are left with isolated findings and little guidance on how and when to employ "proven" techniques.

Implications for the Common Core Era

In the current college- and career-ready standards era, both students' skill efficiency and conceptual understanding are valued outcomes. As noted by Hiebert and Grouws (2007) and

others (Ruiz-Primo et al., 2002; Correnti et al., 2013) some measures of student outcomes, especially those assessing higher cognitive-demand thinking and reasoning and conceptual understanding, may be more sensitive to differences in instruction than others. Such was the case in this study. Our findings support the claim that teaching that provides opportunities for student struggle *and* explicitly attends to concepts (Quadrant 1) is associated with better performance on assessments of conceptual understanding. Interestingly, students who experienced instruction that was less tolerant of sustained student struggle (and slightly lower in attention to concepts) (Quadrant 2) performed well, but not *as* well as the Quadrant 1 students. This suggests that there could be affordances for learning associated with struggle but that some forms of bounded struggle might be worth exploring as well (given the high performance of Quadrant 2 students). Overall, the existence of Quadrant 2 forms of teaching opens the door to a discussion about how much and what kind of struggle is worthwhile, as well as further exploration of the role explicit attention to concepts plays amidst bounded struggle. Student struggle is usually associated with student-centered "reform" instruction. The findings herein lead to questions regarding what kinds of structures might mitigate the potentially poor effects of struggle (Quadrant 3) and bring out the positive aspects of bounded struggle.

Potential Implications for Measurement At-Scale

As we move to the next iterative refinement of our theoretical framework and measures, a logical next step is to explore whether our survey measures can be put to use in large-scale studies of teaching change. Generalizable empirical studies of instructional change in the field of mathematics education have been exceedingly rare (Munter & Correnti, 2014). To the extent our measures allow us to detect changes in teaching, as we propose to do as part of a (separately

funded) continuous improvement study of coaching, we can gain insight into how interventions produce changes in EAC and SOS.

Additionally, we might also benefit from thinking about how measures can be incorporated into a measurement system (Bryk et al., 2015). For example, how can the theory and aligned measures be put to use by teachers and instructional leaders for improvement purposes? We believe that an important test of the theory and measures is whether they are useful in monitoring and supporting large scale instructional improvement in this new Common Core standards-based era. We have reason to be optimistic on this front. The quadrants have resonated deeply with our partners in the Tennessee Department of Education because of their potential to communicate a vision for instruction with practitioners; we are currently exploring whether associated patterns of more specific teaching practices (see Stein, Kelly, Moore, Correnti, & Russell, 2016) can be combined with the information provided by surveys to carry even more specific and actionable guidelines for how to teach differently and how to support and monitor large scale instructional improvement efforts.

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		Opportunities for Student Struggle	
		High	Low
Explicit Attention to Concepts	High	Quadrant 1	Quadrant 2
	Low	Quadrant 3	Quadrant 4

Figure 1: 2x2 matrix displaying our profiles of teaching along two dimensions (EAC and SOS)

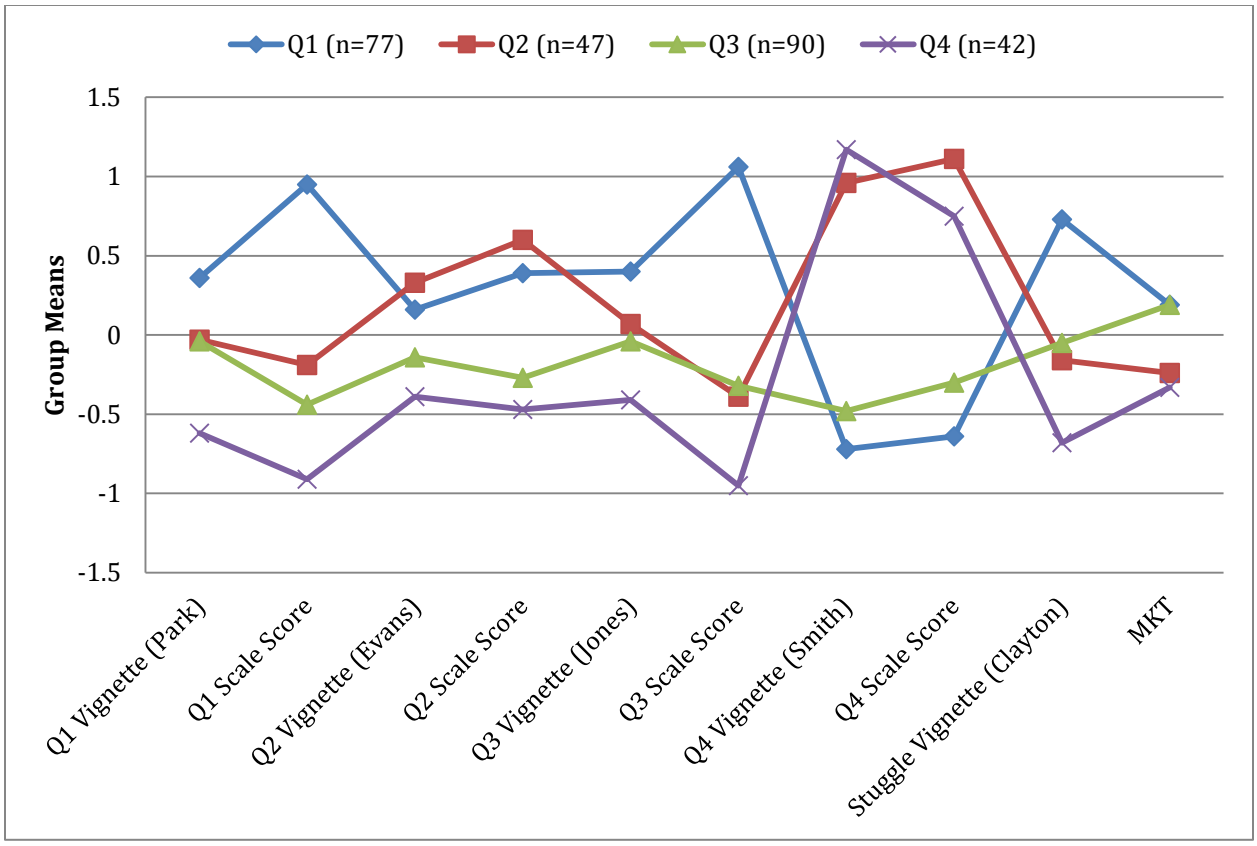
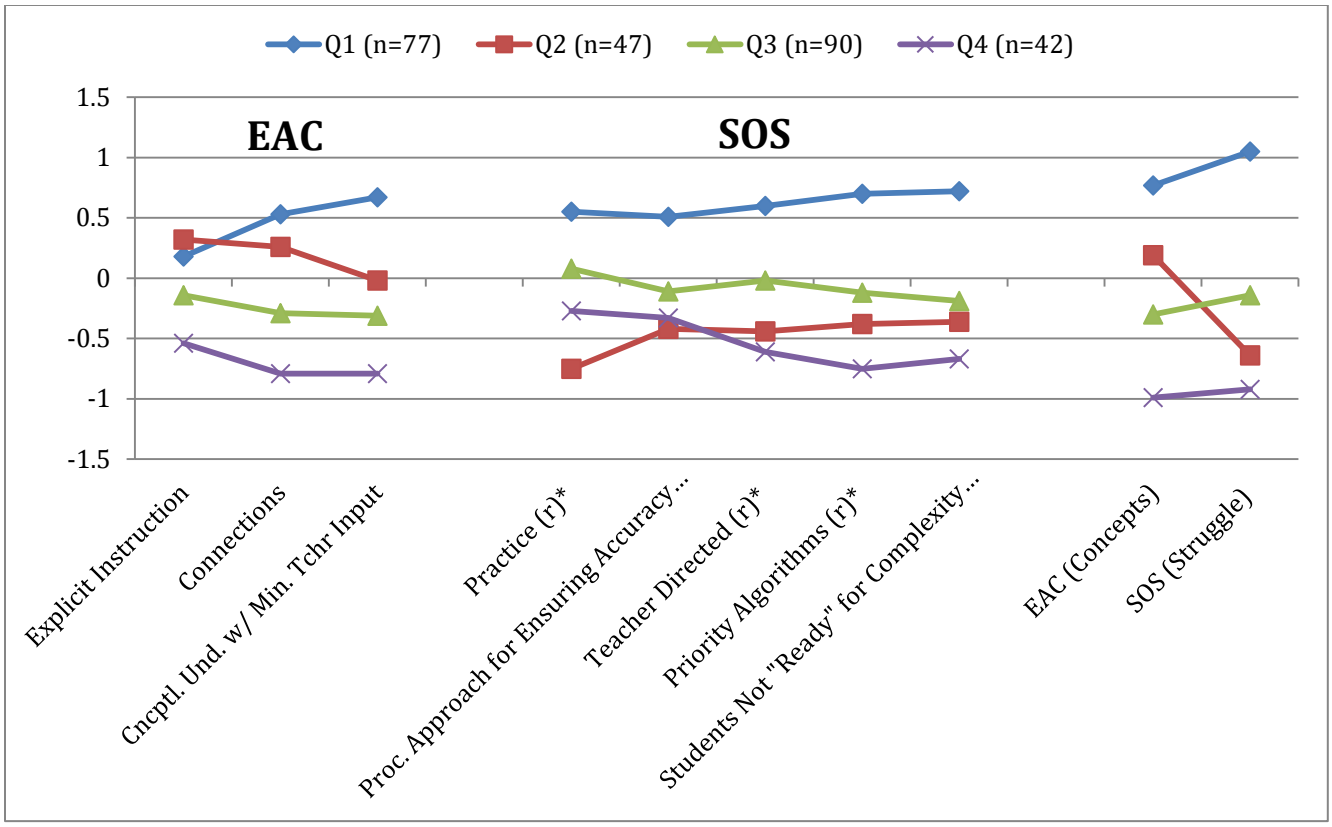


Figure 2: Between-quadrant differences in group means for the variables contained in the LPA (Claim 1)



* Note, these scales have been reverse-scored to indicate high opportunities for productive struggle because each sub-construct indicates high proclivity for low struggle.

Figure 3: Between-quadrant differences in group means for sub-construct and construct scores generated from the CFA (Claim 2)

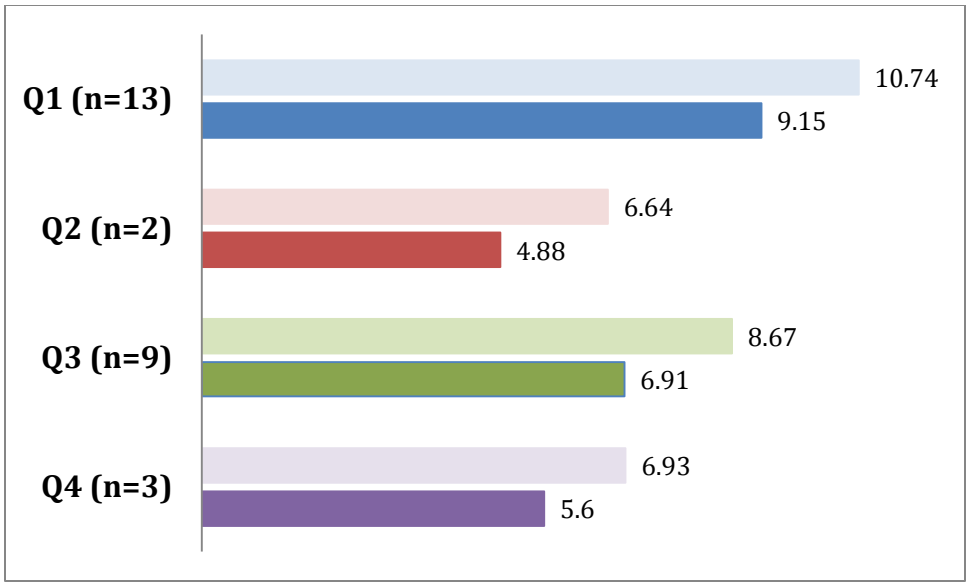


Figure 4: Between-quadrant group means for scores from video observations of mathematics teaching (Claim 3). Graphic displays mean of EAC in light shade and mean of SOS in dark shade for each quadrant.

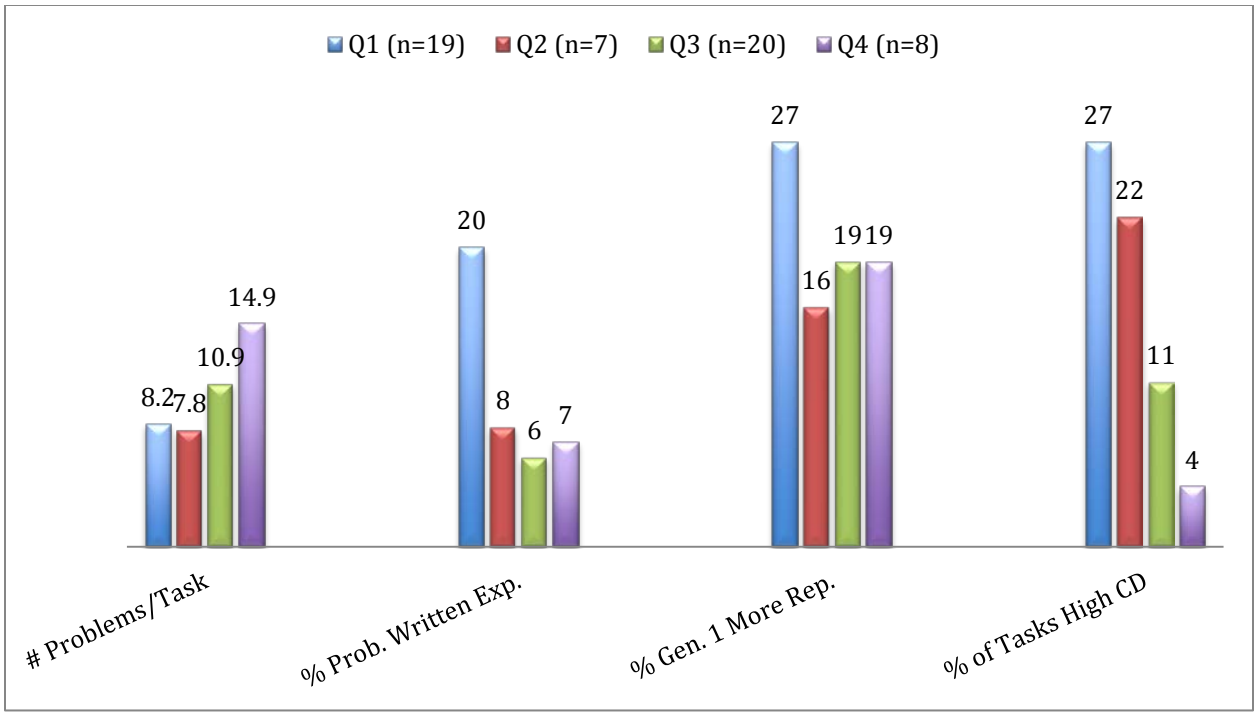


Figure 5: Between-quadrant group means for features of students' mathematics assignments (Claim 3)

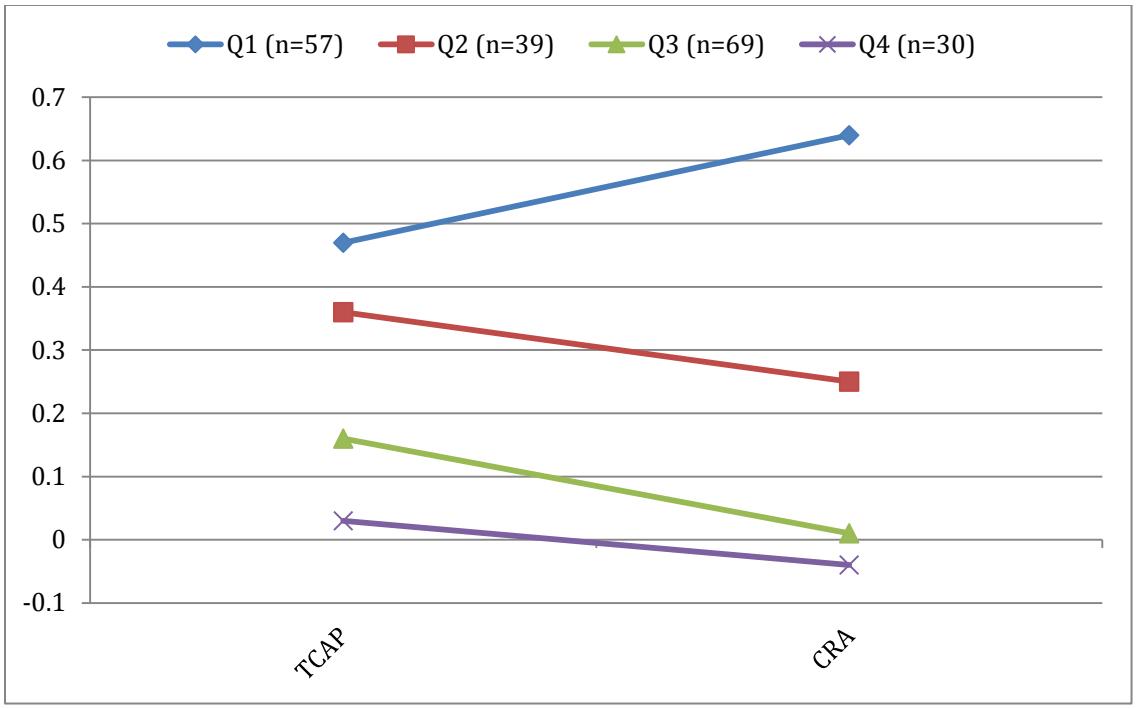


Figure 6: Between-quadrant differences in group means for covariate-adjusted value-added scores on two different assessments – the TCAP and CRA (Claim 4)

Table 1: Demographic Characteristics of Teachers		
Variables	N	Percent of Sample
Grade Level		
4	55	21.5
5	65	25.4
6	49	19.1
7	40	15.6
8	42	16.4
missing	5	2.0
Race		
White	231	90.2
Black	20	7.8
Asian	2	.8
Native American	1	.4
Hawaiian/Pacific Isl.	1	.4
Female	229	89.5
Self Contained Classroom	31	12.2
Mean (sd)		
Years Experience	11.5 (6.0)	
# Minutes Math/Week	302.9 (153.6)	

Table 2: Fit indexes for competing models of the structure of mathematics teaching practice from survey responses								
	df	RMSEA	CFI	TLI	SPMR	AIC	χ^2	$\Delta \chi^{2\ \text{from previous}}$
First-order single-factor	665	.090	.524	.497	.113	23,846	2,046.79	---
Second-order single-factor	657	.070	.718	.698	.112	23,290	1,474.84	571.95
Second-order two-factor (correlated)	656	.060	.794	.779	.086	23,072	1254.10	220.74